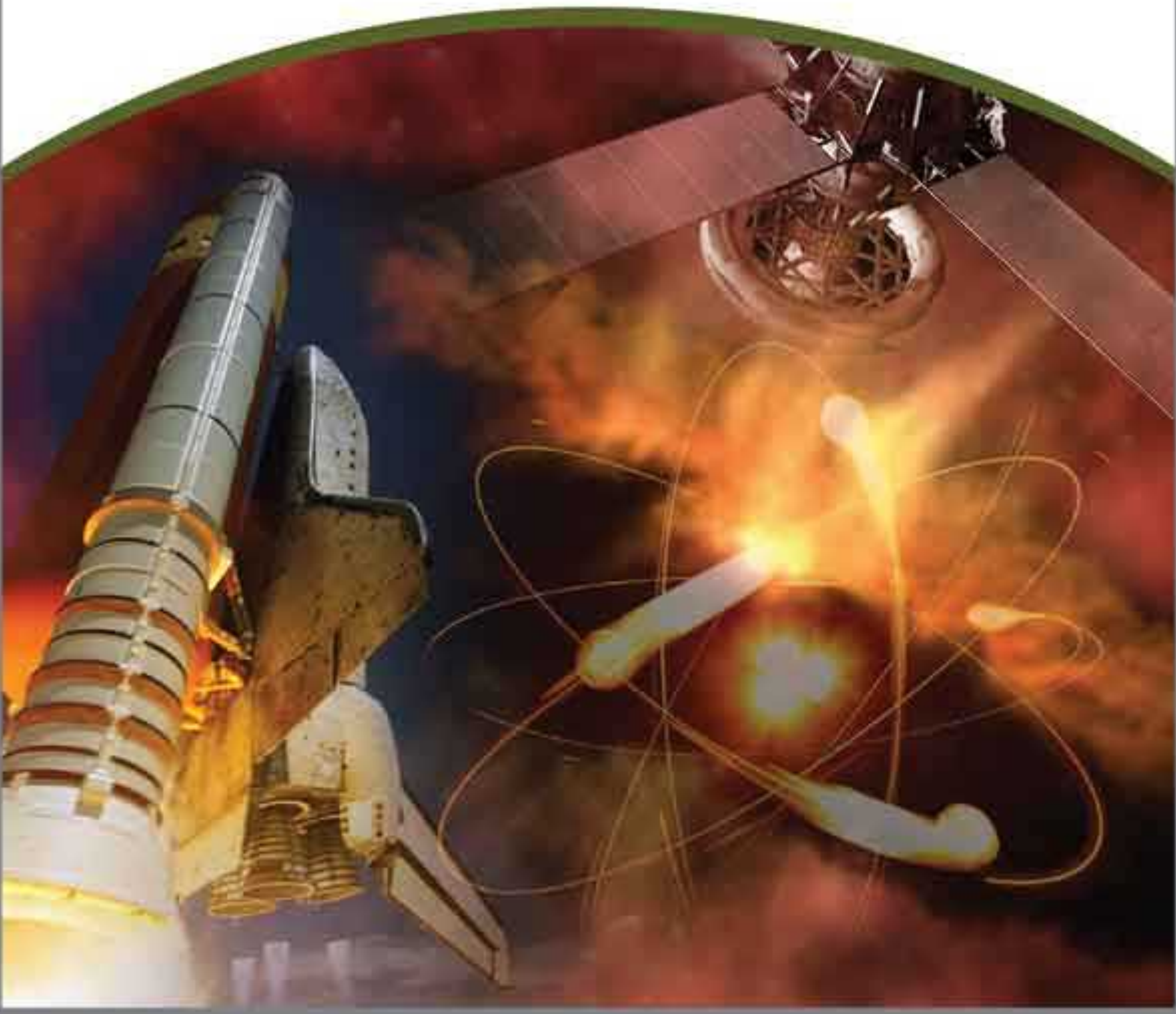
11

EASY MARKS

NCERT EXEMPLAR

PROBLEMS-SOLUTIONS

Physics



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E-mail: info@fcepl.in

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NEW EDITION

Note from the Publisher

National Council of Educational Research and Training (NCERT) developed Exemplar Problems in Science and Mathematics. The prime objective is to provide the students with number of quality problems to facilitate the concept of learning.

Easy Marks NCERT Solutions to Exemplar Problems Physics-XI is mainly based on the idea to present the considerable requirements of the Exemplar Problems in a simple and detailed manner.

Salient features of the book:

- Scientific and methodological solutions to the textual questions are provided.
- Multiple Choice Questions (MCQs) with explanation for understanding the concept better.
- The explanation of the answers are provided with diagram, wherever needed.
- Very Short, Short and Long Answer Type Questions are given to provide students with more practical problems.

It has always been our endeavour to provide better quality material to the students. If there are any suggestions for the betterment of the book, we will certainly try to incorporate them.

CONTENTS

1 Introduction

A NOTE TO STUDENTS

A good number of problems have been provided in this book. Some are easy, some are of average difficult level, some difficult and some problems will challenge even the best amongst you. It is advised that you first master the concepts covered in your textbook, solve the examples and exercises provided in your textbook and then attempt to solve the problems given in this book. There is no single prescription which can help you in solving each and every problem in physics but still researches in physics education show that most of the problems can be attempted if you follow certain steps in a sequence. The following prescription due to Dan Styer presents one such set of steps :

1. Strategy design
- (*a*) Classify the problem by its method of solution.
- (*b*) Summarise the situation with a diagram.
- (*c*) Keep the goal in sight (perhaps by writing it down).

2. Execution tactics
- (*a*) Work with symbols.
- (*b*) Keep packets of related variables together.
- (*c*) Be neat and organised.
- (*d*) Keep it simple.

3. Answer checking
- (*a*) Dimensionally consistent?
- (*b*) Numerically reasonable (including sign)?
- (*c*) Algebraically possible? (Example: no imaginary or infinite answers)
- (*d*) Functionally reasonable? (Example: greater range with greater initial speed)
- (*e*) Check special cases and symmetry.
- (*f*) Report numbers with units specified and with reasonable significant figures.

We would like to emphasise that the problems in this book should be used to improve the quality of teaching-learning process of physics. Some can be directly adopted for evaluation purpose but most of them should be suitably adapted according to the time/marks assigned. Most of the problems included under SA and LA can be used to generate more problems of VSA or SA categories, respectively.

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2 Units and Measurement

MULTIPLE CHOICE QUESTIONS-I

Q2.1. The number of significant figures in 0.06900 is:

(*a*) 5 (*b*) 4 (*c*) 2 (*d*) 3

Main concept used: In a number less than one (i.e., decimal).

The zeroes on the left of non zero number are not significant figures, and zeroes of right side of non zero number are significant figures.

Ans. (*b*): $0.0\underline{6}\underline{9}\underline{0}\underline{0}$ two zeroes before six are not significant figure and two zero on right side of 9 are significant figures. Significant figures are underlined figures so verifies option (*b*).

Q2.2. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is

(*a*) 663.821 (*b*) 664 (*c*) 663.8 (*d*) 663.82

Main concept used: In addition the result will be in least number of places after decimal and minimum number of significant figure.

Ans. (*c*): On adding the given numbers result is 663.821, but in given number the minimum number of places is one so result in significant figure is 663.8. (upto one place of decimal after rounding off).

Q2.3. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is:

(*a*) 1.6048 g cm^{-3} (*b*) 1.69 g cm^{-3} (*c*) 1.7 g cm^{-3} (*d*) 1.695 g cm^{-3}

Main concept used: The final result in either division or multiplication retain as many minimum numbers of significant figures (after rounding off) as there in the original numbers.

Ans. (*c*): The significant figures in given numbers 4.237 g and 2.5 cm^3 are four and two respectively so result must have only two significant figures.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{4.237 \text{ g}}{2.5 \text{ cm}^3}, \text{ Density} = 1.6948 = 1.7 \text{ g cm}^{-3}$$

rounding off upto 2 significant figures.

Q2.4. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give.

(*a*) 2.75 and 2.74 (*b*) 2.74 and 2.73

(*c*) 2.75 and 2.73 (*d*) 2.74 and 2.74

Main concept used: (*i*) If the preceding digit of dropped out digit 5 is **even** the no change in rounding off.

(*ii*) If the preceding digit of dropped out digit 5 is odd then preceding digit is increased by one.

Ans. (*d*): (*i*) In given number 2.745 it is round off upto 3 significant figure, IVth digit is 5 and its preceding is even, so no change in 4 i.e., answer is 2.74.

(*ii*) Given figure 2.735 is round off upto 3 significant figure here IVth i.e., next digit is 5 and its preceding digit is 3 (odd). So 3 is increased by 1 and answer becomes 2.74.

Hence, verifies the option (*d*).

Q2.5. The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm respectively. The area of the sheet in appropriate significant figures and error is

(*a*) 164 ± 3 cm^2 (*b*) 163.62 ± 2.6 cm^2

(*c*) 163.6 ± 2.6 cm^2 (*d*) 163.62 ± 3 cm^2

Main concept used: (*i*) Significant figures in the result multiplication (or division) is the minimum number of significant figures in given number.

(*ii*) If Δx is error in quantity x then relative error or error is $\frac{\Delta x}{x}$.

Ans. (*a*):

$$l = 16.2 \text{ cm} \qquad \Delta l = 0.1$$
$$b = 10.1 \text{ cm} \qquad \Delta b = 0.1$$
$$l = 16.2 \pm .1$$
$$b = 10.1 \pm 0.1$$
$$\text{A} = \text{Area} = l \times b = 16.2 \times 10.1 = \underline{1}\underline{6}\underline{3}.62 \text{ cm}^2$$
$$= 164 \text{ cm}^2 \qquad \text{(in significant figures)}$$
$$\frac{\Delta \text{A}}{\text{A}} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$
$$\frac{\Delta \text{A}}{\text{A}} = \frac{.1}{16.2} + \frac{.1}{10.1} = \frac{1.01 + 1.62}{16.2 \times 10.1} = \frac{2.63}{16.2 \times 10.1}$$
$$\Delta \text{A} = 2.63 \text{ cm}^2.$$

Now rounding off upto significant figures in Δl and Δb i.e., one

$$\Delta \text{A} = 3 \text{ cm}^2$$

$\therefore$ A = (164 ± 3) cm^2. Hence, verifies the option (*a*).

Q2.6. Which of the following pairs of physical quantities does not have same dimensional formula?

(*a*) Work and torque

(*b*) Angular momentum and Planck's constant

(*c*) Tension and surface tension

(*d*) Impulse and linear momentum.

Main concept used: In a formula dimensions of each term are same.

Ans. (*a*): Work = Force × displacement = $[\text{MLT}^{-2}][\text{L}] = [\text{ML}^2\text{T}^{-2}]$

Torque = Force × distance = $[\text{MLT}^{-2}][\text{L}] = [\text{ML}^2\text{T}^{-2}]$

R.H.S has same dimensions.

(*b*) Angular momentum $\text{L} = mvr = [\text{M}][\text{LT}^{-1}][\text{L}] = [\text{ML}^2\text{T}^{-1}]$

Planck's constant $h = \frac{\text{E}}{\nu} = \frac{\text{F}.s}{\nu}$ $(\because \text{E} = h\nu)$

$$= \frac{[MLT^{-2}][L]}{[T^{-1}]} = [ML^2T^{-1}]$$

Dimensions of h and L are equal.

(*c*) $$\text{Tension} = \text{Force} = [MLT^{-2}]$$

$$\text{Surface tension} = \frac{\text{Force}}{l} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$$

(*d*) $$\text{Impulse} = F \times t = [MLT^{-2}][T] = [MLT^{-1}]$$

$$\text{Momentum} = mv = [MLT^{-1}]$$

R.H.S. has same dimensions.

Hence, verify the option (*c*).

Q2.7. Measure of two quantities along with the precision of respective measuring instrument is $A = 2.5\ ms^{-1} \pm 0.5\ ms^{-1}$ and $B = 0.10\ s \pm 0.01\ s$. The value of AB will be:

(*a*) (0.25 ± 0.08) m (*b*) (0.25 ± 0.5) m

(*c*) (0.25 ± 0.05) m (*d*) (0.25 ± 0.135) m

Main concept used: Rules of significant figure in multiplication and addition.

Ans. (*a*): $A = (2.5 \pm 0.5)\ ms^{-1}$

$B = (0.10 \pm 0.01)\ s$

$x = AB = 2.5 \times 0.10 = 0.25\ m$

$$\frac{\Delta x}{x} = \frac{\Delta A}{A} + \frac{\Delta B}{B} = \frac{0.5}{2.5} + \frac{0.01}{0.10} = \frac{0.05 + 0.025}{2.5 \times 0.10}$$

$$\frac{\Delta x}{x} = \frac{0.075}{0.25},\ \Delta x = 0.075 \cong 0.08$$

(Rounding off upto 2 significant figures)

$\therefore$ $AB = (0.25 \pm 0.08)$ m.

Hence, verifies the option (*a*).

Q2.8. You measure two quantities as $A = (1.0 \pm 0.2)$ m, $B = 2.0\ m \pm 0.2\ m$. We report correct value of $\sqrt{AB}$ as:

(*a*) 1.4 m ± 0.4 m (*b*) 1.41 m ± 0.15 m

(*c*) 1.4 m ± 0.3 m (*d*) 1.4 m ± 0.2 m

Main concept used: In significant figures of measured quantities zeroes are included in significant figures.

Ans. (*d*): Quantities A and B are measured quantities so number of significant figures in 1.0 m and 2.0 m are two.

$$\sqrt{AB} = \sqrt{1.0 \times 2.0} = \sqrt{2} = 1.414 \text{ m.}$$

rounding off upto minimum numbers of significant figure in 1.0 and 2.0 result must be in 2 significant figures

$$x = \sqrt{AB} = 1.4$$

$$\frac{\Delta x}{x} = \frac{1}{2}\left[\frac{\Delta A}{A} + \frac{\Delta B}{B}\right] = \frac{1}{2}\left[\frac{0.2}{1.0} + \frac{0.2}{2.0}\right]$$

$$= \frac{1}{2} \times 0.2 \left[\frac{1}{1.0} + \frac{1}{2.0} \right] = 0.1 \left[\frac{2.0 + 1.0}{1.0 \times 2.0} \right]$$

$$\Delta x = \frac{0.3}{1.0 \times 2.0} \times x = \frac{0.3 \times 1.414}{1.0 \times 2.0} = 0.2121$$

Δx = 0.2 m rounding off upto 1 place of decimal.

$\therefore \quad \sqrt{AB} = (1.4 \pm 0.2)$ m

Verifies the option (*d*).

Q2.9. Which of the following measurements is most precise?

(*a*) 5.00 mm (*b*) 5.00 cm (*c*) 5.00 m (*d*) 5.00 km

Main concept used: In these problems unit must be least and in digits number of digits including zeroes after decimal must be zero.

Ans. (*a*): All the measurements are upto two places of decimal, least unit is mm. So 5.00 mm measurement is most precise.

Hence, verifies answer (*a*).

Q2.10. The mean length of an object is 5 cm. Which of the following measurements is most accurate?

(*a*) 4.9 cm (*b*) 4.805 cm (*c*) 5.25 cm (*d*) 5.4 cm

Main concept used: Absolute error $(\overline{a} - a_i)$ must be minimum for more accuracy.

Ans. (*a*): Error or absolute error

$$|\Delta a_1| = |5 - 4.9| = 0.1 \text{ cm}, \; |\Delta a_2| = |5 - 4.805| = 0.195 \text{ cm}$$

$$|\Delta a_3| = |5 - 5.25| = 0.25 \text{ cm}, \; |\Delta a_4| = |5 - 5.4| = 0.4 \text{ cm}$$

$|\Delta a_1|$ is minimum. Hence verifies option (*a*).

Q2.11. Young's modulus of steel is 1.9×10^{11} N/m^2. When expressed in CGS units of dynes/cm^2. It will be equal to (1 N = 10^5 dynes, and 1 m^2 = 10^4 cm^2).

(*a*) 1.9×10^{10} (*b*) 1.9×10^{11} (*c*) 1.9×10^{12} (*d*) 1.9×10^{13}

Ans. (*c*): $Y = 1.9 \times 10^{11}$ N/m^2

or $$Y = \frac{1.9 \times 10^{11} \text{ N}}{1 \text{ m}^2} = \frac{1.9 \times 10^{11} \times 10^5 \text{ dynes}}{10^4 \text{ cm}^2}$$

$$Y = 1.9 \times 10^{11+5-4}$$

$$Y = 1.9 \times 10^{12} \text{ dyne/cm}^2$$

Verifies the option (*c*).

Q2.12. If the momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has dimensional formula.

(*a*) $[P^1A^{-1}T^1]$ (*b*) $[P^2A^1T^1]$ (*c*) $[P^1A^{-1/2}T^1]$ (*d*) $[P^1A^{1/2}T^{-1}]$

Ans. (*d*): Let the dimensional formula for energy in fundamental quantities P, A and T is $[P^a A^b T^c]$.

Dimensional formula of E = $[P^a A^b T^c]$ dimensional formula of momentum $P = mv = [MLT^{-1}]$.

$$\text{Area A} = [L^2]$$
$$\text{Time T} = [T^1]$$
$$\therefore \quad \text{Energy} = F.s = [MLT^{-2}L] = [ML^2T^{-2}]$$
$$\therefore \quad [ML^2T^{-2}] = [MLT^{-1}]^a[L^2]^b\,[T]^c$$
$$[M^1L^2T^{-2}] = [M^a\,L^{a+2b}\,T^{-a+c}]$$

Comparing the powers

$$a = 1 \qquad a + 2b = 2 \qquad -a + c = -2$$
$$1 + 2b = 2 \qquad -1 + c = -2$$
$$2b = 2 - 1 \qquad c = -2 + 1$$
$$b = \frac{1}{2} \qquad c = -1$$

$\therefore$ Dimensional formula of energy is $[P^1A^{1/2}T^{-1}]$.

Verifies the option (*d*).

MULTIPLE CHOICE QUESTIONS-II

Q2.13. On the basis of dimensions, decide which of the following relations for the displacement of particle undergoing simple harmonic motion is *not* correct.

(*a*) $y = a\sin\left(\frac{2\pi t}{T}\right)$ (*b*) $y = a\sin vt$

(*c*) $y = \frac{a}{T}\sin\left(\frac{t}{a}\right)$ (*d*) $y = a\sqrt{2}\left[\sin\frac{2\pi t}{T} - \cos\frac{2\pi t}{T}\right]$

Main concept used: Displacement y and amplitude a has same dimensions, angle of sin and cos is dimensionless. So dimensions in both side must be equal by the principle of homogeneity.

Ans. (*b*, *c*): In (*a*) and (*d*) option the dimensions of y and a in L.H.S. and R.H.S. are equal to L and angles of sin, cos are dimensionless.

In option (*b*) angle is $v.t$ (where v is velocity) $\therefore$ dimension of $v.t$ is $[LT^{-1}][T] = [L]$.

So sin vt is not dimensionless so option (*b*) is wrong. In option (*c*) in R.H.S. dimension of amplitude $\frac{a}{T} = \frac{[L]}{[T]} = [LT^{-1}]$ which not equal to the dimension of y i.e., L and angle $\frac{t}{a} = \frac{[T]}{[L]} = [L^{-1}T]$ is not dimensionless.

Hence, verifies the option (*b*) and (*c*).

Q2.14. P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?

(*a*) $\frac{(P - Q)}{R}$ (*b*) $(PQ - R)$ (*c*) $\frac{PQ}{R}$ (*d*) $\frac{PR - Q^2}{R}$ (*e*) $\frac{(R + Q)}{P}$

Main concept used: Addition or subtraction may be possible of the same physical quantity. But multiplication can be possible for different physical quantities or different dimensions. After multiplication or division of two quantities their dimension may be equal to IIIrd and can be added or subtracted.

Ans. (*a*, *e*): In option (*a*) and (*e*) there is term (P – Q) and (R + Q) as different physical quantities can never be added or subtracted so option (*a*) and (*e*) can never be meaningful.

In option (*b*), the dimension of PQ may be equal to dimension of R so option (*b*) can be possible. Similarly dimensions of PR and Q^2 may be equal and gives the possibility of option (*d*).

In option (*c*), there is no addition subtraction gives the possibilities of option (*c*). Hence, verifies the right option (*a*) and (*e*).

Q2.15. Photon is quantum of radiation with energy E = $h\nu$, where ν is frequency and h is Planck's constant. The dimensions of h are the same as that of:

(*a*) Linear impulse (*b*) Angular impulse

(*c*) Linear momentum (*d*) Angular momentum

Main concept used: Dimensional formulae.

Ans. (*b*, *d*): $\because$ $E = h\nu$

$$h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

$$\text{Linear impulse} = F.t = \frac{dp}{dt}.dt = dp$$

$$= mv = [MLT^{-1}]$$

$$\text{Angular impulse} = \tau.dt = \frac{dL}{dt}.dt = dL = mvr$$

$$= [MLT^{-1}L] = [ML^2T^{-1}]$$

$$\text{Linear momentum} = mv = [MLT^{-1}]$$

$$\text{Angular momentum } L = mvr = [ML^2T^{-1}].$$

So the dimensional formulae of h, Angular impulse and Angular momentum are same.

Hence, verifies the option (*b*) and (*d*).

Q2.16. If the Planck's constant (h) and the speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities.

(*a*) Mass of electron (m_e) (*b*) Universal gravitational constant (G)

(*c*) Charge of electron (e) (*d*) Mass of proton (m_p)

Ans. (*a*, *b*, and *d*): dimension of

$$h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

$$c = \frac{s}{t} = [LT^{-1}]$$

$$G = \frac{Fr^2}{M_1M_2} = \frac{[ML^3T^{-2}]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

$$hc = [ML^2T^{-1}] \times [LT^{-1}] = [ML^3T^{-2}]$$

$$\frac{hc}{G} = \frac{[ML^3T^{-2}]}{[M^{-1}L^3T^{-2}]} = [M^2]$$

$$M = \sqrt{\frac{hc}{G}} = [h^{1/2}\ c^{1/2}\ G^{-1/2}]$$

$$\frac{h}{c} = \frac{[ML^2T^{-1}]}{[LT^{-1}]} = [ML] = \sqrt{\frac{hc}{G}} \times L$$

$$L = \frac{h}{c} \times \sqrt{\frac{G}{hc}} = \frac{\sqrt{Gh}}{c^{3/2}} = [G^{1/2}\ h^{1/2}\ c^{-3/2}]$$

$$c = [LT^{-1}] = [G^{1/2}\ h^{1/2}\ c^{-3/2}\ T^{-1}]$$

$$T = [G^{1/2}\ h^{1/2}\ c^{\frac{-3}{2}-1}] = [G^{1/2}\ h^{1/2}\ c^{-5/2}]$$

Hence, physical quantities (*a*, *b* and *d*) can be used to represent L, M, T in terms of the chosen fundamental quantities.

Q2.17. Which of the following ratios express pressure?

(*a*) Force/Area (*b*) Energy/Volume
(*c*) Energy/Area (*d*) Force/Volume

Main concept used: Ratio can be express as pressure (P) if the dimension of ratio is same as P.

Ans. (*a*, *b*): Dimension of pressure $P = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]}$

Dimension of pressure $P = [ML^{-1}T^{-2}]$

(*a*) Dimension of $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$ = dimension of P

(*b*) Dimension $\frac{E}{V} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$ = dimension of P

(*c*) Dimension of $\frac{E}{A} = \frac{[ML^2T^{-2}]}{[L^2]} = [M^1L^0T^{-2}] \neq [ML^{-1}T^{-2}]$ dimension of P

(*d*) Dimension of $\frac{F}{V} = \frac{[MLT^{-2}]}{[L^3]} = [ML^{-2}T^{-2}] \neq [ML^{-1}T^{-2}]$ dimension of P

Hence, required option are (*a*) and (*b*).

Q2.18. Which of the following are *not* unit of time?

(*a*) Second (*b*) Parsec (*c*) Year (*d*) Light year

Main concept used: Dimension of time is [T].

Ans. (*b*, *d*): The second, and year measures the time so their dimension is $[M^0L^0T^1]$ or unit of time.

But Parsec and light year measures the distance and dimension of distance [L] is not equal to the dimension of Time [T].
Hence, (*b*) and (*d*) are not unit of time.

VERY SHORT ANSWER TYPE QUESTIONS

Q2.19. Why do we have different units for same physical quantity?

Ans. Same physical quantity measures the different order of same physical quantity e.g., velocity is physical quantity it measures the speed of light 10^8 m/s, speed of a person 10^0 m/s, speed of bus 10^1 m/s speed of α, e, p particle measured in terms of speed of light.
So, we have the different units of same physical quantity. So we have different units of velocity cm/s, m/s, km/hr velocity of light.

Q2.20. The radius of atom is of the order of 1 Å and radius of nucleus is of the order fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus?

Ans. Radius (R) of atom = 1 Å = 10^{-10} m
Radius (r) of nucleus = 1 fermi = 10^{-15} m
Ratio of volume of atom to nucleus

$$= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{R^3}{r^3} = \left(\frac{10^{-10}}{10^{-15}}\right)^3 = (10^5)^3 = 10^{15}.$$

Q2.21. Name the device used for measuring the mass of atoms and molecules.

Ans. Deflection of a charge particle or ionised atom or molecule depends on the magnitude of either magnetic or electric field, mass and charge of particle by using this principle spectrograph or spectrometer measures the mass of atom and molecules.

Q2.22. Express unified atomic mass unit in kg.

Ans. Unified atomic mass unit (amu) or (u) = $\frac{1}{12}$ the mass of one carbon (C^{12}) atom.
By Avogadro's number 6.023×10^{23} we know that mass of 6.023×10^{23} atoms of carbon ${}_6C^{12}$ is equal to 12 gm.

So mass of one atom of ${}_6C^{12} = \frac{12}{6.023 \times 10^{23}}$ gm

$$1 \text{ amu} = \frac{1}{12} \text{ mass of one } {}_6C^{12} \text{ atom} = \frac{12}{12 \times 6.023 \times 10^{23}} \text{ gm}$$

$$= 1.67 \times 10^{-24} \text{ gm}$$

$$1 \text{ amu} = 1.67 \times 10^{-27} \text{ kg.}$$

Q2.23. A function $f(\theta)$ is defined as:

$$f(\theta) = 1 - \theta + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} + \ldots$$

Why is it necessary for $f(\theta)$ to be a dimensionless quantity?

Ans. θ is represented by angle which is equal to $\dfrac{\text{arc}}{\text{radius}}$ so angle θ is dimensionless physical quantity.

First term is 1 which is dimensionless, next term contain only powers of θ, as θ is dimensionless so their power will be dimensionless.

Hence, each term in R.H.S. expression are dimensionless i.e. R.H.S is dimensionless so left hand side $f(\theta)$ must be dimensionless.

Q2.24. Why length, mass and time are chosen as base quantities in mechanics?

Ans. The length, mass and time cannot be derived from any other physical quantity and all physical quantities of mechanic can be represented in terms of only length, mass and time.

So length, mass and time are chosen as the base quantities in mechanics.

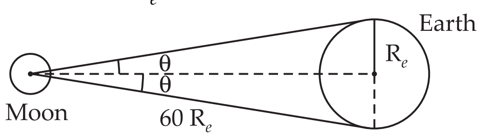
SHORT ANSWER TYPE QUESTIONS

Q2.25. (*a*) The earth-moon distance is about 60 earth radius. What will be the diameter of earth (approximately in degrees) as seen from the moon.

(*b*) Moon is seen to be of $(1/2)^0$ diameter from the earth. What must be the relative size of compared to the earth.

(*c*) From parallax measurement, the sun is found to be at a distance of about 400 times of earth-moon distance. Estimate the ratio of sun-earth diameters.

Ans. (*a*): $\theta = \dfrac{\text{arc}}{\text{radius}} = \dfrac{R_e}{60R_e} = \dfrac{1}{60}\text{ rad}.$



The angle from the moon to the diameter of earth

$$2\theta = 2.\frac{1}{60}\times\frac{180}{\pi} = \frac{6^\circ}{\pi}$$

$$\cong \frac{6}{3.14} \cong 2^\circ$$

(*b*) As moon is seen from earth (diametrically) angle = $\dfrac{1}{2}^\circ$

If earth is seen from moon the angle = 2°

$$\therefore \frac{\text{size of moon}}{\text{size (diameter) of earth}} = \frac{1/2^\circ}{2^\circ} = \frac{1}{4}$$

$\therefore$ Size of moon is $\dfrac{1}{4}$ the size (diameter) of earth.

(*c*) Let the distance between earth and moon $r_{em} = x$ m

Then the distance between earth and sun $r_{se} = 400x$ m

On the complete solar eclipse sun disappear or covered by moon completely. So the angle formed by the diameters of moon and sun on earth are equal $\therefore$ $\theta_m = \theta_s$. Where angle from earth to moon = $\theta°_m$.

Angle from earth to sun = $\theta°_s$

$$\theta_m = \theta_s \quad \Rightarrow \frac{D_m}{r_{em}} = \frac{D_s}{r_{se}}$$

$$\frac{D_m}{x} = \frac{D_s}{400x}$$

$$\frac{D_s}{D_m} = \frac{400\cancel{x}}{\cancel{x}}$$

$$\because \qquad 4D_m = D_e \quad \text{or} \quad D_m = \frac{D_e}{4}$$

$$\therefore \qquad \frac{D_s}{D_e/4} = 400 \quad \Rightarrow \quad \frac{4D_s}{D_e} = 400$$

$$\therefore \qquad \frac{D_s}{D_e} = \frac{400}{4} \quad \Rightarrow \quad \frac{D_s}{D_e} = 100$$

$$D_s = 100D_e$$

Q2.26. Which of the following time measuring devices is most precise?

(*a*) A wall clock (*b*) A stop watch

(*c*) A digital watch (*d*) An atomic clock

Give reason for your answer.

Ans. (*d*): The least count of a wall clock, stop watch, digital watch and atomic clock are 1 sec, $\frac{1}{10}$ sec, $\frac{1}{100}$ sec and $\frac{1}{10^{13}}$ sec. So atomic clock is most precise.

Q2.27. The distance of galaxy is of the order 10^{25} m. Calculate the order of magnitude of time taken by light to reach us from the galaxy.

Ans. Distance travelled by light from galaxy to earth = 10^{25} m

Speed of light = 3×10^8 m/s

So, $\quad$ Required time = $\frac{\text{distance}}{\text{speed}} = \frac{10^{25}}{3 \times 10^8}$ sec. = $\frac{1}{3} \times 10^{17}$ seconds

Required time = $\frac{10}{3} \times 10^{16} = 3.3\overline{3} \times 10^{16}$ seconds.

Q2.28. The vernier scale of travelling microscope has 50 divisions which coincide with 49 divisions of main scale divisions. If each main scale division is 0.5 mm, calculate the minimum inaccuracy in measurement of distance.

Main concept used: Inaccuracy or L.C. of measuring student when all n parts of Vernier scale coincide with $(n-1)$ part of M.S. (main scale)

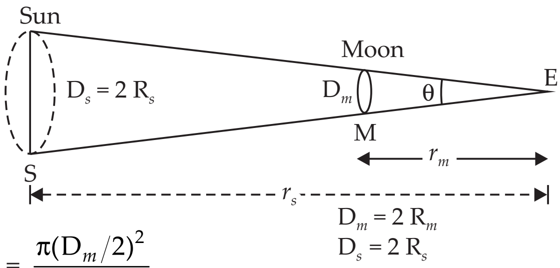
$$\text{L.C.} = \frac{\text{L.C. of M. scale}}{n}$$

Ans. Here, parts on Vernier scale = n = 50 parts
No. of division of M.S. coinciding with n parts of V.S = $(n-1)$

$$\therefore \quad \text{L.C. of instrument} = \frac{\text{L.C. of Main scale}}{\text{No. of parts on V.S.}} = \frac{0.5 \text{ mm}}{50}$$

or minimum inaccuracy = 0.01 mm.

Q2.29. During a total solar eclipse the moon almost covers the sphere of the sun. Write the relation between the distances and sizes of the sun and moon.



Ans. On total solar eclipse, solid or areal angle of sun and moon on earth will be equal

$\therefore \; \Omega_s = \Omega_m$

$$\frac{\pi(D_s/2)^2}{r_s^2} = \frac{\pi(D_m/2)^2}{r_m^2}$$

$$\frac{D_s^2}{4r_s^2} = \frac{D_m^2}{4r_m^2}$$

$$\frac{(2R_s)^2}{4r_s^2} = \frac{(2R_m)^2}{4r_m^2} \Rightarrow \frac{4R_s^2}{4r_s^2} = \frac{4R_m^2}{4r_m^2}$$

Taking square roots both side

$$\frac{R_s}{r_s} = \frac{R_m}{r_m} \quad \text{or} \quad \frac{R_s}{R_m} = \frac{r_s}{r_m}$$

or the ratio of size of sun to moon is equal to the ratio of the distances of sun to moon from earth.

Q2.30. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s. What is the unit of mass in this system of units?

Main concept used: After writing dimensions of given physical quantities, then co-relate them.

Ans. Dimension of force = $[M^1L^1T^{-2}]$ = 100 N ...(*i*)
Dimension of length = $[L^1]$ = 10 m ...(*ii*)
Dimension of time = $[T^1]$ = 100 s (*iii*)

Substituting (*ii*), (*iii*) in (*i*)

$$M\,10 \times (100)^{-2} = 100$$

$$\frac{10M}{100 \times 100} = 100$$

$$M = 10^5 \text{ kg} \qquad L = 10^1 \text{ m}$$
$$F = 10^2 \text{ N} \qquad T = 10^2 \text{ sec}$$

Q2.31. Given an example of:

(*a*) a physical quantity which has a unit but no dimensions.
(*b*) a physical quantity which has neither unit nor dimensions.
(*c*) a constant which has a unit.
(*d*) a constant which has no unit.

Ans. (*a*): Plane angle = $\frac{\text{arc}}{\text{radius}}$ in radians and solid angle-unit, Steradian plane angle has unit radian but no dimensions.

(*b*) Relative density, μ_r, strain, are ratios of same physical quantity so does not have any unit or dimension
(*c*) Gravitational constant 'G' = 6.67×10^{-11} has unit N-m^2 per kg^2
Dielectric constant $k = 9 \times 10^9$ (vacuum) has unit N-m^2 per Coulomb^2
(*d*) Reynold number, Avogadro's number are constant and has no units.

Q2.32. Calculate the length of the arc of a circle of radius 31.0 cm which subtends an angle of $\frac{\pi}{6}$ at the centre.

Ans.
$$\text{Angle} = \frac{\text{length of arc}}{\text{radius of arc}}$$
$$\frac{\pi}{6} = \frac{x}{31}$$
$$x = \frac{31 \times \pi}{6} = \frac{31 \times 3.14}{6} = 16.22 \text{ cm.}$$

Q2.33. Calculate the solid angle subtended by the periphery of an area 1 cm^2 at a point situated symmetrically at a distance of 5 cm from the area.

Ans. Solid angle $\Omega = \frac{\text{Area}}{(\text{dist})^2} = \frac{1 \text{ cm}^2}{(5 \text{ cm})^2} = \frac{1}{25} = 4 \times 10^{-2}$ steradian.

Q2.34. The displacement of a progressive wave is represented by $y = A \sin(\omega t - kx)$, where x is distance and t is time. Write the dimensional formula of (*i*) ω (*ii*) k.

Main concept used: By principle of homogeneity and trigonometrical ratio has no dimensions.

Ans. Dimensional formula in L.H.S. and R.H.S. by principle of homogeneity are equal

$\therefore$ Dimension of y = dimensions of $A \sin(\omega t - kx)$
$[L] = [L] \times$ dimensions of $(\omega t - kx)$

as $\omega t - kx$ are angle of sin (Trigonometrical rate)

So $\omega t - kx$ = No dimension in each term

or dimensions of ωt = dimensions of kx

$$\frac{2\pi}{T}t = k.x \Rightarrow [M^0L^0T^0] = k[L]$$

$$\therefore \quad \text{Dimension of } k = \frac{[M^0L^0T^0]}{[L]} = [M^0L^{-1}T^0].$$

Q2.35. Time for 20 oscillations of a pendulum is measured as $t_1 = 39.6$ s; $t_2 = 39.9$ s; and $t_3 = 39.5$ s. What is the precision in the measurements? What is the accuracy of measurement?

Ans. $t_1 = 39.6$ s, $t_2 = 39.9$ s, and $t_3 = 39.5$ s

the least count of instrument is 0.1 s

Hence precision (LC) = 0.1 s

Mean value of time for 20 oscillations

$$= \frac{39.6 + 39.9 + 39.5}{3} = \frac{118.0}{3} = 39.7 \text{ s}$$

Absolute errors in measurement

$$|\Delta t_1| = |\bar{t} - t_1| = |39.7 - 39.6| = |0.1| = .1 \text{ s}$$

$$|\Delta t_2| = |\bar{t} - t_2| = |39.7 - 39.9| = 0.2 \text{ s}$$

$$\Delta t_3 = |\bar{t} - t\ | = |39.7 - 39.5| = 0.2 \text{ s}$$

$$\therefore \quad \text{Mean absolute error} = \frac{0.1 + 0.2 + 0.2}{3} = \frac{0.5}{3} \cong 0.2 \text{ s}$$

$\therefore$ Accuracy of measurement = ± 0.2 s.

LONG ANSWER TYPE QUESTIONS

Q2.36. A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s. How much will 5 J measure in this new system?

Ans. Dimension of Energy = $[ML^2T^{-2}]$

$$n_2u_2 = n_1u_1$$

$$n_2 = n_1\frac{u_1}{u_2} = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}$$

n_2 = New system of unit = ? $\qquad n_1$ = S.I system of unit = 5 J

$M_2 = \alpha$ kg $\qquad M_1$ = 1 kg

$L_2 = \beta$ m $\qquad T_1$ = 1 second

$T_2 = \gamma$ s $\qquad L_1$ = 1 m

$$n_2 = 5\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ sec}}{\gamma\text{ sec}}\right]^{-2}$$

$$n_2 = 5\left[\alpha^{-1}\beta^{-2}\gamma^2\right]$$

$$\text{New system} = \frac{\gamma^2}{\alpha\beta^2} \quad \text{or} \quad \left[\alpha^{-1}\beta^{-2}\gamma^2\right].$$

Q2.37. The volume of a liquid flowing out per second of a pipe of length l and radius r is written by a student as $v = \dfrac{\pi P r^4}{8\eta l}$ where P is the

pressure difference between two ends of pipe and η is coefficient of viscosity of the liquid having dimensional formula $[ML^{-1}T^{-1}]$.
Check whether the equation is dimensionally correct.

Main Concept used: If the dimensions of equation are equal in both side and also each term.

Ans. Dimension v = Volume per second = $\frac{V}{T} = [L^3T^{-1}]$

$$\text{Dimension of P} = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

Dimension of r = [L]

Dimension of η = $[ML^{-1}T^{-1}]$

Dimension of l = [L]

$$\therefore \quad \text{Dimension of R.H.S.} = \frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]} = [M^0L^3T^{-1}]$$

Dimension of L.H.S. v = $[M^0L^3T^{-1}]$.

As dimension of both sides are equal. Therefore, the equation is dimensionally correct.

Q2.38. A physical quantity X is related to four measurable quantities a, b, c and d as follows: $X = a^2 b^3 c^{5/2} d^{-2}$.
The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4% respectively. What is percentage error in quantity X? If the value of X calculated on the basis of the above relation is 2.763, to what value should you round off the result?

Ans. $\because$

$$\frac{\Delta X}{X} \times 100 = \pm\left[2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{5}{2}\frac{\Delta c}{c} + 2\frac{\Delta d}{d}\right] \times 100$$

$$\frac{\Delta X}{X} \times 100 = \pm\left[\frac{2\times 1}{100} + \frac{3\times 2}{100} + \frac{5}{2}\times\frac{3}{100} + \frac{2\times 4}{100}\right]100$$

$$= \pm\frac{100}{100}\left[2 + 6 + \frac{15}{2} + 8\right]$$

$$\frac{\Delta X}{X} \times 100 = \pm\left[16 + \frac{15}{2}\right] = \pm\left[\frac{32+15}{2}\right] = \pm\frac{47}{2} = \pm 23.5\%$$

$$\text{Mean absolute error} = \pm\frac{23.5}{100} = \pm 0.235$$

= 0.24 (rounding off in significant figure)

Again rounding off X = 2.763 in two significant figure = 2.8.

Q2.39. In the expression $P = E\, l^2\, m^{-5}\, G^{-2}$, E, m, l and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.

Ans. Since E, l and G have dimensional formulas:

$E \rightarrow [ML^2T^{-2}]$

$l \rightarrow [ML^2T^{-1}]$

$G \rightarrow [M^{-1}L^3T^{-2}]$

Hence, P = E l^2 m^{-5} G^{-2} will have dimensions:

$$[P] = \frac{[ML^2T^{-2}][M^2L^4T^{-2}][M^2T^4]}{[M^5][L^6]} = [M^0\,L^0\,T^0]$$

Thus, P is a dimensionless quantity.

Q2.40. If velocity of light C, Planck's constant h and gravitational constant G are taken as fundamental quantities then express mass, length and time in terms of dimensions of these quantities.

Main concept used: Homogeneity of dimensions in R.H.S. and L.H.S. of equation separately in each terms.

Ans. (i) Dimension of $h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

Dimensions of C = $[LT^{-1}]$

Dimension of G = N-m^2 kg^{-2} = $[MLT^{-2}][L^2][M^{-2}] = [M^{-1}L^3T^{-2}]$

Let Mass M = $[h]^a\,[C]^b\,[G]^c$

$$[M^1L^0T^0] = [ML^2T^{-1}]^a\,[LT^{-1}]^b\,[M^{-1}L^3T^{-2}]^c$$

$$[M^1L^0T^0] = [M^{a-c}\,L^{2a+b+3c}\,T^{-a-b-2c}]$$

Comparing the powers of M, L, T both side by principle of homogeneity powers must be equal.

$\therefore$ $a - c = 1$

$a = c + 1$ ...(i)

$2a + b + 3c = 0$

from (i)

$2(c+1) + b + 3c = 0$

$2c + 2 + b + 3c = 0$

$-a - b - 2c = 0$

from (i)

$-(c+1) - b - 2c = 0$

$-b - 3c = 1$ ...(iii)

$b + 5c = -2$ ...(ii)

$-b - 3c = 1$ ...(iii)

adding $2c = -1$

$c = -\frac{1}{2}$ then $a = -\frac{1}{2} + 1 = \frac{1}{2}$

$$b + 5c = -2 \Rightarrow b + 5\left(-\frac{1}{2}\right) = -2$$

$$b = -2 + \frac{5}{2}$$

$b = \frac{1}{2}$, $a = \frac{1}{2}$ and $c = -\frac{1}{2}$

$$\therefore \quad m = kh^{1/2}\,C^{+\frac{1}{2}}\,G^{-\frac{1}{2}}$$

$$m = k\sqrt{\frac{hC}{G}}$$

(ii) Let L $\propto$ $C^a\,h^b\,G^c$

$$[M^0L^1T^0] = k[LT^{-1}]^a\,[ML^2T^{-1}]^b\,[M^{-1}L^3T^{-2}]^c$$

$$[M^0L^1T^0] = k[M^{b-c}\,L^{a+2b+3c}\,T^{-a-b-2c}]$$

By the principle of homogeneity powers of M, L and T in both side must be equal.

$$\therefore \quad b - c = 0 \qquad\qquad a + 2b + 3c = 1 \qquad\qquad -a - b - 2c = 0$$

$$b = c \quad ...(i) \qquad \text{from } (i) \qquad a + b + 2c = 0$$

$$a + 2b + 3b = 1 \qquad\qquad \text{from } (i)$$

$$a + 5b = 1 \quad ...(ii) \qquad a + b + 2b = 0$$

$$\underset{-}{}a \underset{-}{+} 3b = \underset{-}{} 0 \quad ...(iii) \qquad a + 3b = 0 \; ...(iii)$$

Subtracting (*iii*) from (*ii*) $\quad 2b = 1$

$$b = \frac{1}{2} \Rightarrow c = \frac{1}{2}$$

$$a + 3 \times \frac{1}{2} = 0 \qquad \text{from } (iii)$$

$$a = -\frac{3}{2}$$

$$\therefore \qquad L = k\,C^{-\frac{3}{2}}\,h^{\frac{1}{2}}\,G^{\frac{1}{2}}$$

$$L = k\sqrt{\frac{hG}{C^3}}$$

(*iii*) $T \propto C^a\, G^b\, h^c$

$$[M^0L^0T^1] = k[LT^{-1}]^a\,[M^{-1}L^3T^{-2}]^b\,[ML^2T^{-1}]^c$$

$$[M^0L^0T^1] = k[M^{-b+c}\,L^{a+3b+2c}\,T^{-a-2b-c}]$$

By the principle of homogeneity powers of M, L and T in both side are equal.

$$\therefore -b + c = 0 \qquad\qquad a + 3b + 2c = 0 \qquad\qquad -a - 2b - c = 1$$

$$\text{from } (i) \qquad\qquad \text{from } (i)$$

$$b = c \quad ...(i) \qquad a + 3b + 2b = 0 \qquad a + 2b + b = -1$$

$$a + 5b = 0 \quad ...(ii) \qquad a + 3b = -1 \quad ...(iii)$$

$$\underset{-}{}a \underset{-}{+} 3b = \underset{+}{-}1 \quad ...(iii)$$

Subtracting (*iii*) from (*ii*) $\quad 2b = +1$

$$b = +\frac{1}{2} \qquad c = +\frac{1}{2}$$

$$a + 5 \times \frac{1}{2} = 0 \qquad \text{from } (ii)$$

$$a = -\frac{5}{2}$$

$$\therefore \qquad T = k\,C^{-\frac{5}{2}}\,G^{\frac{1}{2}}\,h^{\frac{1}{2}}$$

$$T = k\sqrt{\frac{hG}{C^5}}\,.$$

Q2.41. An artificial satellite is revolving around a planet of mass M and radius R in a circular orbit of radius *r*. From Kepler's Third law about the period of satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit *r*. Show using dimensional analysis, that:

$$T = \frac{k}{R}\sqrt{\frac{r^3}{g}},$$

where k is a dimensionless constant and g is acceleration due to gravity.

Main concept used: Principle of homogeneity.

Ans. By Kepler's third law of planetary motion

$$T^2 \propto r^3 \quad \text{or} \quad T \propto r^{3/2}$$

We know T also depend on radius R and g

$\therefore\ T \propto g^a R^b r^{3/2}$

or $$T = k[LT^{-2}]^a [L]^b [L]^{3/2}$$

$$[M^0L^0T^1] = k[M^0 L^{a+b+\frac{3}{2}} T^{-2a}]$$

By principle of homogeneity comparing powers of M, L and T

$$a + b + \frac{3}{2} = 0 \qquad -2a = +1$$

$$-\frac{1}{2} + b + \frac{3}{2} = 0 \qquad a = -\frac{1}{2}$$

$$b + 1 = 0$$

$$b = -1$$

$\therefore$ $$T = k\, g^{-\frac{1}{2}} R^{-1} r^{3/2}$$

$$T = \frac{k}{R}\sqrt{\frac{r^3}{g}}.$$ Hence proved.

Q2.42. In an experiment to estimate the size of a molecule of oleic acid, 1 mL of oleic acid is dissolved in 19 mL of alcohol. Then 1 mL of this solution is diluted to 20 mL by adding alcohol. Now 1 drop of this diluted solution is placed on water in a shallow trough. The solution spreads over the surface of water forming one molecule thick layer. Now, Lycopodium powder is sprinkled evenly over the film and its diameter is measured. Knowing the volume of the drop and area of the film we can calculate the thickness of the film, which will give us the size of oleic acid molecule.

Read the passage carefully and answer the following questions:

(*a*) Why we dissolve oleic acid in alcohol?

(*b*) What is the role of lycopodium powder?

(*c*) What would be the volume of oleic acid in each mL of solution prepared.

(*d*) How will you calculate the volume of n drops of this solution of oleic acid?

(*e*) What will be the volume of oleic acid in one drop of this solution?

Ans. (*a*): To get a molecular level we have to reduce the concentration of oleic acid by dissolving it in a proper solvent. Oleic acid is an organic compound so cannot be dissolve in ionic solvent water. It can dissolve in organic solvent alcohol.

(*b*) Lycopodium prevent to mix oleic acid in water, when drop of oleic acid is poured on water. So lycopodium powder spread on water surface first and then thin layer of diluted oleic acid is made on surface of lycopodium spread on water.

(*c*) The concentration of oleic acid in solution of alcohol in V volume solution is $\frac{1}{20} \times \frac{1}{20} V = \frac{1}{400} V$ ml if V = 1 ml then required concentration in one ml solution = $\frac{1}{400}$ ml as given in question.

(*d*) Volume of *n* drop solution can be calculate by burette, by dropping 1 ml, solution drop by drop in a beaker and counting its number of drops. If *n* drops are in 1 ml, then volume of 1 drop = $\frac{1}{n}$ ml.

(*e*) The volume of 1 drop of solution is $\frac{1}{n}$ ml. If *n* drops are measured in one ml in part (*d*). Then concentration of oleic acid in one drop solution = $\frac{1}{400} V = \frac{1}{400} . \frac{1}{n}$ ml $= \frac{1}{400n}$ ml oleic acid.

Q2.43. (*a*) How many astronomical units (A.U.) make 1 parsec?

(*b*) Consider a sun like star at a distance of 2 parsecs. When it is seen through a telescope with magnification, what should be angular size of the star? Sun appears to be $\left(\frac{1}{2}\right)^\circ$ from the earth due to atmospheric fluctuations, eye cannot resolve objects smaller than 1 arc minute.

(*c*) Mars has approximately half of earth's diameter. When it is closest to earth, it is at about $\frac{1}{2}$ A.U. from the earth. Calculate what size it will appear when seen through the same telescope.

(Comment: This is to illustrate why a telescope can magnify planets but not stars.)

Ans. (*a*): One parsec is the distance at which 1 A.U. long arc subtends angle of 1 s or 1 arc sec.

$$\therefore \quad \text{Angle 1 sec} = \frac{1 \text{ A.U.}}{1 \text{ parsec}} \sqrt{b^2 - 4ac}$$

$$\therefore \quad 1 \text{ parsec} = \frac{1 \text{ A.U.}}{1 \text{ arc sec}}$$

$$1 \text{ arc sec} = \frac{\pi}{180 \times 3600} \text{ radian}$$

$$1 \text{ parsec} = \frac{1 \times 180 \times 3600}{\pi} \text{ A.U.} = \frac{7 \times 180 \times 3600}{22}$$

$$1 \text{ parsec} = \frac{630 \times 3600}{11} \text{ A.U.} = \frac{2268000}{11} = 206181.8$$

$$1 \text{ parsec} = 2 \times 10^5 \text{ A.U.}$$

(*b*) 1 A.U. subtends angle of sun's diameter = $\left(\frac{1}{2}\right)^\circ$

As the distance from sun increases angle subtended in the same ratio.

So 2×10^5 A.U. (one parsec) distant star will form an angle of

$$\theta' = \left(\frac{1/2}{2 \times 10^5}\right)^\circ = \left(\frac{1}{4 \times 10^5}\right)^\circ$$

So angle subtend by 1 parsec on earth of size (diameter of sun) is $\frac{1}{4 \times 10^5}$ as the sunlike star's diameter is same as sun. If sun like star is at 2 parsec then angle at earth by star becomes

$$= \frac{1}{4 \times 10^5} \times \frac{1}{2} = \frac{1}{8} \times 10^{-5} = (1.25 \times 10^{-6})^\circ$$

$$\text{Angle} = (1.25 \times 60 \times 10^{-6}) \text{ minute}$$

$$= (75 \times 10^{-6}) \text{ min}$$

When these sun like star is seen by telescope of magnification 100, then angle formed by sun like star becomes $75 \times 10^{-6} \times 100 = 75 \times 10^{-4} = 7.5 \times 10^{-3}$ minute which is very less than 1 minute as eyes cannot resolve object or image smaller than one minute. So cannot be observed by given telescope.

Q2.44. Einstein's mass energy relation emerging out from this famous theory of relativity relates mass (m) to energy (E) as E = mc^2, where c is speed of light in vacuum. At the nuclear level, the magnitude of energy at nuclear level is usually measured in MeV where 1 MeV = 1.6×10^{-13} J; the mass are measured in unified atomic mass unit (u) where 1 u = 1.67×10^{-27} kg.

(*a*) Show the energy equivalent of 1 u is 931.5 MeV.

(*b*) A student writes the relation as 1 u = 931.5 MeV.

The teacher points out that the relation is dimensionally incorrect. Write the correct relation.

Ans. (*a*) $m = 1 \text{ u} = 1.67 \times 10^{-27}$ kg

$c = 3 \times 10^8$ m/s

By mass energy relation given by Einstein $E = mc^2$

$$\therefore \quad E = 1.67 \times 10^{-27} \times 3 \times 10^8 \times 3 \times 10^8 \text{ J}$$

$$= 1.67 \times 10^{-27+16} \times 9 \text{ J}$$

$$= \frac{1.67 \times 9 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV} = \frac{15.03 \times 10^{-11+13}}{1.6} \text{ MeV}$$

$$E = \frac{1503}{1.6} = 939.4 \text{ MeV}$$

$$\cong 931.5 \text{ MeV.}$$

(*b*) 1 amu = 931.5 MeV is dimensionally incorrect, but if in general represents 931.5 MeV energy will released if 1 u mass converted totally into energy

$$E = mc^2$$

$$\Rightarrow \quad 1 \text{ u } c^2 = 931.5 \text{ MeV}$$

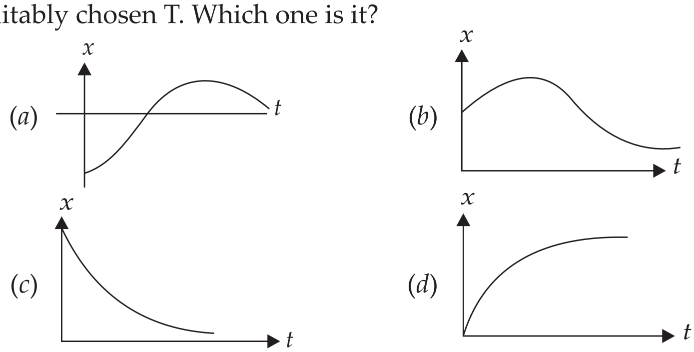
is dimensionally correct.

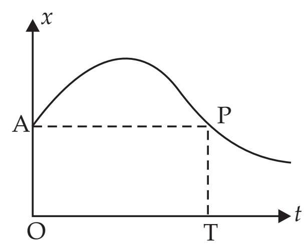
❑❑❑

3 Motion in a Straight Line

MULTIPLE CHOICE QUESTIONS-I

Q3.1. Among the four graphs given below, there is only one graph for which average velocity over the time interval (0, T) can vanish for a suitably chosen T. Which one is it?





Main concept used: Average velocity of body will be zero when displacement is zero any time interval-T in *x*-*t* graph.

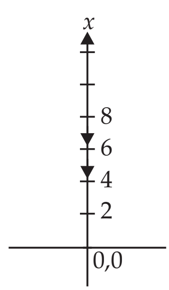
Ans. (*b*): If we draw a line parallel to time axis from the point (A) on graph at $t = 0$ sec. This line can intersect graph again at P in only option (*b*) as shown in the figure.

The change in displacement (O-T) time is zero i.e., displacement at A and P are equal so as change in displacement zero so velocity of body vanishes to zero.

Q3.2. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?

(*a*) $x < 0, v < 0, a > 0$ (*b*) $x > 0, v < 0, a < 0$

(*c*) $x > 0, v < 0, a > 0$ (*d*) $x > 0, v > 0, a < 0$



Ans. (*a*): As the lift moving downward so displacement is in negative or $\vec{x} < 0$.

As displacement is negative $\vec{v} < 0$. As the lift is just to reach 4th floor so it's motion is retarded ($-a$) downward ($-$).

So net acceleration $-(-a) = +\text{ve}$ i.e. $a > 0$.

Hence, verifies the option (*a*).

Q3.3. In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$.

(*a*) The displacement in time T must always takes non-negative values.

(*b*) The displacement x in time T satisfies $-v_0\text{T} < x < v_0\text{T}$.

(*c*) The acceleration is always a non-negative number.

(*d*) The motion has no turning points.

Ans. (*b*): For maximum and minimum displacement we have the magnitude and direction of maximum velocity.

As maximum velocity in positive direction is v_0 maximum velocity in opposite direction is also v_0.

Maximum displacement in one direction = $v_0\text{T}$

Maximum displacement in opposite direction = $-v_0\text{T}$

Hence, $-v_0\text{T} < x < v_0\text{T}$.

Q3.4. A vehicle travels half the distance L with speed v_1 and the other half with speed v_2, then its average speed is

(*a*) $\dfrac{v_1 + v_2}{2}$ (*b*) $\dfrac{2v_1 + v_2}{v_1 + v_2}$ (*c*) $\dfrac{2v_1 v_2}{v_1 + v_2}$ (*d*) $\dfrac{\text{L}(v_1 + v_2)}{v_1 v_2}$

Ans. (*c*): Time t_1 taken in half distance $= t_1 = \dfrac{\text{L}}{v_1}$

Time t_2 taken in half distance $t_2 = \dfrac{\text{L}}{v_2}$

$$\text{Total time } (t) \text{ taken in distance } (\text{L} + \text{L}) = \frac{\text{L}}{v_1} + \frac{\text{L}}{v_2} = \frac{\text{L}(v_2 \quad v_1)}{v_1 v_2}$$

$$\text{Total distance} = \text{L} + \text{L} = 2\text{L}$$

$$\text{Average speed } v_{av} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2\text{L}}{\dfrac{\text{L}(v_2 + v_1)}{v_1 v_2}} = \frac{2v_1 v_2}{(v_1 + v_2)}$$

Q3.5. The displacement of a particle is given by $x = (t - 2)^2$ where x is in metres and t in seconds. The distance covered by the particle in first 4 seconds is:

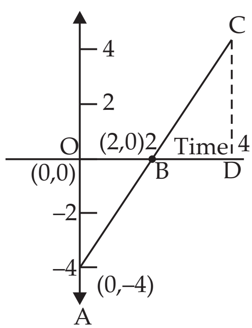
(*a*) 4 m (*b*) 8 m (*c*) 12 m (*d*) 16 m

Ans. (*b*):

$$x = (t-2)^2$$

$$v = \frac{dx}{dt} = 2(t-2) \text{ m/s}$$

$$a = \frac{d^2x}{dt^2} = 2(1-0) = 2 \text{ m s}^{-2}$$



at $t = 0$ $v_0 = 2(0-2) = -4$ m/s

$t = 2$ $v_2 = 2(2-2) = 0$ m/s

$t = 4$ $v_4 = 2(4-2) = 4$ m/s

Distance = Area between time axis and (v-t) graph

$$= \text{ar }\Delta OAB + \text{ar }\Delta BCD$$

$$= \frac{1}{2} OB \times OA + \frac{1}{2} BD \times CD$$

$$= \frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 2 \times 4$$

Distance = 8 m verifies option (*b*).

If displacement

$$= \frac{1}{2} OB \times OA + \frac{1}{2} BD \times CD$$

$$= \frac{1}{2} \times 2 \times (-4) + \frac{1}{2} \times 2 \times 4 = 0$$

Displacement = Zero.

Q3.6. At metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be:

(*a*) $\frac{(t_1 + t_2)}{2}$ (*b*) $\frac{t_1 t_2}{(t_2 - t_1)}$ (*c*) $\frac{t_1 t_2}{(t_1 + t_2)}$ (*d*) $(t_1 - t_2)$

Ans. (*c*): Let L be the length of escalator.

Velocity of girl w.r.t. ground $v_g = \frac{L}{t_1}$

Velocity of escalator w.r.t. ground $v_e = \frac{L}{t_2}$

Velocity of girl on moving escalator with respect to ground

$$= v_g + v_e = \frac{L}{t_1} + \frac{L}{t_2} = L\left[\frac{1}{t_1} + \frac{1}{t_2}\right]$$

v_{gG} on moving escalator = V = $L\left[\frac{t_1 + t_2}{t_1 t_2}\right]$

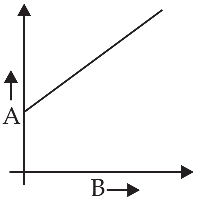
∴ Time t taken by girl on moving escalator in going up the distance L is

$$t = \frac{\text{distance}}{\text{speed}} = \frac{L}{L\left(\frac{t_1 + t_2}{t_1 t_2}\right)} = \frac{t_1 t_2}{t_1 + t_2}$$

Hence, verifies the option (*c*).

MULTIPLE CHOICE QUESTIONS-II

Q3.7. The variation of quantity A with quantity B, plotted in figure. Describe the motion of a particle in straight line.



(*a*) Quantity B may represent time.

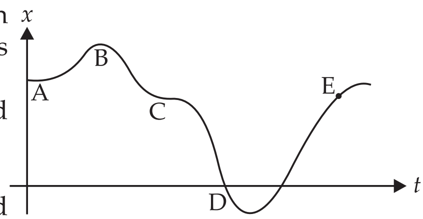
(*b*) Quantity A is velocity if motion is uniform.

(*c*) Quantity A is displacement if motion is uniform.

(*d*) Quantity A is velocity if motion is uniformly accelerated.

Ans. (*a, c, d*): If B represents time and A represents velocity then graph become (*v*-*t*). *v*-*t* graph is straight line so it is uniformly accelerated motion, so motion is not uniform. Verifies option (*a*), (*d*).
If B represents time and A represents displacement, then graph become (*s*-*t*) graph. Here *s*-*t* graph is straight line which represents uniform motion, so verifies the option (*c*).

Q3.8. A graph of *x* versus *t* shown in figure. Choose correct alternatives from below.



(*a*) The particle was released from rest at $t = 0$
(*b*) At B, the acceleration $a > 0$
(*c*) At C, the velocity and acceleration vanishes.
(*d*) Average velocity for the motion A and D is positive.
(*e*) The speed at D exceeds that at E.

Ans. (*a, c, e*): **Main concept used:** Slope of *x*-*t* graph gives $v = \dfrac{dx}{dt}$ At A graph (*x*-*t*) is parallel to time axis, so $\dfrac{dx}{dt}$ is zero or particle is at rest.
After A slope $\dfrac{dx}{dt}$ increases so velocity increases. Verifies option (*a*).
Tangent at B and C is graph (*x*-*t*) is parallel to time axis, so $\dfrac{dx}{dt} = 0$ or $v = 0$.
It implies that acceleration $a = 0$ so $a \ngtr 0$ discards option (*b*) and verifies the option (*c*).
From graph the slope at D is greater than at E. So speed at D is greater than at E. Verifies the option (*e*).
Velocity at A is zero as *x*-*t* parallel to time axis so average velocity at A is zero. At D displacement or slope is negative. So average velocity at D is negative not positive discards option (*d*).

Q3.9. For the one-dimensional motion, described by $x = t - \sin t$
(*a*) $x(t) > 0$ for all $t > 0$ (*b*) $v(t) > 0$ for all $t > 0$
(*c*) $a(t) > 0$ for all $t > 0$ (*d*) $v(t)$ lies between 0 and 2

Ans. (*a, d*):
$$\boxed{x = t - \sin t}$$
$$v = \frac{dx}{dt} = (1 - \cos t) \quad \Rightarrow \quad \boxed{v = (1 - \cos t)}$$
$$a = \frac{dv}{dt} = \frac{d}{dt}(1 - \cos t) = + \sin t$$
$$\boxed{a = \sin t}$$
For v_{max} at $\cos t$ minimum i.e., $\cos t = -1$.
$\therefore \quad v_{max} = 1 - (-1) = 2$

For v_{min} at cos t maximum i.e., cos $t = 1$

$$v_{min} = 1 - 1 = 0$$

Hence, v lies between 0 to 2. Verifies the option (*d*).

$$x = t - \sin t$$

sin t varies between 1 and – 1 for t > 0.

x will be always positive $x(t) > 0$. Verifies answer (*a*).

$$v = 1 - \cos t$$

cos t also varies from – 1 to 1,

at $\cos t = +1$

$$v = 1 - 1 = 0$$

$v(t) > 0$ is false discards option (*b*).

$$a = \sin t$$

sin t varies from – 1 to 1.

So a will varies from – 1 to 1 or a can be (–). So discards option (*c*).

Hence, verifies option (*a*) and (*d*).

Q3.10. A spring with one end attached to a mass m and other end to a rigid support is stretched and released.

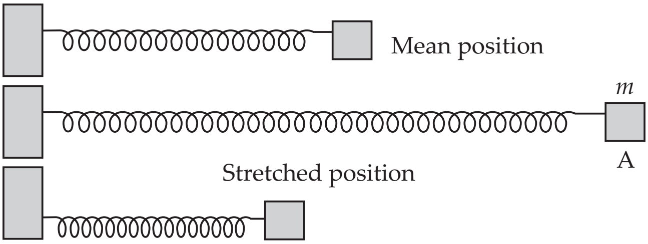
(*a*) Magnitude of acceleration, when just released is maximum.

(*b*) Magnitude of acceleration, when at equilibrium position is maximum.

(*c*) Speed is maximum when mass is at equilibrium position.

(*d*) Magnitude of displacement is always maximum, whenever speed is minimum.

Ans. (*a, c, d*): Consider a spring of spring constant k is attached to mass m at one end and other end is fixed at right support. Spring is lying on a frictionless table.



Now spring is stretched by a force F by x displacement then F = – kx (–) sign shows that displacement x is opposite to the direction of force applied, when a force F acts on spring also applies equal to opposite force. P.E. at A = $\frac{1}{2}kx^2$ the restoring force is directly proportional to the x so execute Simple Harmonic Motion (SHM)

$$\therefore \quad a = \frac{-F}{m}$$

$$a = \frac{-kx}{m}$$

at $x = 0$ then $a = 0$

at $x = x$, $a = \frac{-kx}{m}$

Magnitude of a is maximum at x when released. Verifies the option (*a*).

At mean position where P.E. is converted into KE = $\frac{1}{2}mv^2$.

So the speed of mass is maximum at $x = 0$.

Verifies the option (*c*).

Magnitude of $a = 0$ at $x = 0$. So (*b*) is incorrect when mass (m) is at its maximum displacement then it returns at this point and momentarily $v = 0$. So it verifies answer (*d*) also.

Q3.11. A ball is bouncing elastically with a speed of 1 m/s between walls of a railway compartment of size 10 m in the direction perpendicular to walls. The train is moving at a constant speed of 10 m/s parallel to the direction of motion of the ball. As seen from the ground,

(*a*) the direction of motion of the ball changes every 10 seconds.

(*b*) speed of the ball changes every 10 seconds.

(*c*) average speed of the ball over any 20 seconds interval is fixed

(*d*) the acceleration of ball is the same as from the train.

Main concept used: Motion of ball with respect to observer and relative velocity of body.

Ans. (*a, c, d*): As the motion is observed from ground, time to strike ball with walls will be after every 10 sec. So the direction speed of ball changes (direction) every 10 sec, but change in speed is zero.

i.e., speed of ball remains always 1 m/s so option (*a*) is correct and (*b*) is incorrect.

As speed of ball is uniform so average speed at any time remain same or 1 m/s with respect to train or ground. So option (*c*) is correct.

Speed of ball changes when it strike to wall initial speed of ball in the direction of moving train with respect to ground = 10 + 1 = 11 m/s.

v_{BG} (opposite to the direction of train) = 10 – 1 = 9 m/s.

$\therefore$ Change in velocity on collision will be in magnitude = 11 – 9 = 2 m/s.

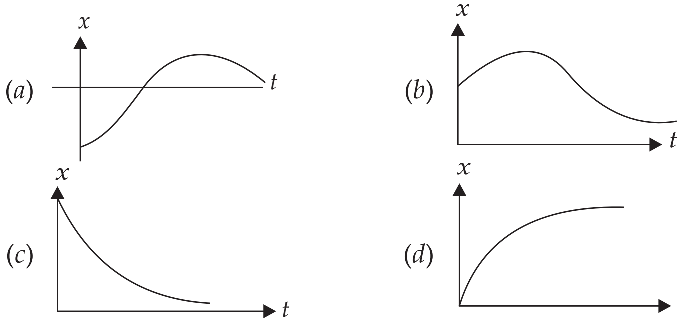
So magnitude of acceleration on both walls of compartment is same but direction will be opposite. Hence, right option are (*a, c, d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q3.12. Refer to graph of Question 3.1. Match the following.

Graph	**Characteristic**
(*a*)	(*i*) has $v > 0$ and $a < 0$ throughout.
(*b*)	(*ii*) has $x > 0$ throughout and has a point with $v = 0$, and a point $a = 0$.
(*c*)	(*iii*) has a point with zero displacement for $t > 0$.
(*d*)	(*iv*) has $v < 0$ and $a < 0$

Ans.



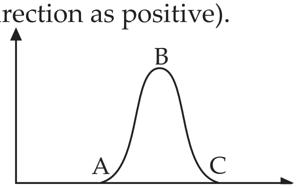
(i) $v > 0$ indicate that slope is always positive i.e., between 0° to 90° (tan θ) it matches with graph (d). Hence, (i) part matches with graph (d).

(ii) $x > 0$ throughout and $v = 0$, $v = 0$ matches with graph (ii). At point A slope is zero so $v = 0$, $a = 0$ graph lies in + x direction always so verifies the answer. So (ii) part matches with (b).

(iii) Zero displacement where $y = 0$ is only in graph (a). So part (iii) matches with graph (a).

(iv) $v < 0$ i.e. slope is (–)ve it is in graph (c). So part (iii) matches with graph (c).

Q3.13. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time. (Take acceleration in backward direction as positive).



Ans. When ball is hitted by bat its acceleration decreases till its velocity becomes zero, so acceleration is in **backward** direction which here taken is positive as shown in graph from A to B part.

Now after when the velocity of ball decreased to zero its velocity increases in forward direction so acceleration in forward direction is negative (here) show by part BC in graph.

Q3.14. Give examples of a one dimensional motion where

(a) the particle moving along positive x-direction comes to rest periodically and moves *forward*.

(b) the particle moving along positive x-direction comes to rest periodically and moves *backward*.

Ans. (i) Consider a motion $x(t) = \omega t - \sin \omega t$

$$v = \frac{dx}{dt} = \omega - \omega \cos \omega t$$

$$a = \frac{dv}{dt} = \omega^2 \sin \omega t$$

at $\omega t = 0$ $\quad x(t) = 0;\quad v = 0 \quad a = 0$

at $\omega t = \pi$ $\quad x(t) = \pi > 0;$ $\quad v = \omega - \omega \cos \pi = 2\omega > 0$ $\quad a = 0$

at $\omega t = 2\pi$ $\quad x(t) = 2\pi > 0;$ $\quad v = 0$ $\quad a = 0$

(*ii*) Consider a function of motion

$$x(t) = -a \sin \omega t$$

at $t = 0$ $\quad x(t) = -a \sin 0 = 0$

at $t = \frac{T}{4}$ $\quad x(t) = -a \sin \frac{2\pi}{T}.\frac{T}{4} = -a \sin \frac{\pi}{2} = -a$

at $t = \frac{T}{2}$ $\quad x(t) = -a \sin \frac{2\pi}{T}.\frac{T}{2} = -a \sin \pi = 0$

at $t = \frac{3T}{4}$ $\quad x(t) = -a \sin \frac{2\pi}{T}.\frac{3T}{4} = -a \sin \frac{3\pi}{2} = -a \sin\left(\pi + \frac{\pi}{2}\right)$

$$= -a\left(-\sin \frac{\pi}{2}\right) = a$$

at $t = T$ $\quad x(t) = -a \sin \frac{2\pi}{T}.T = -a \sin 2\pi = +0$

Hence, the particle comes to rest periodically and displacement is in negative direction.

Hence, periodic function is $-a \sin \omega t$

$$v = \frac{dx(t)}{dt} = \frac{d}{dt}(-a \sin \omega t) = -a\omega \cos \omega t$$

at, $t = 0$ $\quad v = -a\omega \cos 0° = -\omega a$

at, $t = \frac{T}{4}$ $\quad v = -\omega a \cos \frac{2\pi}{T}.\frac{T}{4} = -\omega a \cos \frac{\pi}{2} = 0$

at, $t = \frac{T}{2}$ $\quad v = -\omega a \cos \frac{2\pi}{T}.\frac{T}{2} = -a\omega \cos \pi = +\omega a$

at, $t = \frac{3T}{4}$ $\quad v = -\omega a \cos \frac{2\pi}{T}.\frac{3T}{4} = -\omega a \cos \frac{3\pi}{2} = -\omega a \cos\left(\pi + \frac{\pi}{2}\right)$

$$v = +a\omega \cos \frac{\pi}{2} = a\omega \times 0 = 0$$

at, $t = T$ $\quad v = -a\omega \cos \frac{2\pi}{T}.T = -\omega a \cos 2\pi = -\omega a$

Hence, the velocity after zero displacement changes periodically.

So required function of motion = $x(t) = -a \sin \omega t$.

(*i*) Consider a function of motion of time periods and amplitude a, $x(t) = a \sin \omega t$.

$$x(0) = 0 = 0$$

$$x\left(\frac{T}{4}\right) = a \sin \frac{2\pi}{T}.\frac{T}{4} = a \sin \frac{\pi}{2} = a$$

$$x\left(\frac{T}{2}\right) = a \sin \frac{2\pi}{T}.\frac{T}{2} = a \sin \pi = 0$$

$$x\left(\frac{3T}{2}\right) = a \sin \frac{2\pi}{T}.\frac{3T}{4} = a \sin \frac{3\pi}{2} = a \sin\left(\pi + \frac{\pi}{2}\right) = -a$$

$$x(\text{T}) = a \sin \frac{2\pi}{\text{T}}.\text{T} = a \sin 2\pi = 0$$

Hence, particle is moving with displacement zero periodically and moves in +ive direction i.e., in forward.

Hence, require function is $x(t) = a \sin \omega t$.

Q3.15. Give example of a motion where $x > 0$, $v < 0$ and $a > 0$ at a particular instant.

Ans. Let us consider function of motion

$$x(t) = \text{A} + \text{B}e^{-\gamma t} \quad ...(i)$$

where γ and A, is a constant B is amplitude

$x(t)$ is displacement at time t, where $\text{A} > \text{B}$ and $\gamma > 0$

$$v(t) = \frac{dx(t)}{dt} = 0 + (-\gamma)\,\text{B}e^{-\gamma t} = -\gamma\,\text{B}e^{-\gamma t} \quad ...(ii)$$

$$a(t) = \frac{d[v(t)]}{dt} = \frac{d}{dt}(-\gamma\,\text{B}e^{-\gamma t}) = +\gamma^2\text{B}e^{-\gamma t} \quad ...(iii)$$

From (i) $\because$ $\text{A} > \text{B}$ so x is always +ve i.e., $x > 0$

From (ii) v is always negative from (ii) $v < 0$

From (iii) a is always again positive $a > 0$

As the value of $\gamma^2\text{B}e^{-\gamma t}$ can varies from 0 to $+\infty$.

Q3.16. An object falling through a fluid is observed to have acceleration given by $a = g - bv$ where g = gravitational acceleration and b is a constant. After a long time of release, it is observed to fall with constant speed. What must be the value of constant speed?

Ans. After long time of released the velocity becomes constant i.e.,

$$\frac{dv(t)}{dt} = 0 \quad \text{or} \quad a = 0 \quad ...(i)$$

Given acceleration is $a = g - bv$

$$0 = g - bv \quad \text{[from } (i)\text{]}$$

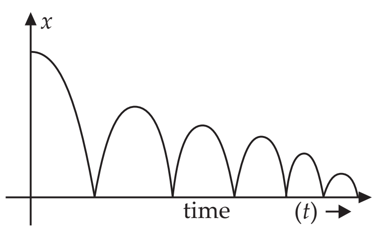
$$bv = g$$

$$v = \frac{g}{b}$$

Hence, the constant speed after long time of release is $\left(\frac{g}{b}\right)$.

SHORT ANSWER TYPE QUESTIONS

Q3.17. A ball is dropped and its displacement verses time graph is shown (Displacement x is from ground and all quantities are positive upwards.)



(*a*) Plot qualitatively velocity verses time graph.

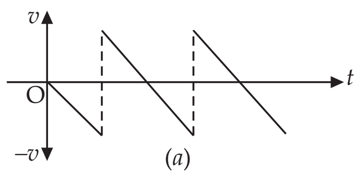
(*b*) Plot qualitatively acceleration verses time graph.

Ans. It is clear from graph, displacement (x) is always positive. Velocity of body increases till the x becomes zero then velocity becomes in opposite direction and velocity (slope of x-t graph) decreases to zero till it reaches maximum value of x but smaller than earlier.

When velocity increases and body reaches towards $x = 0$ acceleration is in downward direction. When body moves upward i.e., $x > 0$ then velocity decreases so direction of 'a' is again downward.

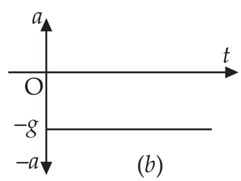
Hence the $a = -g$ always.

(*a*) at $t = 0$, $v = 0$ it increases in downward direction with constant accele-ration 'g'. When $x = 0$ body after it bounces upward but its velocity decreases with constant $a = g$ if it again come back (downward) with acceleration $(-g)$. So v-t graph.



(*a*)

(*b*) The (a-t) graph



(*b*)

Q3.18. A particle length executes the motion described by $x(t) = x_0(1 - e^{-\gamma t})$; $t \geq 0$, $x_0 > 0$

(*a*) where does the particle start and with what velocity?

(*b*) find the maximum and minimum values of $x(t)$, $v(t)$, $a(t)$. Show that $x(t)$ and $a(t)$ increases with time and $v(t)$ decreases with time.

Main concept used: By calculating $v(t)$ and $a(t)$ with the help of $x(t)$, then determining the maximum and minimum value of $x(t)$, $v(t)$ and $a(t)$.

Ans.

$$x(t) = x_0\left[1 - e^{-\gamma t}\right] \quad ...(i)$$

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_0(1 - e^{-\gamma t})\right] = +\, x_0\gamma\, e^{-\gamma t} \quad ...(ii)$$

$$a(t) = \frac{dv}{dt} = \frac{d\left[+x_0\, \gamma\, e^{-\gamma t}\right]}{dt} = -\, x_0\, \gamma^2\, e^{-\gamma t} \quad ...(iii)$$

(*i*) At, $t = 0$

$$x(0) = x_0\left[1 - e^0\right] = x_0(1 - 1) = 0$$

$$v(0) = x_0\, \gamma\, e^0 = x_0\, \gamma$$

Hence, the particle start from $x = 0$ with velocity $v_0 = x_0\, \gamma$.

(*ii*) (*a*) $x(t)$ is minimum at $t = 0$

$x(t)$ is maximum $t = \infty$ $\quad \because e^{-\gamma t} = \infty$ at $t = \infty$.

(*b*) $v(t)$ at $t = 0$ is v_0 $(= x_0\, \gamma)$ (maximum)

at $t = \infty$ $\quad v(t) = x_0\, \gamma\, (0) = 0$ (minimum)

$a(t)$ at $t = 0$ is $a(0) = -\, x_0\, \gamma^2$ (minimum)

$a(t)$ at $t = \alpha$ is $a(\infty) = 0$ (maximum)

Q3.19. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has a speed of 18 km/hr while the other has the speed of 27 km/hr. The bird starts moving from first car towards the other and is moving with the speed of 36 km/hr and when the two cars were separated by 36 km. What is the total distance covered by the bird? What is the total displacement of the bird?

Main concept used: Bird will fly to and fro till both the cars meet together. So the total distance covered by bird during the time speed of bird × time to meet the cars together.

Ans. Time to meet the two cars together (t)

$$t = \frac{\text{distance between cars}}{\text{relative speed of cars}} = \frac{36 \text{ km}}{(27 + 18) \text{ km/hr}} = \frac{36}{45} = \frac{4}{5} \Rightarrow t = \frac{4}{5} \text{ hours}$$

$\therefore$ Distance covered by bird in $\frac{4}{5}$ hours $= 36 \times \frac{4}{5} = 28.8$ km.

Q3.20. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is of a lower height than the first. If his speed is 9 m/s, the (horizontal) distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building? (take $g = 10$ m/s^2)

Main concept: During fall freely 9 m the horizontal distance covered by man should be atleast 10 m.

Ans. Vertical motion

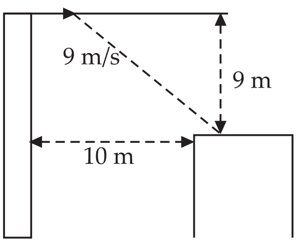
$u_y = 0,\qquad a = 10$ m/s^2

$s = 9$ m $\qquad t = t$

$$s = u_y t + \frac{1}{2}at^2$$

$$9 = 0 \times t + \frac{1}{2} \times 10 \times t^2$$

$$t = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} \text{ sec}$$



Horizontal distance covered by person is

$$\frac{3}{\sqrt{5}} \text{ sec} = 9 \text{ m/s} \times \frac{3}{\sqrt{5}} \text{ sec} = \frac{27}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{27 \times 2.236}{5} = \frac{60.37}{5} = 12.07 \text{ m}$$

As 12.07 m covered during free falling of 9 m. So he reach on the building next farther the first edge by $12.07 - 10 = 2.07$ m.

Q3.21. A ball is dropped from a building of height 45 m. Simultaneously another ball is thrown up with a speed 40 m/s. Calculate the relative speed of the balls as a function of time.

Ans. For the first ball falling from top

$v = v_1 = ?\qquad u = 0\qquad h = 45$ m $\qquad a = g\qquad t = t$

$$v = u + at$$

$$v_1 = 0 + gt \quad \text{or} \quad v_1 = gt \quad \text{downward} \quad \therefore v_1 = -gt$$

For the second ball thrown upward

$v = v_2\qquad u = 40$ m/s $\qquad a = -g\qquad t = t$

$$\therefore \qquad v = u + at$$

$$v_2 = (40 - gt) \quad \text{upward} \quad \therefore v_2 = +(40 - gt)$$

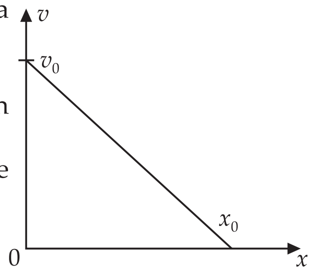
Relative velocity of ball Ist with respect to IInd

$$v_{12} = v_1 - v_2 = -gt - (40 - gt)$$
$$= -gt - 40 + gt = -40 \text{ m/s (downward)}$$

Relative velocity of ball first with the respect to second is 40 m/s downward.

In this problem due to acceleration the speed of one increases and of other decreases with same rate. So their relative speed remains $(40 - 0) = 40$ m/s.

Q3.22. The velocity-displacement graph of a particle is shown in figure.

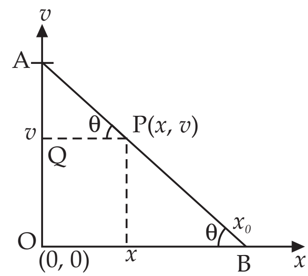


(*a*) Write the relation between v and x.

(*b*) Obtain the relation between acceleration and displacement and plot it.

Ans. (*a*) Consider a point P(x, v) at any time t on graph. Let $\angle$ABO is θ then

$$\tan\theta = \frac{AQ}{QP} = \frac{v_0 - v}{x} = \frac{v_0}{x_0}$$



As velocity decrease from v_0 to zero during displacement zero to x.

So acceleration is negative

$$a = -\tan\theta = \frac{-(v_0 - v)}{x} = \frac{-v_0}{x_0}$$

$$v_0 - v = \frac{v_0}{x_0} x$$

$$v = v_0 - \frac{v_0}{x_0} . x$$

$$v = v_0 \left[1 - \frac{x}{x_0}\right] \text{ is the relation between } v \text{ and } x.$$

(*b*) $$a = \frac{dv}{dt} = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{dv}{dx} . \frac{dx}{dt}$$

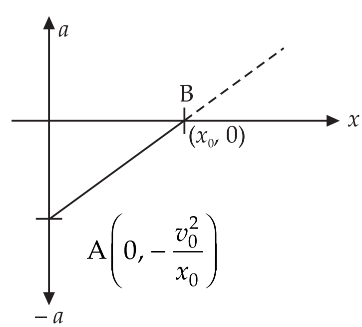
$$a = \frac{-v_0}{x_0} . v = \frac{-v_0}{x_0} . \left[v_0\left(1 - \frac{x}{x_0}\right)\right]$$

$$= \frac{-v_0^2}{x_0}\left(1 - \frac{x}{x_0}\right)$$

$$a = \frac{v_0^2 x}{x_0^2} - \frac{v_0^2}{x_0}$$

at $x = 0$ $$a = \frac{v_0^2}{x_0^2} \times 0 - \frac{v_0^2}{x_0}$$

$$a = -\frac{v_0^2}{x_0}$$



at $a = 0$ $\qquad 0 = \dfrac{v_0^2}{x_0^2}x - \dfrac{v_0^2}{x_0}$

$$\frac{v_0^2}{x_0} = \frac{v_0^2}{x_0^2}x \Rightarrow x = x_0$$

$\therefore$ A$\left(0, \dfrac{-v_0^2}{x_0}\right)$ and B$(x_0, 0)$.

LONG ANSWER TYPE QUESTIONS

Q3.23. It is a common observation that rain clouds can be at about a kilometre altitude above the ground.

(*a*) If a rain drop falls from such a height freely under gravity, what will be its speed? Also calculate in km/h. ($g = 10$ m/s^2)

(*b*) A typical rain drop is about 4 mm diameter. Momentum is mass × speed in magnitude. Estimate its momentum when it hits ground.

(*c*) Estimate the time required to flatten the drop.

(*d*) Rate of change of momentum is force. Estimate how much force such a drop would exert on you.

(*e*) Estimate the order of magnitude force on umbrella. Typical lateral separation between two rain drops is 5 cm.
(Assume that umbrella is circular and has a diameter of 1 m and cloth is not pierced through!!)

Main concept used: F = ma, $\dfrac{dp}{dt} = \text{F}$, eqn. of motion, $p = mv$

Ans. Given $\quad h = 1$ km $= 1000$ m, $\qquad g = 10$ m/s^2

$u = 0$ m/s $\qquad d = 4$ mm $\Rightarrow r = \dfrac{4}{2}$ mm $= 2 \times 10^{-3}$ m

(*a*) Velocity of rain drop on ground

$$v^2 = u^2 + 2gh$$
$$v^2 = 0^2 + 2 \times 10 \times 1000$$
$$v = 100\sqrt{2} \text{ m/s Ans. } (i)$$
$$v = \overset{20}{\cancel{100}}\sqrt{2} \times \frac{18}{\cancel{5}} \text{ km/hr} = 360\sqrt{2} \text{ km/hr Ans. } (ii)$$

(*b*) Mass of drop = Volume × density = $\dfrac{4}{3}\pi r^3 \rho$

$$= \frac{4}{3}\pi(2 \times 10^{-3})^3 \times 1000 \text{ kg/m}^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 \times 10^{-9} \times 10^3$$
$$= \frac{32 \times 22}{21} \times 10^{-6} \text{ kg}$$
$$m = \frac{704}{21} \times 10^{-6} = 33.5 \times 10^{-6} \text{ kg}$$

$\therefore$ Momentum $= mv = 33.5 \times 10^{-6} \times 100\sqrt{2}$

$$= 33.5 \times 1.414 \times 10^{-4} \text{ kg ms}^{-1}$$
$$= 47.37 \times 10^{-4} \text{ kg ms}^{-1}$$
$$= 4.7 \times 10^{-3} \text{ kg ms}^{-1}$$

(*c*) Time to required a drop of 4 mm diameter spherical drop i.e., time to reach upper part of spherical drop on ground i.e., distance covered by upper part of drop to reach ground = diameter (d) = 4 mm = 4×10^{-3} m

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{4 \times 10^{-3} \text{ m}}{100\sqrt{2}} = \frac{\overset{2}{\cancel{4}} \times \sqrt{2}}{100 \times \cancel{2}} \times 10^{-3}$$

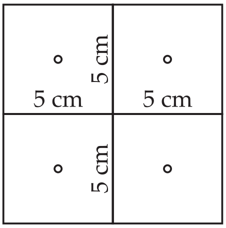
$$\text{Time } (t) = \frac{2 \times 1.414}{100} \times 10^{-3} = \frac{2.828}{100} \times 10^{-3} = 2.8 \times 10^{-5} \text{ sec}$$

(*d*) $$\text{Force} = \frac{dp}{dt} = \frac{mv - 0}{t - 0} = \frac{4.7 \times 10^{-3}}{2.8 \times 10^{-5}} = 1.68 \times 10^2$$

$$= 168 \text{ N } \textbf{Ans.}$$

(it equivalent to 16 kg which is not actually force exerted by drop on ground or man)

(*e*) Area of umbrella $= \pi R^2 = \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \text{ m}^2$



Square area covered by one drop

$$= (5 \times 10^{-2})^2 = 25 \times 10^{-4} \text{ m}^2$$

Number of drops falling on umbrella

$$= \frac{\pi R^2}{25 \times 10^{-4}}$$

$$= \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{25 \times 10^{-4}} \text{ drops}$$

$$= \frac{11 \times 10^4}{14 \times 25} = \frac{11 \times 10^4}{350} = 0.0314 \times 10^4$$

$$n = 314 \text{ drops}$$

$\therefore$ Net force on umbrella by 314 drops = 314×168 N = 52752 N

It is equivalent to 5,275 kg wt which again not possible on umbrella. Velocity of drop decreased to terminal velocity due to retarding force of friction of air molecules.

Q3.24. A motor car moving at a speed of 72 km/h cannot come to stop in less than 3.0 second while for a truck this time interval is 5.0 second. On a highway the car is behind the truck both moving 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bump onto (collide with) the truck? Human response time is 0.5 s.

(Comment: This is to illustrate why vehicles carry the message on the rear side. "Keep Safe Distance").

Ans. For truck $u = 72 \times \frac{5}{18}$ m/s = 20 m/s

$v = 0 \qquad a = ? \qquad t = 5$ sec.

$$v = u + at$$
$$0 = 20 + a_T \times 5$$
$$a_T = \frac{-20}{5} = -4 \text{ m/s}^2$$

For car $\quad t = 3$ s $\quad u = 20$ m/s $\quad v = 0 \quad a = a_C$

$$v = u + at$$
$$0 = 20 + a_C 3$$
$$a_C = \frac{-20}{3} \text{ m/s}^2$$

Let car is at distance x metre behind the truck. Car takes time 't' to stop after observing the signal given by truck to stop.
Time of response for human = 0.5 second
Time t includes the time to stop car and responding time both. So time taken by car to stop after applying breaks is (t – 0.5) seconds.

$$v_C = u + a_c t$$
$$0 = 20 - \frac{20}{3}(t - 0.5) \qquad ...(i)$$

For truck driver there is no responding time he applies breaks with passing signal to car back side, so

$$v_T = u + at$$
$$0 = 20 - 4t \qquad ...(ii)$$

Equating (i) and (ii) equation.

$$20 - 4t = 20 - \frac{20}{3}(t - 0.5)$$
$$-4t = -\frac{20}{3}(t - 0.5)$$
$$12t = 20t - 10$$
$$-20t + 12t = -10$$
$$-8t = -10$$
$$t = \frac{10}{8} = \frac{5}{4} = 1.25 \text{ seconds}$$

Distance travelled by car and truck in $\frac{5}{4}$ sec

$$s = 20 \times \frac{5}{4} + \frac{1}{2}(-4)\frac{5}{4} \times \frac{5}{4} \qquad \left(\because s = ut + \frac{1}{2}at^2\right)$$
$$s_T = 25 - \frac{25}{8} = 25 - 3.125 = 21.875 \text{ m.}$$

Car travel first 0.5 sec with speed of uniform but after this responding time 0.5 sec breaks are applied and then retarding motion starts for car

$$s_C = (20\times.5)+20(1.25-0.5)+\frac{1}{2}\times\left(\frac{-20}{3}\right)(1.25-0.5)^2$$
$$= 10 + 20\times 0.75 - \frac{10}{3}\times 0.75\times 0.75 = 10 + 15.0 - 7.5\times .25$$
$$s_C = 25 - 1.875 = 23.125 \text{ m}$$
$$s_C - s_T = 23.125 - 21.875 = 1.25 \text{ m.}$$

So avoid bump onto truck, the car must be behind atleast 1.25 m.

Q3.25. A monkey climbs up a slippery pole for 3 seconds and subsequently slips for 3 seconds. Its velocity at time 't' is given by

$$v(t) = 2t\,(3-t);\quad 0 < t < 3 \text{ seconds}$$

and $v(t) = -(t-3)(6-t)$ for $3 < t < 6$ s in m/s. It repeats this cycle till it reaches the height of 20 m.

(*a*) At what time its velocity is maximum?

(*b*) At what time its average velocity is maximum?

(*c*) At what time its acceleration is maximum in magnitude?

(*d*) How many cycles (counting fractions) are required to reach the top?

Ans. (*a*) For maximum velocity $v(t)$

$$\frac{dv(t)}{dt} = 0$$
$$\frac{d[2t(3-t)]}{dt} = 0$$
$$\frac{d(6t-2t^2)}{dt} = 0$$
$$6 - 4t = 0$$
$$4t = 6 \;\Rightarrow\; t = \frac{6}{4} = \frac{3}{2} = 1.5 \text{ second} \quad \textbf{Ans.}$$

(*b*) For average velocity $= \dfrac{\text{Total distance}}{\text{Total time}}$

$$\because \quad v(t) = 6t - 2t^2 \qquad ...(i)$$
$$\frac{ds(t)}{dt} = 6t - 2t^2$$
$$ds = (6t - 2t^2)\,dt$$

Integrating B.S. from 0 to 3 sec

$$\int_0^s ds = \int_0^3 (6t-2t^2)\,dt$$
$$s = \left[6\frac{t^2}{2} - 2\times\frac{t^3}{3}\right]_0^3 = \left[3t^2 - \frac{2}{3}t^3\right]_0^3$$
$$= \left[3\times 9 - \frac{2}{3}\times 27\right] = 27 - 18$$
$$s = 9 \text{ m} \qquad ...(ii)$$

$$\text{Average velocity } v_{av} = \frac{9\text{ m}}{3} = 3\text{ m/s}$$

$$v(t) = 6t - 2t^2 \qquad \text{(given } 0 < t < 3)$$

$$3 = 6t - 2t^2 \qquad v_{av} = 3$$

$$2t^2 - 6t + 3 = 0 \qquad a = 2,\ b = -6,\ c = 3$$

$$t = \frac{+6 \pm \sqrt{(-6)^2 - 4(2)(3)}}{2 \times 2}$$

$$t = \frac{6 \pm \sqrt{36 - 24}}{2 \times 2} = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$t = \frac{2\left(3 \pm \sqrt{3}\right)}{4} = \frac{3 \pm \sqrt{3}}{2}$$

[Taking +ve] $$t = \frac{3 + 1.732}{2} = \frac{4.732}{2} = 2.31\text{ sec}$$

[Taking –ve] $$t = \frac{3 - 1.732}{2} = \frac{1.268}{2} = .634\text{ sec} \quad \text{rejected.}$$

As $v = 3$ is not equal to 3 m/s from (i).
So average velocity is maximum at 2.31 sec.

(c) Time for maximum acceleration in periodic motion acceleration is maximum when body returns at its mean position or changes the direction of motion it at $v = 0$

$$v(t) = 6t - 2t^2$$

For maximum acceleration $v = 0$

$$0 = 6t - 2t^2$$

$$2t(3 - t) = 0$$

$t \neq 0 \quad \therefore$ at $t = 3$ second acceleration is maximum.

(d) Distance covered from 0-3 sec

$$s = 9\text{ m} \qquad \text{[from (}ii\text{) in (}b\text{) part]}$$

For 3 to 6 seconds

$$v(t) = -(t - 3)(6 - t)$$

$$\frac{ds}{dt} = (t - 3)(t - 6)$$

$$ds = (t^2 - 9t + 18)\,dt$$

Integrating both sides from 3 s – 6 s

$$s_2 = \int_3^6 (t^2 - 9t + 18)\,dt = \left[\frac{t^3}{3} - \frac{9}{2}t^2 + 18t\right]_3^6$$

$$s_2 = \frac{(6)^3}{3} - \frac{9}{2}(6)^2 + 18 \times 6 - \left[\frac{(3)^3}{3} - \frac{9}{2}(3)^2 + 18 \times 3\right]$$

$$= \frac{6 \times 6 \times 6}{3} - \frac{9 \times 6 \times 6}{2} + 108 - \frac{3 \times 3 \times 3}{3} + \frac{9 \times 3 \times 3}{2} - 54$$

$$= 72 - 162 + 108 - 9 + \frac{81}{2} - 54$$

$$= 180 - 162 - 63 + 40.5 = 18 - 22.5$$

$$s_2 = -4.5 \text{ m}$$

s_2 distance is downward $\quad \therefore s_2 = -4.5$ m

So net distance = 9 – 4.5 = 4.5 m.

Height climb up in three cycles = 4.5 × 3 = 13.5 m

Now remaining height = 20 – 13.5 = 6.5 m

Remaining height to climb is 6.5 m but monkey can climb 9 m up without slip. So in 4th cycle it will slip as it reaches on the top of pole.

Net number of cycle to climb 20 m high pole is 4.

Q3.26. A man is standing on the top of building 100 m high. He throws two balls vertically, one at $t = 0$ and other after a time interval (less than 2 seconds). The later ball is thrown at a velocity of half the first. The vertical gap between first and second ball is + 15 m at $t = 2$ s. The gap is found to remain constant. Calculate the velocity with which balls were thrown and the exact time interval between their throw.

Ans. Let the speed of ball 1 = $u_1 = 2u$ m/s

Then the speed of ball 2 = $u_2 = u$ m/s

Let the height covered by ball 1 before coming to rest = h_1

Let the height covered by ball 2 before coming to rest = h_2

$$\because \quad v^2 = u^2 + 2gh$$

At top their velocities becomes zero

$$\therefore \quad v^2 = 2gh \quad \text{or} \quad h = \frac{u^2}{2g}$$

$$\Rightarrow \quad h_1 = \frac{u_1^2}{2g} = \frac{4u^2}{2g} \quad \text{and} \quad h_2 = \frac{u^2}{2g}$$

According to question $h_1 - h_2 = 15$ m (given)

$$\therefore \quad \frac{4u^2}{2g} - \frac{u^2}{2g} = 15 \quad \text{(given)}$$

$$\frac{u^2}{2g}[4 - 1] = 15$$

$$u^2 = \frac{15 \times 2 \times 10}{3} \quad \Rightarrow \quad u = 10 \text{ m/s}$$

$$h_1 = \frac{4 \times 10 \times 10}{2 \times 10} = 20 \text{ m} \qquad h_2 = \frac{10 \times 10}{2 \times 10} = 5 \text{ m}$$

For 1st ball $\quad v_1 = u_1 + gt$

$$0 = 20 - 10t_1 \quad \Rightarrow \quad t_1 = 2 \text{ sec}$$

For ball 2 $\quad v_2 = u_2 + gt_2$

$$0 = 10 - 10t_2 \quad \Rightarrow \quad t_2 = 1 \text{ sec}$$

Velocities of ball 1 and 2 are 20 m/s and 10 m/s.

Exact time intervals between 2 balls = $t_1 - t_2 = (2 - 1) = 1$ second.

❑❑❑

4 Motion in a Plane

MULTIPLE CHOICE QUESTIONS-I

Q4.1. The angle between $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j}$ is

(*a*) 45° (*b*) 90° (*c*) – 45° (*d*) 180°

Ans. (*b*): Given

$$\vec{A} = \hat{i} + \hat{j}$$

$$\vec{B} = \hat{i} - \hat{j}$$

$$\vec{A}.\vec{B} = |A|\,|B| \cos\theta = \sqrt{1^2+1^2}\,\sqrt{1^2+1^2}\cos\theta$$

$$(\hat{i}+\hat{j}).(\hat{i}-\hat{j}) = \sqrt{2}\,\sqrt{2}\cos\theta$$

$$1 - 1 = 2\cos\theta$$

$$\cos\theta = \frac{0}{2} = \cos 90°$$

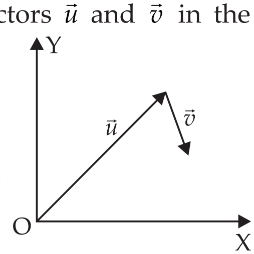
$\theta = 90°$. Hence, verifies the option (*b*).

Q4.2. Which one of the following statements is true?

(*a*) A scalar quantity is the one that is conserved in a process.
(*b*) A scalar quantity is the one that can never take negative values.
(*c*) A scalar quantity is the one that does not vary one point to another in space.
(*d*) A scalar quantity has the same value for observers with different orientations of the axes.

Ans. (*d*): A scalar quantity does not depend on direction so it does not change after orientation of axis. So verifies the option (*d*).

Q4.3. Figure shows the orientation of two vectors $\vec{u}$ and $\vec{v}$ in the X-Y plane.



If
$$\vec{u} = a\hat{i} + b\hat{j} \text{ and}$$
$$\vec{v} = p\hat{i} + q\hat{j}$$

which of the following is correct?

(*a*) *a* and *p* are positive while *b* and *q* are negative.
(*b*) *a*, *p* and *b* are positive while *q* is negative.
(*c*) *a*, *q* and *b* are positive while *p* is negative.
(*d*) *a*, *b*, *p* and *q* are all positive.

Main concept used: Sign of *a*, *b*, *p* and *q* are sign of axis of their resolving components in the X-Y direction.

Ans. (*b*): Components along X and Y axis of vector $\vec{u}$ are both + X and Y direction, so *a*, *b* are positive.

Now if we resolve $\vec{v}$ its X component is in +ve X direction but Y component will be in negative Y direction.

Hence, *a*, *b* and *p* are positive but *q* is negative. Verifies option (*b*).

Q4.4. The component of vector r along X-axis will have maximum value if:

(*a*) $\vec{r}$ is along positive Y-axis.
(*b*) $\vec{r}$ is along positive X-axis.
(*c*) $\vec{r}$ makes an angle of 45° with X-axis.
(*d*) $\vec{r}$ is along negative Y-axis.

Main concept used: On resolving the vector, values of cos θ or sin θ is always less than one, so components have smaller value.

Ans. (*b*): As the vector $\vec{r}$ has maximum value along positive X-axis. So it component along Y-axis is zero or any other value of its component $r \cos \theta$ will be $|\vec{r}| > |\vec{r} \cos \theta|$.

Hence, given vector $\vec{r}$ is along positive X-axis.

Q4.5. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be:

(*a*) 60 m (*b*) 71 m (*c*) 100 m (*d*) 141 m

Ans. (*c*): Projectile is fired θ = 15°, R = 50 m

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$50 = \frac{u^2 \sin 2 \times 15°}{g} \Rightarrow u^2 = 50g \times 2$$

$$u^2 = 100g$$

Now $\theta = 45° \quad u^2 = 100g$

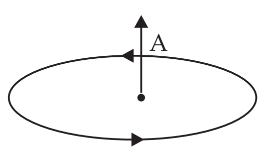
$$\therefore \quad R = \frac{u^2 \sin 2\theta}{g} = \frac{100 \cancel{g} \times \sin 2 \times 45°}{\cancel{g}}$$

R = 100 m. So verifies option (*c*).

Q4.6. Consider the quantities, pressure, power, energy impulse, gravitational potential, electric charge, temperature, area. Out of these, the only vector quantities are:

(*a*) Impulse, pressure and area (*b*) Impulse and area
(*c*) Area and gravitational constant (*d*) Impulse and pressure

Ans. (*b*): $I = F dt = \frac{dp}{dt}.dt = dp$ which is vector.



When current or a charge passing through a path its area of path is also vector, direction of which can be find out by Right hand thumb rule. Hence, verifies the option (*b*).

Q4.7. In a two dimensional motion, instantaneous speed v_0 is positive, constant. Then which of the following are necessarily true?

(*a*) The average velocity is not zero at any time.
(*b*) Average acceleration must always vanish.
(*c*) Displacements in equal time intervals are equal.
(*d*) Equal path lengths are traversed in equal intervals.

Ans. (*d*): As speed is scalar and in option *a*, *b* and *c* are vectors, in (*d*) path length which is scalar is right option.

Q4.8. In two dimensional motion, instantaneous speed v_0 is positive and constant. Then which of the following are necessarily true?

(*a*) The acceleration of a particle is zero.

(*b*) The acceleration of the particle is bounded.

(*c*) The acceleration of the particle is necessarily in the plane of motion.

(*d*) The particle must be undergoing a uniform circular motion.

Ans. (*c*): As we knew that change in acceleration and velocity is in the direction of Force (F) by $\vec{F} = m\vec{a}$ and change in velocity is zero so rate of change of acceleration is again zero and will be in the same planes as that of velocity.

Q4.9. Three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ add up to zero. Find which is false.

(*a*) $(\vec{A} \times \vec{B}) \times \vec{C}$ is not zero unless $\vec{B}$ and $\vec{C}$ are parallel.

(*b*) $(\vec{A} \times \vec{B}).\vec{C}$ is not zero unless $\vec{B}, \vec{C}$ are parallel.

(*c*) If $\vec{A}, \vec{B}, \vec{C}$ define a plane, $(\vec{A} \times \vec{B}) \times \vec{C}$ is in that plane.

(*d*) $(\vec{A} \times \vec{B}).\vec{C} = |\vec{A}||\vec{B}||\vec{C}| \rightarrow C^2 = A^2 + B^2$

Ans. (*c*): $\vec{A} + \vec{B} + \vec{C} = 0$ (Given)

So $\vec{A}, \vec{B}$ and $\vec{C}$ are in a plane and can be represented by the sides of Δ taken in order.

(*a*)
$$\vec{B} \times (\vec{A} + \vec{B} + \vec{C}) = \vec{B} \times 0 = 0$$
$$\vec{B} \times \vec{A} + \vec{B} \times \vec{B} + \vec{B} \times \vec{C} = 0$$
$$\vec{B} \times \vec{A} + 0 + \vec{B} \times \vec{C} = 0$$
$$\vec{B} \times \vec{A} = -\vec{B} \times \vec{C}$$
$$-\vec{A} \times = -\vec{B} \times \vec{C} \qquad ...(i)$$

or $$(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{B} \times \vec{C}) \times \vec{C}$$

$\vec{B} \times \vec{C}$ will be zero if $\vec{B}$ and $\vec{C}$ are parallel or antiparallel.

i.e., $(\vec{A} \times \vec{B}) \times \vec{C} = [BC \sin 0°] \times \vec{C}$.

$$(\vec{A} \times \vec{B}) \times \vec{C} = 0 \text{ only if } \vec{B} \parallel \vec{C}.$$

Hence, option (*a*) is false verified.

(*b*)
$$(\vec{A} \times \vec{B}) = \vec{B} \times \vec{C} \qquad [\text{from } (i)]$$
$$(\vec{A} \times \vec{B}).\vec{C} = (\vec{B} \times \vec{C}).\vec{C}$$

If $\vec{B} \parallel \vec{C}$ $\quad \vec{B} \times \vec{C} = BC \sin 0° = 0$

∴ $\quad (\vec{A} \times \vec{B}).\vec{C} = 0$ if $\vec{B} \parallel \vec{C}$

So option (*b*) is not verified.

(*c*)
$$(\vec{A} \times \vec{B}) = \vec{X}$$

The direction of $\vec{X}$ is perpendicular to both planes containing A and B.

$$(\vec{A} \times \vec{B}) \times \vec{C} = \vec{X} \times \vec{C} = \vec{Y}$$

The direction of $\vec{Y}$ is perpendicular to the plane of $\vec{X}$ and $\vec{C}$ which again become in the plane of $\vec{A}, \vec{B}, \vec{C}$ but perpendicular to the plane of $\vec{X}$ and $\vec{C}$. Hence, option (*c*) is also verified.

(*d*) $|A|^2 + |B|^2 = |C|^2$ given

It shows that angle between $\vec{A}$ and $\vec{B}$ is 90°

$$\therefore \quad (\vec{A} \times \vec{B}).\vec{C} = \left[|A||B| \sin 90°\right].\vec{C} = |A||B|.\vec{C}$$
$$= |A||B||C| \cos\theta \neq |A||B||C|$$
$$(\vec{A} \times \vec{B}).\vec{C} = |A||B||C| \cos\theta$$

does not verified option (*d*).

Q4.10. It is found that $|\vec{A} + \vec{B}| = |\vec{A}|$. This is necessarily implies,

(*a*) $\vec{B} = 0$
(*b*) $\vec{A}, \vec{B}$ are anti-parallel
(*c*) $\vec{A}, \vec{B}$ are perpendicular
(*d*) $\vec{A} . \vec{B} \leq 0$

Ans. (*a*):
$$|\vec{A} + \vec{B}|^2 = |\vec{A}|^2$$
$$|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}| \cos\theta = |\vec{A}|^2$$
$$|\vec{B}|^2 + 2|\vec{A}||\vec{B}| \cos\theta = 0$$
$$|\vec{B}|\left[|\vec{B}| + 2|\vec{A}| \cos\theta\right] = 0$$

$|\vec{B}| = 0$ verifies option (*a*).

or
$$|\vec{B}| + 2|\vec{A}| \cos\theta = 0$$
$$\boxed{\cos\theta = \frac{-|\vec{B}|}{2|\vec{A}|}}$$

(*b*) If A and B are antiparallel then $\theta = 180°$

$$\cos 180° = \frac{-|\vec{B}|}{2|\vec{A}|} \Rightarrow -1 = \frac{-|\vec{B}|}{2|\vec{A}|} \Rightarrow |\vec{B}| = 2|\vec{A}|$$

or A and B are antiparallel if $|\vec{B}| = 2|\vec{A}|$ does not verified answer (*b*) for all condition.

(*c*) If $\vec{A}, \vec{B}$ are perpendicular then $\theta = 90°$

$$\cos\theta = \frac{-|\vec{B}|}{2|\vec{A}|}$$
$$\cos 90° = \frac{-|\vec{B}|}{2|\vec{A}|} \quad \text{or} \quad 0 = \frac{-|\vec{B}|}{2|\vec{A}|}$$
$$\Rightarrow \quad |\vec{B}| = 0$$

$\vec{A}, \vec{B}$ are perpendicular if $|\vec{B}| = 0$.

(*d*)
$$\vec{A}.\vec{B} \leq 0$$
$$|\vec{A}||\vec{B}| \cos\theta \leq 0$$

$|\vec{A}|, |\vec{B}|$ are +ve always cos θ is +ve from 0° → 90° but between 90° to 180° cos θ is negative

∴ $\vec{A}.\vec{B} \le 0$ does not true always.

MULTIPLE CHOICE QUESTIONS-II

Q4.11. Two particles are projected in air with speed v_0 at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices

(*a*) angle of projection: $\theta_1 > \theta_2$ (*b*) time of flight: $T_1 > T_2$

(*c*) horizontal range: $R_1 > R_2$ (*d*) total energy: $U_1 < U_2$

Ans. (*a*, *b*): Maximum height H of projectile $\boxed{H = \dfrac{u^2 \sin^2 \theta}{2g}}$

Here, Projectile 1 Projectile 2

$$\theta_1 = \theta_1 \qquad \theta_2 = \theta_2$$

$$u_1 = v_0 \qquad u_2 = v_0$$

$$H_1 > H_2 \qquad \text{(Given)}$$

$$\frac{v_0^2 \sin^2 \theta_1}{2g} > \frac{v_0^2 \sin^2 \theta_2}{2g}$$

$$\sin^2 \theta_1 > \sin^2 \theta_2$$

$$\sin^2 \theta_1 - \sin^2 \theta_2 > 0$$

$$(\sin \theta_1 - \sin \theta_2)(\sin \theta_1 + \sin \theta_2) > 0$$

$$\sin \theta_1 + \sin \theta_2 > 0 \quad \text{or} \quad \sin \theta_1 - \sin \theta_2 > 0$$

⇒ θ_1 and θ_2 lies between 0° to 90° i.e., acute as given; $\sin \theta_1 > \sin \theta_2$

$$\theta_1 > \theta_2 \qquad ...(i)$$

(*a*) $\boxed{T = \dfrac{2u \sin \theta}{g}}$

$$T_1 = \frac{2v_0 \sin \theta_1}{g} \quad \text{and} \quad T_2 = \frac{2v_0 \sin \theta_2}{g}$$

$$\frac{T_1}{T_2} = \frac{\sin \theta_1}{\sin \theta_2} \quad \Rightarrow \quad T_1 \sin \theta_2 = T_2 \sin \theta_1$$

$$\because \quad \sin \theta_1 > \sin \theta_2$$

$$\therefore \quad T_1 > T_2$$

Hence, verified (*a*, *b*) options.

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

$$\frac{R_1}{R_2} = \frac{\frac{v_0^2 \sin 2\theta_1}{g}}{\frac{v_0^2 \sin 2\theta_2}{g}} = \frac{\sin 2\theta_1}{\sin 2\theta_2}$$

$$= \frac{2\sin\theta_1 \cos\theta_1}{2\sin\theta_2 \cos\theta_2} = \frac{\sin\theta_1 \cos\theta_1}{\sin\theta_2 \cos\theta_2}$$

$$R_1 \sin\theta_2 \cos\theta_2 = R_2 \sin\theta_1 \cos\theta_1$$

$\because \quad \theta_1 > \theta_2$ and

$$\sin\theta_1 > \sin\theta_2 \quad \text{and} \quad \cos\theta_1 < \cos\theta_2$$

$$R_1 \sin\theta_2 < R_2 \sin\theta_1$$

$$\sin\theta_2 < \sin\theta_1 \quad \text{Proved above.}$$

$$R_1 \sin\theta_2 + \sin\theta_2 < R_2 \sin\theta_1 + \sin\theta_1$$

$$\sin\theta_2 (R_1 + 1) < \sin\theta_1 (R_2 + 1)$$

$$\frac{\sin\theta_2}{\sin\theta_1}(R_1 + 1) < (R_2 + 1) \qquad \because \frac{\sin\theta_2}{\sin\theta_1} < 1$$

$\therefore \quad R_1 + 1 < R_2 + 1$

$$R_1 < R_2$$

Does not verifies the option (*c*).

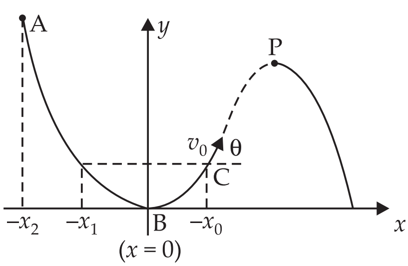
For option (*d*): as $H_1 > H_2$

We know at highest point the total energy is equal to the P.E. If $m_1 = m_2 = m$ then

$$PE_1 > PE_2 \quad \text{or} \quad U_1 > U_2$$

So does not verifies option (*d*).

Q4.12. A particle slides down a frictionless parabolic ($y = x^2$) track (A – B – C) starting from rest at point A as in figure. Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then



(*a*) KE at P = KE at B

(*b*) height at P = height at A

(*c*) total energy at P = total energy at A

(*d*) time of travel from A to B = time of travel from B to P.

Main concept used: As path A, B, C is frictionless with air and plane, so total energy at any point of journey remains same.

Ans. (*c*) (*i*) For option (*a*): At P projectile is at its highest point so velocity at P is zero or KE = 0 but at B it is lowest point where the KE is maximum i.e., so KE at B ≠ KE at P does not verifies option (*a*).

(*ii*) For option (*b*): At A $v_A = 0$ (given)

So $$U_A = PE_A \quad ...(i)$$

Only P is again highest point but has only horizontal component of velocity. Vertical component of velocity is zero.

So $$U_P = PE_P + KE_P \quad ...(ii)$$

∴ As at P particle has PE and KE.

So $$PE_P < PE_A.$$

Hence, the height at P < height at A.

Does not verifies the option (*b*).

(*iii*) For option (*c*): By the law of conservation of energy.

Total energy at P = Total energy at A

Verifies the option (*c*).

(*iv*) For option (*d*): As the path A to B > Path B to P

or height of A > height of P

Hence, time from A to B > time from B to P.

As time of flight is directly proportional to height.

So does not verifies the option (*d*).

Q4.13. Following are four different relations about displacement, velocity and acceleration for the motion of a particle in general. Choose the incorrect one(s).

(*a*) $\vec{v}_{av} = \frac{1}{2}\left[\vec{v}(t_1) + \vec{v}(t_2)\right]$ (*b*) $\vec{v}_{av} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$

(*c*) $\vec{r} = \frac{1}{2}\left[\vec{v}(t_2) - \vec{v}(t_1)\right] \div (t_2 - t_1)$ (*d*) $\vec{a}_{av} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{(t_2 - t_1)}$

Ans. (*a*, *c*): For option (*a*): (*i*) If acceleration is uniform then

$\vec{v}_{av} = \frac{\vec{u} + \vec{v}}{2}$ or $\vec{v}_{av} = \frac{1}{2}\left[\vec{v}(t_2) - \vec{v}(t_1)\right]$ is correct only when acceleration is uniform, but it not given.

So option (*a*) is verified that given relation is incorrect.

(*ii*) For option (*b*): Rate of change in displacement is velocity

or $$\vec{v}_{av} = \frac{\text{final displacement} - \text{initial displacement}}{\text{change in time}}$$

$$\vec{v}_{av} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{(t_2 - t_1)}$$ is correct. So does not verified option (*b*).

(*iii*) For option (*c*): In given relation displacement is half of rate of change of velocity which not possible dimensions in LHS is $[M^0L^1T^0]$ but in RHS. $[M^0L^1T^{-2}]$ are not equal. So relation is incorrect. So verifies option (*c*).

(*iv*) For option (*d*): Rate of change of velocity with time is called acceleration, so

$$a_{av} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$$ is correct and does not verifies option (*d*).

Q4.14. For a particle performing uniform circular motion, choose the correct statement(s) from the following:

(*a*) Magnitude of particle velocity (speed) remains constant.

(*b*) Particle velocity remains directed perpendicular to radius vector.

(*c*) Direction of acceleration keeps changing as particle moves.

(*d*) Angular momentum is constant in magnitude but direction keeps changing.

Ans. (*a, b, c*): (*i*) For option (*a*): In uniform circular motion speed is always constant, so verified option (*a*).

(*ii*) For option (*b*): The direction of velocity is always tangentially and towards motion, if a string breaks up the particle moves tangentially. So verifies the option (*b*).

(*iii*) For option (*c*): The direction of acceleration $\left(\frac{v^2}{r}\right)$ is always along the direction of force applied by Newton's IInd law of motion. The force is applied on the string to rotate the particle in circular motion, hence the direction of acceleration is along the string towards the centre of circular path. As string is moving in circular plane with particle so the direction of acceleration changes always and verifies the option (*c*).

(*iv*) For option (*d*): The angular momentum L = mvr constant as m, v and r does not change in uniform circular motion. The direction of L is perpendicular to the plane containing the vectors $r \times p$ or $r \times v$ by right hand thumb rule. So direction of L does not changes, so does not verifies option (*d*).

Q4.15. For two vectors $\vec{A}$ and $\vec{B}$, $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$ is always true when

(*a*) $|\vec{A}| = |\vec{B}| \neq 0$

(*b*) $\vec{A} \perp \vec{B}$

(*c*) $|\vec{A}| = |\vec{B}| \neq 0$ and $\vec{A}$ and $\vec{B}$ are parallel or antiparallel

(*d*) when either $|\vec{A}|$ or $|\vec{B}|$ is zero.

Ans. (*b, d*): $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$ (given)

$$|\vec{A}+\vec{B}|^2 = |\vec{A}-\vec{B}|^2 \quad \text{(squaring both sides)}$$

$$|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta$$

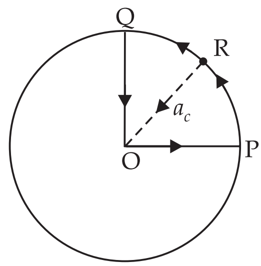
$$4|\vec{A}||\vec{B}|\cos\theta = 0$$

$\Rightarrow 4 \neq 0,\ |\vec{A}| = 0$ or $|\vec{B}| = 0$ or $\cos\theta = 0 \Rightarrow \cos\theta = \cos 90° \Rightarrow \theta = 90°$

$\Rightarrow |A| = |B| = 0$ does not verifies the option (*a*) and (*c*) and verifies the option (*b*) and (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q4.16. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path OPRQO as shown in figure. If he maintains constant speed of 10 ms^{-1}, what is his acceleration at point R in magnitude and direction?

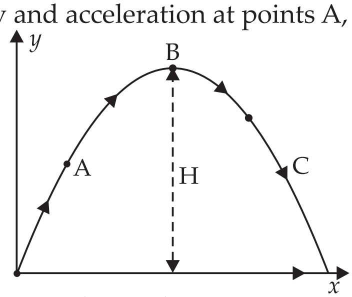


Ans. The path of cyclist at R is circular of constant radius 1 km and he is moving with constant speed 10 m/s. So his motion is uniform circular motion at R.

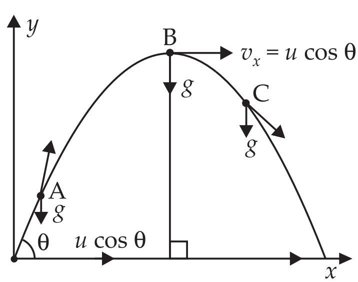
Hence, the R = 1000 m, v = 10 m/s

$$\therefore \qquad a_C = \frac{v^2}{R} = \frac{10 \times 10}{1000} = \frac{1}{10} = 0.1 \text{ m/s}^2$$ towards R to O the centre of circular path.

Q4.17. A particle is projected in air at some angle to the horizontal, moves along the parabola as shown in figure, where *x* and *y* indicate horizontal and vertical directions respectively. Shown in the diagram, direction of velocity and acceleration at points A, B and C.



Ans. Motion of projectile is always parabola or its part. Its velocity at any point of its path in always tangentially toward the direction of motion so velocities at points A, B and C are tangents as shown.



The point B is at its maximum height of trajectory. So the vertical component of B v_y = 0 and horizontal component is $u \cos\theta$.

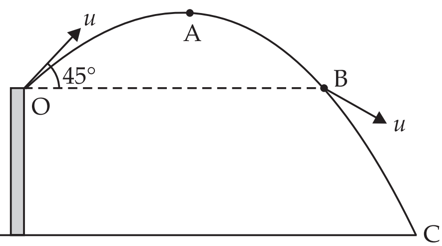
As the direction of acceleration is always in the direction of force acting on it. The gravitational force is acting on the body hence the direction of acceleration is always vertically **downward equal** to acceleration to gravity (g).

Q4.18. A ball is thrown from a roof-top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during its motion, does the ball have

(*a*) greatest speed (*b*) smallest speed

(*c*) greatest acceleration? Explain.

Ans. A ball is projected from O at an angle of 45° with horizontal. O to A body rises up so its KE (speed) decrease and height increases. From A to it's speed again increases as its height decreases equal to it's initial speed at O, because O and B are on same horizontal line.



From B to C, its height again decreases so it speed from B to C increases and become maximum at C v_y maximum. $v_x = u \cos 45° = \frac{u}{\sqrt{2}}$ m/s is always constant at any point.

Hence, (*a*) greatest speed of ball is at C as v_y maximum and $v_x = \frac{u}{\sqrt{2}}$

(*b*) smallest speed will be at A where maximum height and $v_y = 0$ has only horizontal speed of constant value $\frac{u}{\sqrt{2}}$.

(*c*) As the force acting on ball is only due to gravitational force downward and is constant so acceleration is also **constant downward** and always equal to 'g'.

Q4.19. A football is kicked into the air vertically upwards. What is its (*a*) acceleration, and (*b*) velocity at the highest point?

Ans. (*a*) As the motion of body is under gravity and no any external force acts on body, so the direction of acceleration is in the direction of force (gravitational). Hence, the direction of acceleration is always **towards the centre of earth i.e., downward**.

(*b*) As the ball is thrown vertically upward so its component of horizontal velocity become zero. At highest point the velocity of body $v_y = 0$. Hence, net velocity of body at highest point is zero.

Q4.20. $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three non-collinear, non co-planar vectors. What can you say about direction of $\vec{A} \times (\vec{B} \times \vec{C})$?

Ans. The direction of vector $(\vec{B} \times \vec{C})$ will be perpendicular to plane containing by vectors $\vec{B}$ and $\vec{C}$ by Right hand thumb or Right hand grip rule (RHGR).

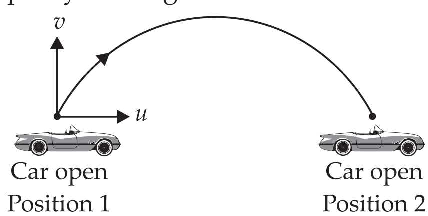
The direction of vector $\vec{A} \times (\vec{B} \times \vec{C})$ will be perpendicular to $\vec{A}$ and in plane containing $\vec{B}$ and $\vec{C}$ by RHGR.

SHORT ANSWER TYPE QUESTIONS

Q4.21. A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.

Ans. v = vertical velocity of ball given by boy

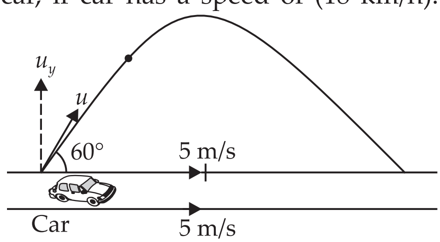
u = velocity of car which is equal to the horizontal velocity of ball.



As the ball has both vertical and horizontal components of velocities it's path will be parabolic as observed by a person standing on footpath.

Path of the ball as seen by boy sitting in the same car will be only vertically up-down under gravity and will catch up by boy if car is moving with constant velocity.

Q4.22. A boy throws a ball in air at 60° to the horizontal along a road with a speed of 10 m/s (36 km/h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18 km/h). Give explanation to support your diagram.



Ans. $u = 36 \text{ km/h} = \dfrac{36 \times 5}{18} \text{ ms}^{-1}$

$= 10 \text{ m/s}$

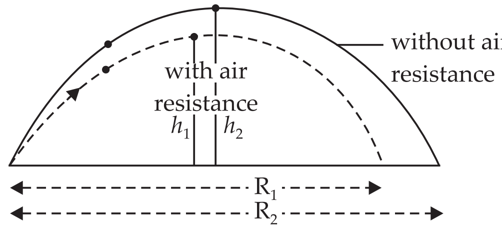
$u_x = u \cos 60° = 10 \times \dfrac{1}{2} = 5 \text{ m/s}$

Speed of car in the direction of motion of ball $= \left(18 \times \dfrac{5}{18}\right) = 5$ m/s.

If the ball is thrown by boy when car just passes him, then the ball and car (boy) travel (covers) equal horizontal distances as their horizontal speeds are same. But due to vertical component of velocity $u_y = u \cos 30° = \dfrac{10\sqrt{3}}{2}$ m/s $= 5\sqrt{3}$ m/s ball rises up and then come back. The motion of ball by boy sitting in car will be simple vertically up-down motion.

Q4.23. In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? Sketch such a trajectory and explain why you have drawn it that way.

Ans. When air resistance acts on projectile then its vertical and horizontal both velocity will decrease due to air resistance. Hence its maximum height h_1 becomes smaller than h_2 when there is no force of friction (resistance) of air. By formula $R = \dfrac{u^2}{g}\sin 2\theta$ and $H_{max} = \dfrac{u^2 \sin^2\theta}{2g}$.



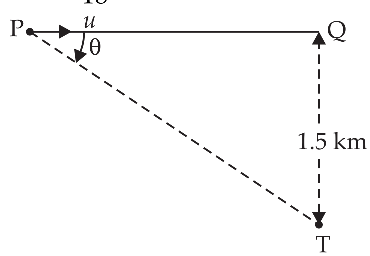
$$\therefore \qquad h_1 < h_2 \quad \text{and} \quad R_1 < R_2$$

But time of flight for both will remain same as the body in case II (with air resistance) $h_1 < h_2$ takes smaller time to rise.

Q4.24. A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/h. At what angle of sight (w.r.t horizontal) when the target is seen, should the pilot drop the bomb in order to attack the target?

Main concept used: $u = 720 \text{ km/h} = 720 \times \dfrac{5}{18} \text{ m/s} = 200 \text{ m/s}$

Ans. Let pilot drops the bomb t sec before the point Q vertically up the target T.



The horizontal velocity of bomb will be equal to the velocity of fighter plane, but vertical component of it is zero. So in time t bomb must cover the vertical distance TQ as free fall with initial velocity zero.

$u = 0$ $\qquad$ $TQ = h = 1.5 \text{ km} = 1500 \text{ m}$ $\qquad$ $g = +10 \text{ m/s}^2$

$$h = ut \quad - gt$$

$$1500 = 0 \times t + \frac{1}{2} \times 10t^2$$

$$t = \sqrt{\frac{1500}{5}} = \sqrt{300} = 10\sqrt{3} \text{ second.}$$

$\therefore$ Distance covered by plane or bomb $PQ = ut$

$$PQ = 200 \times 10\sqrt{3} = 2000\sqrt{3} \text{ m}$$

$$\tan\theta = \frac{TQ}{PQ} = \frac{1500}{2000\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{20 \times 3} = \frac{\sqrt{3}}{4}$$

$$\tan\theta = \frac{1.732}{4} = 0.433 = \tan 23°42'$$

$$\theta = 23°\ 42'.$$

Q4.25. (*a*) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of the earth due to earth's rotation (period 1 day). What is the acceleration of object on the surface of earth (at equator) towards its centre? What is it at latitude θ? How does these accelerations compare with $g = 9.8\ \text{m/s}^2$?
(*b*) Earth also moves in circular orbit around sun once every year with an orbital radius of 1.5×10^{11} m. What is the acceleration of earth (or any object on the surface of the earth) towards the centre of the sun? How does this acceleration compare with $g = 9.8\ \text{m/s}^2$.

$$\left[\text{Hint: acceleration } \frac{V^2}{R} = \frac{4\pi^2 R}{T^2}\right]$$

Ans. (*a*) When an object revolve in circular path of radius R with angular speed ω then it's angular acceleration towards the centre of path is

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

$$\omega = \frac{2\pi}{T} \qquad R = 6400000\ \text{m} = 6.4 \times 10^6\ \text{m}$$
$$T = 24 \times 3600 \text{ second.}$$

Acceleration of the person on surface of earth towards centre of earth

$$a_c = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R = \frac{4\pi^2}{T^2} R$$

$$a_c = \frac{4 \times 22 \times 22 \times 6.4 \times 1000000}{7 \times 7 \times 24 \times 24 \times 3600 \times 3600}$$

$$= \frac{4 \times 22 \times 22 \times 64 \times 100}{7 \times 7 \times 24 \times 24 \times 36 \times 36 \times 10} = \frac{1210}{49 \times 9 \times 81}$$

$$a_c = \frac{1210}{35721} = 0.034\ \text{m/s}^2$$

At equator latitude 0°

$$\frac{a_c}{g} = \frac{0.034}{9.8} = \frac{1}{288}$$

(*b*)For the acceleration earth revolving around the sun.

$$R = 1.5 \times 10^{11}\ \text{m} \quad \omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{T}$$

$$T = 365 \times 24 \times 3600 \text{ sec} = 3.15 \times 10^7 \text{ seconds}$$

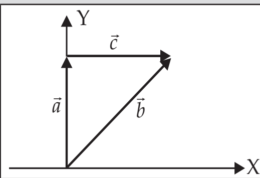
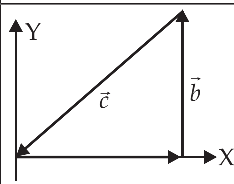
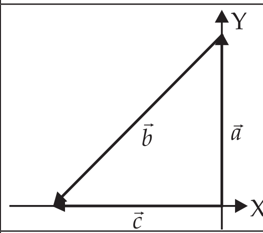
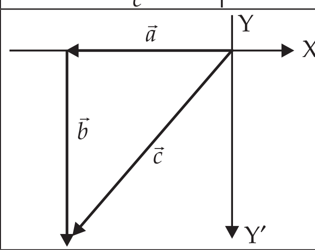
$$\therefore \qquad a = \omega^2 R = \frac{4\pi^2}{T^2} R$$

$$a_c = \frac{4 \times 3.14 \times 3.14 \times 1.5 \times 10^{11}}{3.15 \times 3.15 \times 10^7 \times 10^7} \cong 6 \times 10^{11-14}$$

$$a_c \cong 6 \times 10^{-3}\ \text{m/s}^2$$

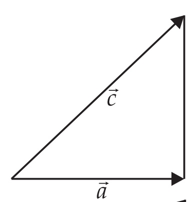
$$\frac{a_c}{g} = \frac{6 \times 10^{-3}}{9.8} \cong \frac{1}{1633}.$$

Q4.26. Given below in column I are the relations between vectors $\vec{a}, \vec{b}$ and $\vec{c}$ and in column II are the orientations of $\vec{a}, \vec{b}$ and $\vec{c}$ in the X-Y plane. Match the relation in column I to correct orientations in column II.

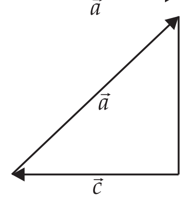
Column I		Column II	
(*a*)	$\vec{a} + \vec{b} = \vec{c}$	(*i*)	
(*b*)	$(\vec{a} - \vec{c}) = \vec{b}$	(*ii*)	
(*c*)	$(\vec{b} - \vec{a}) = \vec{c}$	(*iii*)	
(*d*)	$\vec{a} + \vec{b} + \vec{c} = 0$	(*iv*)	

Ans. Consider $\vec{a} + \vec{b} = \vec{c}$

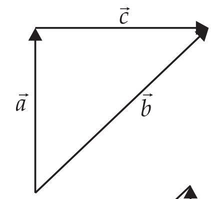
(*a*) $\vec{a} + \vec{b} = \vec{c}$ matches with figure (*iv*)



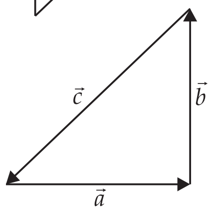
(*b*) $\vec{a} - \vec{c} = \vec{b}$ or $\vec{c} + \vec{b} = \vec{a}$ matches with figure (*iii*)



(*c*) $\vec{b} - \vec{a} = \vec{c}$ or $b = \vec{a} +$ matches with figure (*i*)



(*d*) $\vec{b} + \vec{a} + \vec{c} = 0$ matches with figure (*ii*)



Q4.27. If $|\vec{A}| = 2$ and $|\vec{B}| = 4,$ then match the relations in column I with the angle θ between A and B in column II.

Column I	**Column II**
(*a*) $\vec{A}.\vec{B} = 0$	(*i*) $\theta = 0°$
(*b*) $\vec{A}.\vec{B} = +8$	(*ii*) $\theta = 90°$
(*c*) $\vec{A}.\vec{B} = 4$	(*iii*) $\theta = 180°$
(*d*) $\vec{A}.\vec{B} = -8$	(*iv*) $\theta = 60°$

Ans. (*a*) $\vec{A}.\vec{B} = |\vec{A}||\vec{B}| \cos\theta = 0$

$$2 \times 4 \cos\theta = 0$$

$$\cos\theta = \cos 90°$$

∴ Option of (*a*) of column I matches with option (*ii*) in column II.

(*b*) $\vec{A}.\vec{B} = 8$ (given)

$$|\vec{A}||\vec{B}| \cos\theta = 8$$

$$2 \times 4 \cos\theta = 8$$

∴ $\cos\theta = 1 = \cos 0°$

Hence, $\theta = 0°$

So option (*b*) of column I matches with option (*i*) of column II.

(*c*) $\vec{A}.\vec{B} = |\vec{A}||\vec{B}| \cos\theta = 2 \times 4 \cos\theta$

$$= 8 \cos\theta$$

$8 \cos\theta = 4$ as in option (*c*) of column I

$$\cos\theta = \frac{4}{8} = \frac{1}{2} \Rightarrow \cos\theta = \cos 60° \Rightarrow \theta = 60°$$

Hence, option (*c*) of column I matches with option (*iv*) of column II.

(*d*) $\vec{A}.\vec{B} = -8$

$$|\vec{A}||\vec{B}| \cos\theta = -8$$

$$2 \times 4 \cos\theta = -8$$

$$\cos\theta = -1 \Rightarrow \cos\theta = \cos 180° \Rightarrow \theta = 180°$$

Hence, option (*d*) of column I matches with option (*iii*) of column II.

Q4.28. If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, then match the relations in column I with the angle θ between A and B in column II.

Column I	Column II
(*a*) $|\vec{A}\times\vec{B}| = 0$	(*i*) $\theta = 30°$
(*b*) $|\vec{A}\times\vec{B}| = 8$	(*ii*) $\theta = 45°$
(*c*) $|\vec{A}\times\vec{B}| = 4$	(*iii*) $\theta = 90°$
(*d*) $|\vec{A}\times\vec{B}| = 4\sqrt{2}$	(*iv*) $\theta = 0°$

Ans. Given $|\vec{A}| = 2$ and $|\vec{B}| = 4$

(*a*) $|\vec{A}\times\vec{B}| = 0$ (given)

$$|\vec{A}|\,|\vec{B}| \sin\theta = 0$$
$$2\times 4 \sin\theta = 0$$
$$\sin\theta = \sin 0°$$
$$\theta = 0°$$

Hence, option (*a*) matches with option (*iv*).

(*b*) $|\vec{A}\times\vec{B}| = 8$ (given)

$$|\vec{A}|\,|\vec{B}| \sin\theta = 8$$
$$2\times 4 \sin\theta = 8$$

$\therefore$ $\sin\theta = 1 \Rightarrow \sin\theta = \sin 90° \Rightarrow \theta = 90°$

Hence, option (*b*) of column I matches with option (*iii*) of column II.

(*c*) $|\vec{A}\times\vec{B}| = 4$ (given)

$$|\vec{A}|\,|\vec{B}| \sin\theta = 4$$
$$2\times 4 \sin\theta = 4$$
$$\sin\theta = \frac{1}{2} \Rightarrow \sin\theta = \sin 30° \Rightarrow \theta = 30°$$

Hence, option (*c*) of column I matches with option (*i*) of column II.

(*d*) $|\vec{A}\times\vec{B}| = 4\sqrt{2}$ (given)

$$|\vec{A}|\,|\vec{B}| \sin\theta = 4\sqrt{2} \Rightarrow 2\times 4 \sin\theta = 4\sqrt{2}$$
$$\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \sin\theta = \sin 45°$$
$$\theta = 45°$$

Hence, option (*d*) of column I matches with (*ii*) of column II.

LONG ANSWER TYPE QUESTIONS

Q4.29. A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800 m from the foot of hill and can be moved on the ground at a speed of 2 m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g = 10$ m/s^2.

Ans. Speed of packets = 125 m/s

Height of hill = 500 m

To cross the hill by packet the vertical components of speed of packet (125 ms^{-1}) must be so that it can attain a height of 500 m and distance between hill and canon must be half the range of packet u_y

$$v^2 = u^2 + 2gh$$
$$0 = u_y^2 - 2gh$$
$$u_y = \sqrt{2gh} = \sqrt{2 \times 10 \times 500} = \sqrt{10000}$$
$$\boxed{u_y = 100 \text{ m/s}}$$
$$u^2 = u_x^2 + u_y^2$$
$$(125)^2 = u_x^2 + 100^2 \Rightarrow u_x^2 = 125^2 - 100^2$$
$$u_x^2 = (125 - 100)(125 + 100) = 25 \times 225$$
$$u_x = 5 \times 15 \Rightarrow \boxed{u_x = 75 \text{ m/s}}$$

Vertical motion of packet

$$v_y = u_y + gt$$
$$0 = 100 - 10t$$
$$t = 10 \text{ sec.}$$

$\therefore$ Total time of $\frac{1}{2}$ flight = 10 sec

So the canon must be at $\frac{1}{2}$ the range = horizontal distance in 10 sec

$$= u_x \times 10 = 75 \times 10 \text{ m} = 750 \text{ m}$$

Hence, the distance between hill and canon = 750 m

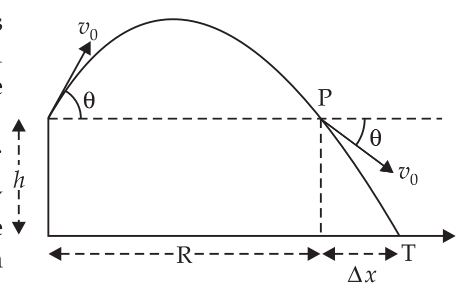
So can must move toward hill = 800 – 750 = 50 m

Time taken to move canon in 50 m = $\frac{\text{distance}}{\text{speed}} = \frac{50}{2} = 25$ sec

Hence, the total time taken by packet from 800 m away from hill to reach other side = 25 s + 10 s + 10 s = 45 seconds.

Q4.30. A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$.

If a target farther away by distance Δx (beyond R) has to be hit with the same gun as shown in figure here. Show that it could be achieved by raising the gun to a height at least $h = \Delta x\left[1 + \frac{\Delta x}{R}\right]$.



Hint: This problem can be approached in two different ways:

(i) Refer to the diagram: target T is at horizontal distance $x = R + \Delta x$ and below point of projection $y = -h$.

(ii) From point P in the diagram: Projection at speed v_0 at an angle θ below horizontal with height h and horizontal range Δx.]

Main concept used: This problem can be solved in two different ways:

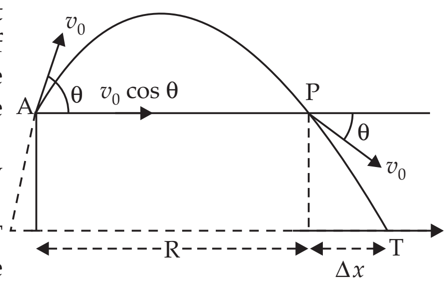
(i) The target is at a horizontal distance (R + Δx) and below the point of projection h metre below i.e., $y = -h$.

(ii) Motion of projectile after point P to T: Projection speed is at an angle (– θ) i.e., θ° below horizontal and vertical height covered it is (– h) and horizontal range Δx.

Ans. $\boxed{R = \frac{v_0^2}{g}}$ ...(i) that it is maximum range of projectile

∴ Angle of projection $\boxed{\theta = 45°}$

Let the gun is raised to a height h from the horizontal level of target T. So that the projectile can hit the target T. Total range of projectile must be = (R + Δx).



Horizontal component of velocity at A = $v_0 \cos\theta$

Motion of projectile from P to T as A and P are on same. So the magnitude of velocity will be same at A and P are equal by law of conservation of energy.

∴ But the direction of velocity will be below horizontal, so horizontal velocity at P = $-v_0 \cos\theta = v_x$

So vertical velocity at P = $v_y = -v_0 \sin\theta$

$$h = ut + \frac{1}{2}at^2$$

$$h = -v_0 \sin\theta\,(t) + \frac{1}{2}gt^2 \qquad ...(ii)$$

Consider horizontal motion from A to T = distance (R + Δx) = $v_0 \cos\theta . t$

$$\boxed{t = \frac{R + \Delta x}{v_0 \cos\theta}}$$

Substitute t in (ii)

$$h = -v_0 \sin\theta \left[\frac{R + \Delta x}{v_0 \cos\theta}\right] + \frac{1}{2}g\frac{(R + \Delta x)^2}{v_0^2 \cos^2\theta}$$

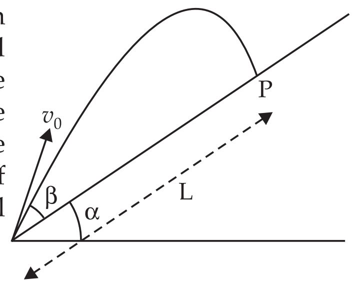
$$h = -\tan\theta\,(R + \Delta x) + \frac{1}{2}\left(\frac{g}{v_0^2}\right)\frac{(R + \Delta x)^2}{1/2} \qquad \theta = 45°$$

$$h = -(R + \Delta x) + \frac{1}{R}(R^2 + \Delta x^2 + 2R\Delta x) \qquad \left[\because \frac{g}{v_0^2} = \frac{1}{R}\right]$$

$$= -R - \Delta x + R + \frac{\Delta x^2}{R} + 2\Delta x = \Delta x + \frac{\Delta x^2}{R}$$

$$h = \Delta x\left[1 + \frac{\Delta x}{R}\right] \quad \text{Hence proved.}$$

Q4.31. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal as in figure



(*a*) Find an expression of range on the plane surface [distance on the plane from the point of projection at which particle will hit the surface.]

(*b*) Time of flight.

(*c*) β at which range will be maximum.

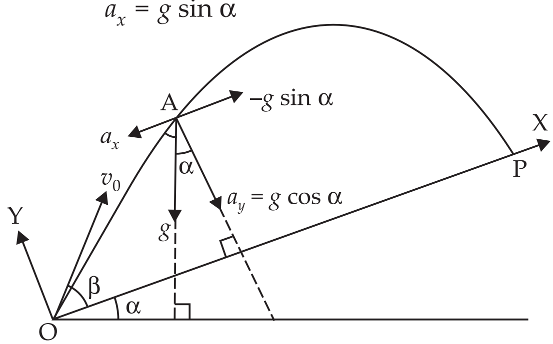
[*Hint:* This problem can be solved in two different ways:

(*i*) Point P at which particle hits the plane can be seen as intersection of its trajectory (parabola) and straight line. Remember particle is projected at an angle (α + β) w.r.t. horizontal.

(*ii*) We can take *x*-direction along the plane and *y*-direction perpendicular to the plane. In that case resolve *g* (acceleration due to gravity) in two different components, g_x along the plane and g_y perpendicular to the plane. Now the problem can be solved as two independent motions in *x* and *y* directions respectively with time as a common parameter.]

Ans. (*a*) Consider a new Cartesian coordinates in which X-axis is along inclined plane OP and OY-axis perpendicular to it as shown in figure. Consider the motion of projectile from OAP.

$$a_y = -g\cos\alpha$$
$$a_x = g\sin\alpha$$



At O and P $\quad y = 0$

$u_y = v_0 \sin\beta, \quad t = T$

(*b*) Motion of projectile along **New OY axis**.

$$s = ut + \frac{1}{2}gt^2$$

$s = 0, \quad u = u_y = v_0 \sin\beta \quad g = g_y = -g\cos\alpha \quad t = T$

$$0 = v_0 \sin\beta(T) + \frac{1}{2}(-g\cos\alpha)\,T^2$$

$$0 = v_0 \sin\beta(T) - \frac{g}{2}\cos\alpha\,(T)^2$$

$$T\left[v_0 \sin\beta - T\frac{g}{2}\cos\alpha\right] = 0$$

Either $T = 0$ or $v_0 \sin\beta - \frac{gT}{2}\cos\alpha = 0$

$$\frac{gT}{2}\cos\alpha = v_0 \sin\beta$$

∴ Time of flight from O to P is $\boxed{T = \dfrac{2v_0 \sin\beta}{g\cos\alpha}}$

At T = 0 projectile is at O and $T = \dfrac{2v_0 \sin\theta}{g}$ it is at P.

(*a*) Consider motion along OX axis

$x = L \quad u = v_0 \cos\beta, \quad a_x = -g\sin\alpha$

$$t = T = \frac{2v_0 \sin\beta}{g\cos\alpha}$$

$$\therefore \quad s = ut + \frac{1}{2}gt^2$$

$$L = v_0 \cos\beta(T) + \frac{1}{2}(-g\sin\alpha)T^2 = T\left[v_0\cos\beta - \frac{1}{2}g\sin\alpha.T\right]$$

$$= \frac{2v_0 \sin\beta}{g\cos\alpha}\left[v_0\cos\beta - \frac{1}{2}g\sin\alpha.\frac{2v_0 \sin\beta}{g\cos\alpha}\right]$$

$$= \frac{2v_0^2 \sin\beta}{g\cos^2\alpha}[\cos\beta.\cos\alpha - \sin\beta\sin\alpha]$$

$$\Rightarrow \boxed{L = \frac{2v_0^2 \sin\beta}{g\cos^2\alpha}\cos(\alpha+\beta)}$$

(*b*) Time of flight done before part (*a*).

(*c*) For L will be maximum or maximum range along new OX axis. From above relation of L, it will be maximum when $\sin\beta \cos(\alpha+\beta)$ is maximum as $\cos^2\alpha$ is constant angle of inclination of plane.

Consider $Z = \sin\beta\cos(\alpha+\beta)$

$= \sin\beta\,[\cos\alpha\cos\beta - \sin\alpha\sin\beta]$

$$= \frac{1}{2}\ [\cos\alpha\ 2\sin\beta\cos\beta - \sin\alpha\ 2\sin^2\beta]$$

$$= \frac{1}{2}\ [\cos\alpha.\sin 2\beta - \sin\alpha.(1 - \cos 2\beta)]$$

$$= \frac{1}{2}\ [\cos\alpha\sin 2\beta - \sin\alpha + \sin\alpha.\cos 2\beta]$$

$$= \frac{1}{2}\ [\cos\alpha\sin 2\beta + \sin\alpha\cos 2\beta - \sin\alpha]$$

$$Z = \frac{1}{2}\ [\sin(2\beta + \alpha) - \sin\alpha]$$

For Z maximum

$$\sin(2\beta + \alpha) = 1$$

$$\sin(2\beta + \alpha) = \sin 90°$$

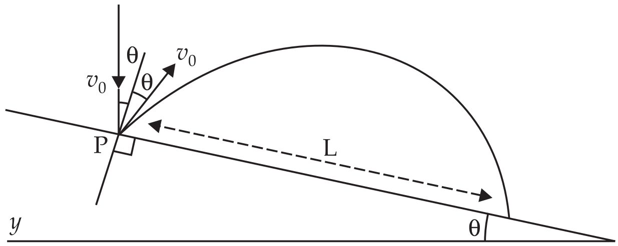
$$2\beta + \alpha = 90°$$

$$2\beta = 90° - \alpha$$

$$\beta = \frac{90°}{2} - \frac{\alpha}{2} = 45° - \frac{\alpha}{2}$$

$$\boxed{\beta = \frac{\pi}{4} - \frac{\alpha}{2}} \text{ radian.}$$

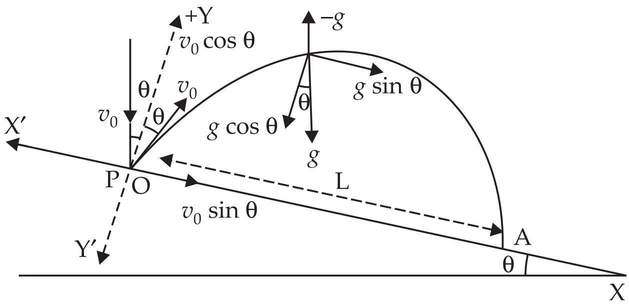
Q4.32. A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_0 and rebounds elastically as shown in figure. Find the distance along the plane where it will hit second time.



[*Hint:* (*i*) After rebound, particle still has speed V_o to start.
(*ii*) Work out angle particle speed has with horizontal after it rebounds.
(*iii*) Rest is similar to if particle is projected up the incline.]

Ans. Particle rebounces from P so it will be elastic collision. As it strike plane inclined at v_0 speed so speed of particle after rebounces will be v_0.

Again consider the new axis X′OX and YOY′ axis at P as origin 'O'.
The components of g and v_0 in new OX and OY axis are:



$v_x = v_0 \sin\theta$ and $v_y = v_0 \cos\theta$

$g_x = g \sin\theta$ $g_y = g \sin\theta$ vertically downward to plane

Consider the motion of particle from O to A in new YOY′ axis.

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$s_y = 0$ $v_y = v_0 \cos\theta$ $a_y = -g \sin\theta$ (upward)

$\therefore$ $t = T$ (time of flight)

$$0 = v_0 \cos\theta\, T - \frac{1}{2} g \sin\theta\, T^2$$

$$0 = T\left[v_0 \cos\theta - \frac{1}{2} g \sin\theta . T\right]$$

$$T = 0 \quad \text{or} \quad v_0 \cos\theta - \frac{g \cos\theta (T)}{2} = 0$$

$$\Rightarrow \quad T = \frac{2v_0 \cos\theta}{g \cos\theta}$$

$$\boxed{T = \frac{2v_0}{g}}$$

Now consider the motion along OX axis

$s_x = L$ $u_x = v_0 \sin\theta$, $a_x = g \sin\theta$ $t = T = \dfrac{2v_0}{g}$

$$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$L = \left[\frac{2v_0}{g}\right] v_0 \sin\theta + \frac{1}{2} g \sin\theta \left[\frac{2v_0}{g}\right]^2$$

$$L = \frac{2v_0^2}{g} \sin\theta + \frac{1}{2} g \sin\theta . \frac{4v_0^2}{g^2}$$

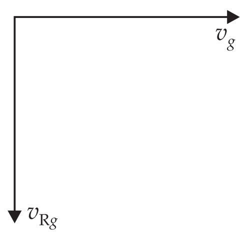
$$= \frac{2v_0^2}{g} [\sin\theta + \sin\theta] = \frac{2v_0^2}{g} 2 \sin\theta$$

$$L = \frac{4v_0^2}{g} \sin\theta .$$

Q4.33. A girl riding a bicycle with speed of 5 ms^{-1} towards north direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45° to the vertical. What is the speed of rain? In what direction does rain fall as observed by a ground based observer?

[*Hint:* Assume north to be $\hat{\mathbf{i}}$ direction and vertically downward to be $-\hat{\mathbf{j}}$. Let the rain velocity $\mathbf{v}_r$ be $a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. The velocity of rain as observed by the girl is always $\mathbf{v}_r - \mathbf{v}_{\text{girl}}$. Draw the vector diagram/s for the information given and find a and b. You may draw all vectors in the reference frame of ground based observer.]

Ans. Consider north direction as $+\hat{i}$ and downward direction $-\hat{j}$.



Case I: When $v_g = 5\hat{i}$

Let $v_R = a\hat{i} + b\hat{j}$

$$v_{Rg} = v_R - v_g = a\hat{i} + b\hat{j} - 5\hat{i}$$

Rain appear her vertical downward.

$\therefore$ Horizontal component of v_{Rg} is zero

$\Rightarrow$ $a - 5 = 0$ or $\boxed{a = 5}$

Case II: Now $v_g = 10 \text{ m/s} = 10\hat{i}$

$$v_R = a\hat{i} + b\hat{j} = 5\hat{i} + b\hat{j}$$

$$v_{RG} = 5\hat{i} + b\hat{j} - 10\hat{i} = -5i + b\hat{j}$$

Now rain appears to her at 45° with vertical or with $\hat{j}$.

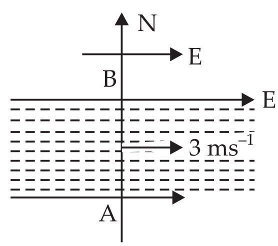
$$\tan 45° = \frac{b}{a} = \frac{b}{-5}$$

$$1 = \frac{b}{-5} \Rightarrow b = -5$$

$\therefore$ Hence $v_R = +5\hat{i} - 5\hat{j}$

$$|v_R| = \sqrt{5^2 + (-5)^2} = \sqrt{2 \times 25} = 5\sqrt{2} \text{ m/s}$$

Q4.34. A river is flowing due east with a speed of 3 m/s. A swimmer can swim in still water at a speed of 4 m/s as shown in figure.



(*a*) If swimmer starts swimming due north, what will be his resultant velocity (magnitude and direction)?

(*b*) If he wants to start from point A on south bank and reaches the opposite point B on north bank.

(*i*) Which direction should he swim?

(*ii*) What will be his resultant speed?

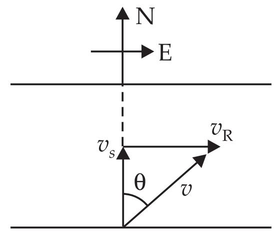
(*c*) From two different cases as mentioned in (*a*) and (*b*) above, in which case will he reach opposite bank in shorter time?

Ans. $v_s = 4$ m/s

$v_R = 3$ m/s towards East

(*a*) $v_s = 4$ m/s due North

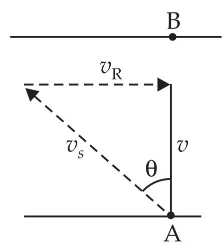
$v_R = 4$ m/s due East



As both are perpendicular.

So
$$v^2 = v_s^2 + v_R^2 = 4^2 + 3^2$$
$$v = \sqrt{25} = 5 \text{ m/s}$$
$$\tan\theta = \frac{v_R}{v_s} = \frac{3}{4} = 0.75$$
$$\tan\theta = \tan 36^\circ 54'$$
$$\theta = 36^\circ 54' \text{ with north direction.}$$

(*b*) Let swimmer strike to swim with angle θ with north direction towards West as again in ⊥Δ then from figure



$$v^2 = v_s^2 - v_R^2 = 4^2 - 3^2$$
$$v^2 = 16 - 9 = 7$$
$$v = \sqrt{7} \text{ m/s}$$
$$\therefore \quad \tan\theta = \frac{v_R}{v} = \frac{3}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$
$$\tan\theta = \frac{3 \times 2.64}{7} = \frac{7.92}{7}$$
$$\tan\theta = 1.13 = \tan\ 48^\circ 29' 30''$$
$$\theta = 48^\circ 29' 30'' \text{ from N to W.}$$

(*c*) Time taken by swimmer when strike at North direction, the component of velocity perpendicular to river is 4 m/s.

Let width of river = d_R

Time taken when strike towards North = $\dfrac{d_R}{4}$ sec = t_1

When swimmer strike at angle 48° 29′ 30″ in part (*b*) its resultant velocity along perpendicular to river is $v = \sqrt{7}$ m/s

∴ Time in (*b*) part = $\dfrac{d_R}{\sqrt{7}} = t_2$

$$\frac{t_1}{t_2} = \frac{\dfrac{d_R}{4}}{\dfrac{d_R}{\sqrt{7}}} = \frac{\sqrt{7}}{4}$$
$$4t_1 = \sqrt{7}t_2 \quad \text{as } 4 > \sqrt{7}$$

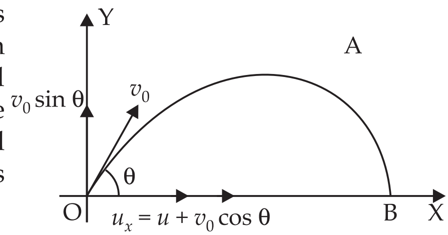
∴ $t_1 < t_2$. Hence, he will reach the opposite bank in shorter time in the case (*a*).

Q4.35. A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal, find

(*a*) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator.
(*b*) what will be the time of flight?
(*c*) what is the distance (horizontal range) from the point of projection at which the ball will land?
(*d*) find θ at which he should throw the ball that would maximise the horizontal range as found in (*c*).
(*e*) how does θ for maximum range change if $u > v_0$, $u = v_0$, $u < v_0$?
(*f*) how does θ in (*e*) compare with that for $u = 0$ (i.e., 45°)?

Ans.

(*a*) As the cricket fielder runs with velocity u (in horizontal direction) and he throws the ball while running. So the horizontal component of ball includes his speed u.



So $u_x = u + v_0 \cos\theta$

As he runs horizontally so vertical component of velocity of ball does not affect.

$$\therefore \qquad u_y = v_0 \sin\theta$$

$$\tan\theta = \frac{u_y}{u_x} = \frac{v_0 \sin\theta}{u + v_0 \cos\theta}$$

$$\theta = \tan^{-1}\left[\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right]$$

(*b*) For time of flight motion, $t = \mathrm{T}$

Vertical motion $s_y = u_y t + \frac{1}{2} g t^2$

$s_y = 0$ (as ball return in same position of Y-axis)

$u_y = v_0 \sin\theta \qquad a_y = -g \qquad t = \mathrm{T}$

$$0 = v_0 \sin\theta\,(\mathrm{T}) - \frac{1}{2} g\mathrm{T}^2$$

$$\mathrm{T}\left[v_0 \sin\theta - \frac{1}{2} g\mathrm{T}\right] = 0$$

$$\mathrm{T} = 0 \quad \text{or} \quad v_0 \sin\theta - \frac{1}{2} g\mathrm{T} = 0$$

$$\boxed{\mathrm{T} = \frac{2\, v_0 \sin\theta}{g}}$$

(*c*) Horizontal Range $= u_x \times \mathrm{T} = [u + v_0 \cos\theta]\,\mathrm{T}$

$$\mathrm{R} = [u + v_0 \cos\theta]\left[\frac{2v_0 \sin\theta}{g}\right]$$

$$= \frac{v_0}{g}\,[2u \sin\theta + v_0\, 2 \sin\theta \cos\theta]$$

$$\boxed{R = \frac{v_0}{g}[2u \sin\theta + v_0 \sin 2\theta]}$$

(*d*) For maximum range $= \dfrac{dR}{d\theta} = 0$

$$\frac{d\,\frac{v_0}{g}[2u \sin\theta + v_0 \sin 2\theta]}{d\theta} = 0$$

$$\frac{v_0}{g}[2u \cos\theta + 2v_0 \cos 2\theta] = 0$$

$$\frac{v_0}{g} \neq 0 \quad \therefore\ 2u \cos\theta + 2v_0\,(2 \cos^2\theta - 1) = 0$$

$$2u \cos\theta + 4v_0 \cos^2\theta - 2v_0 = 0$$

$$2v_0 \cos^2\theta + u \cos\theta - v_0 = 0$$

By quadratic formula

$$\cos\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 2v_0,\ b = u,\ c = -v_0$$

$$\cos\theta = \frac{-u \pm \sqrt{u^2 - 4.2v_0(-v_0)}}{2.2v_0}$$

$$\cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0} \qquad ...(i)$$

$$\theta = \cos^{-1}\left[\frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}\right]$$

(*e*)
$$\cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$$

If $u = v_0$, $\cos\theta = \dfrac{-v_0 \pm \sqrt{[v_0^2 + 8v_0^2]}}{4v_0} = \dfrac{-v_0 \pm 3v_0}{4v_0}$

$$\cos\theta = \frac{-1 \pm 3}{4}$$

As θ acute angle, θ is angle of projection.

$$\cos\theta = \frac{-1+3}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

$$\theta = 60^\circ$$

If $u \ll v_0$ then $\cos\theta = \dfrac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$ [from (*i*)]

$$\cos\theta = \frac{-u \pm 2\sqrt{2}v_0}{4v_0}$$

For θ is acute angle

$$\cos\theta = \frac{-u \pm 2\sqrt{2}v_0}{4v_0} = \frac{2\sqrt{2}v_0}{4v_0} - \frac{u}{4v_0}$$

$$\cos\theta = \frac{1}{\sqrt{2}} - \frac{u}{4v_0}$$

$\because u << v_0$ so neglecting $\frac{u}{4v_0}$

$$\cos\theta = \frac{1}{\sqrt{2}} = \cos 45°$$

$$\therefore \qquad \boxed{\theta = \frac{\pi}{4}}$$

For $u >> v_0$ from (*i*)

$$\cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$$

$$\cos\theta = \frac{-u \pm u}{4v_0}$$

As $u >> v_0 \quad \therefore \frac{-u \pm u}{4v_0} \to 0$

$$\therefore \qquad \cos\theta = 0 = \cos 90°$$

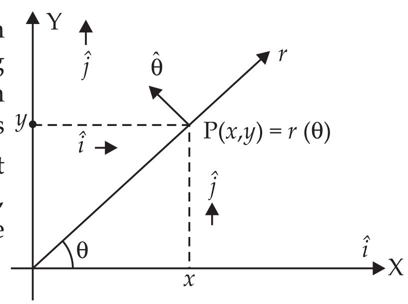
$$\boxed{\theta = \frac{\pi}{2}}$$

(*f*) If $u = 0$ $\quad \cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$

$$\cos\theta = \frac{\pm\sqrt{8v_0^2}}{4v_0} = \frac{2\sqrt{2}v_0}{4v_0} = \frac{1}{\sqrt{2}} = \cos 45°$$

$$\boxed{\theta = \frac{\pi}{4}}$$

Q4.36. Motion in two dimensions, in a plane can be studied by expressing position, velocity and acceleration as vectors in Cartesian co-ordinates $\vec{A} = A_x\hat{i} + A_y\hat{j}$ where $\hat{i}, \hat{j}$ are unit vectors along X and Y directions, respectively and A_x and A_y are corresponding components of $\vec{A}$



(figure). Motion can also be studied by expressing vectors in circular polar co-ordinates as $\vec{A} = A_r\hat{r} + A_\theta\hat{\theta}$ where $\hat{r} = \frac{\vec{r}}{|r|} = \cos\theta\hat{i} + \sin\theta\hat{j}$ and $\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$ are unit vectors along direction in which 'r' and 'θ' are increasing.

(*a*) Express $\hat{i}$ and $\hat{j}$ in terms of $\hat{r}$ and $\hat{\theta}$.

(*b*) Show that both $\hat{r}$ and $\hat{\theta}$ are unit vectors and are perpendicular to each other.

(*c*) Show that $\frac{d}{dt}\hat{r} = \omega\hat{\theta}$ where $\omega = \frac{d\theta}{dt}$ and $\frac{d\hat{\theta}}{dt} = -\omega\hat{r}$.

(*d*) For a particle moving along a spiral given by $\vec{r} = |\hat{a}||\hat{\theta}|\,\hat{r}$ where $a = 1$ (unit), find dimension of 'a'.

(*e*) Find velocity and acceleration in polar vector representation for particle moving along spiral described in (*d*) above.

Ans. (*a*)

$$\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j} \quad \text{(given)} \quad ...(i) \times \sin\theta$$

$$\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j} \quad \text{(given)} \quad ...(ii) \times \cos\theta$$

$$\hat{r}\sin\theta = \sin\theta\cos\theta\hat{i} + \sin^2\theta\hat{j}$$

$$\hat{\theta}\cos\theta = -\sin\theta\cos\theta\hat{i} + \cos^2\theta\hat{j}$$

Adding above both equations

$$\hat{r}\sin\theta + \hat{\theta}\cos\theta = (\sin^2\theta + \cos^2\theta)\hat{j}$$

$$\hat{r}\sin\theta + \hat{\theta}\cos\theta = \hat{j} \quad ...(iii)$$

Multiplying equation (*i*) by $\cos\theta$ and equation (*ii*) by $\sin\theta$

$$\hat{r}\cos\theta = \cos^2\theta\hat{i} + \sin\theta\cos\theta\,\hat{j} \quad ...(iv)$$

$$\hat{\theta}\sin\theta = -\sin^2\theta\hat{i} + \sin\theta\cos\theta\hat{j} \quad ...(v)$$

Subtracting eqn. (*v*) from (*iv*)

$$\hat{r}\cos\theta - \hat{\theta}\sin\theta = (\cos^2\theta + \sin^2\theta)\,\hat{i} + 0$$

$$\left.\begin{aligned} \hat{r}\cos\theta - \hat{\theta}\sin\theta &= \hat{i} \\ \hat{r}\sin\theta + \hat{\theta}\cos\theta &= \hat{j} \end{aligned}\right]$$

(*b*) From (*i*), (*ii*) by dot product

$$\hat{r}.\hat{\theta} = (\cos\theta\hat{i} + \sin\theta\hat{j}).(-\sin\theta\hat{i} + \cos\theta\hat{j})$$

$$|\hat{r}|.|\hat{\theta}|\cos\theta = -\sin\theta\cos\theta + \sin\theta\cos\theta$$

$$|\hat{r}|.|\hat{\theta}|\cos\theta = 0$$

$|\hat{r}| \neq 0$ and $|\hat{\theta}| \neq 0$ so $\cos\theta = \cos 90°$

So $\theta = 90° = \frac{\pi}{2}$

So angle between $\hat{r}$ and $\hat{\theta}$ is $\frac{\pi}{2}$.

(*c*)
$$\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$$
$$\frac{d\hat{r}}{dt} = \frac{d}{dt}(\cos\theta\hat{i} + \sin\theta\hat{j}) = -\sin\theta\frac{d\theta}{dt}\hat{i} + \cos\theta\frac{d\theta}{dt}\hat{j}$$
$$\frac{d\hat{r}}{dt} = (-\sin\theta\hat{i} + \cos\theta\hat{j})\frac{d\theta}{dt}$$
$$\frac{d\hat{r}}{dt} = \omega\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right] \qquad \because \omega = \frac{d\theta}{dt}$$

(*d*)
$$\vec{r} = |\hat{a}||\hat{\theta}|\,\hat{r}\,; \quad \text{(given)}$$
$$[a] = \frac{[\vec{r}]}{[\hat{\theta}]\,[\hat{r}]} = \frac{[M^0L^1T^0]}{[M^0L^0T^0][M^0L^0T^0]} = [M^0L^1T^0]$$
$$\because \quad \hat{\theta} = \frac{\vec{\theta}}{|\vec{\theta}|} = [M^0L^0T^0] \qquad \because\ \hat{r} = \frac{\vec{r}}{|\vec{r}|} = [M^0L^0T^0]$$

$\therefore$ Dimension of [*a*] constant = $[M^0L^1T^0]$.

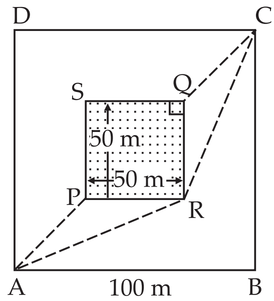
(*e*) $a = 1,\ \vec{r} = \theta\hat{r} = \theta\left[\cos\theta\hat{i} + \sin\theta\hat{j}\right]$
$$v = \frac{d\vec{r}}{dt} = \frac{d\theta}{dt}\hat{r} + \theta\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{r} + \theta\frac{d}{dt}\left[\cos\theta\hat{i} + \sin\theta\hat{j}\right]$$
$$v = \frac{d\theta}{dt}\hat{r} + \theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]\frac{d\theta}{dt}$$
$$\boxed{v = \omega\hat{r} + \theta.\hat{\theta}\omega} \qquad \left[\begin{array}{l}\because\ \hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j} \\ \dfrac{d\theta}{dt} = \omega\end{array}\right]$$
$$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}\left[\omega\hat{r} + \theta.\hat{\theta}\omega\right] = \frac{d}{dt}\left[\frac{d\theta}{dt}\hat{r} + \frac{d\theta}{dt}(\theta.\hat{\theta})\right]$$
$$\vec{a} = \frac{d^2\theta}{dt^2}\hat{r} + \frac{d\theta}{dt}\frac{d\hat{r}}{dt} + \frac{d^2\theta}{dt^2}(\theta.\hat{\theta}) + \frac{d\theta}{dt}\frac{d}{dt}(\theta.\hat{\theta})$$
$$\vec{a} = \frac{d^2\theta}{dt^2}\hat{r} + \omega\frac{d}{dt}(\cos\theta\hat{i} + \sin\theta\hat{j}) + \frac{d^2\theta}{dt^2}(\theta.\hat{\theta}) + \omega\frac{d}{dt}(\theta.\hat{\theta})$$
$$= \frac{d^2\theta}{dt^2}\hat{r} + \omega\left(-\sin\theta\hat{i}\frac{d\theta}{dt} + \cos\theta\frac{d\theta}{dt}\hat{j}\right) + \frac{d^2\theta}{dt^2}(\theta.\hat{\theta}) + \omega\frac{d}{dt}(\theta.\hat{\theta})$$

From (*ii*)
$$\vec{a} = \frac{d^2\theta}{dt^2}\hat{r} + \omega(\hat{\theta})\omega + \frac{d^2\theta}{dt^2}(\theta.\hat{\theta}) + \omega\left[\frac{d\theta}{dt}.\hat{\theta} + \theta\frac{d\hat{\theta}}{dt}\right]$$
$$= \frac{d^2\theta}{dt^2}\hat{r} + \omega^2\hat{\theta} + \frac{d^2\theta}{dt^2}(\theta.\hat{\theta}) + \omega\left[\omega\hat{\theta} + \theta.\frac{d}{dt}\frac{\vec{\theta}}{|\vec{\theta}|}\right]$$

$$\vec{a} = \frac{d^2\theta}{dt^2}\hat{r} + \omega^2\hat{\theta} + \frac{d^2\theta}{dt^2}(\theta.\hat{\theta}) + \omega^2\hat{\theta} + \omega^2(-\hat{r})$$

$$\vec{a} = \left[\frac{d^2\theta}{dt^2} - \omega^2\right]\hat{r} + \left[2\omega^2 + \frac{d^2\theta}{dt^2}\theta\right]\hat{\theta}$$

Q4.37. A man wants to reach from A to the opposite corner of the square C (as in figure). The sides of square are 100 m. A central square of 50 m× 50 m is filled with sand. Outside this square, he can walk at a speed of 1 ms^{-1}. In the central square, he can walk at speed of v m/s (v < 1). What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?



Ans.

$$PQ = \sqrt{50^2 + 50^2} = \sqrt{50^2 \times 2} = 50\sqrt{2}$$

$$AC = \sqrt{[100^2 + 100^2]} = \sqrt{100^2 \times 2} = 100\sqrt{2}$$

Time (t_1) taken through path A → P → Q → C

$$= \frac{(AP + QC)}{1\ m/s} + \frac{PQ}{v}$$

$$t_1 = \frac{AC - PQ}{1} + \frac{PQ}{v} = \frac{100\sqrt{2} - 50\sqrt{2}}{1} + \frac{50\sqrt{2}}{v} = 50\sqrt{2}\left[1 + \frac{1}{v}\right] s$$

Time taken $A \to R \to C = \dfrac{(AR + RC)}{1} = 2AR = t_2$

$$AR^2 = OC^2 + OR^2 = \left(\frac{100\sqrt{2}}{2}\right)^2 + \left(\frac{100\sqrt{2}}{2}\right)^2 = (25)^2(\sqrt{2})^2\left[2^2 + 1^2\right]$$

$$AR = 25\sqrt{2}\sqrt{5} = 25\sqrt{10}$$

$$\therefore \quad t_2 = 2 \times 25\sqrt{10}\ s$$

$$t_1 < t_2$$

$$50\sqrt{2}\left[1 + \frac{1}{v}\right] < 2 \times 25\sqrt{5}\sqrt{2}$$

$$\left[1 + \frac{1}{v}\right] < \sqrt{5}$$

$$\frac{1}{v} < \sqrt{5} - 1$$

$$v < \frac{1}{(\sqrt{5} - 1)} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1} = \frac{2.3 + 1}{5 - 1} = \frac{3.3}{4}$$

$$v < 0.82\ m/s.$$

❑❑❑

5 Laws of Motion

MULTIPLE CHOICE QUESTIONS-I

Q5.1. A ball is travelling with uniform translatory motion. This means that:

(*a*) It is at rest.

(*b*) The path can be a straight line or circular and the ball travels with uniform speed.

(*c*) All parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.

(*d*) The centre of the ball moves with constant velocity and the ball spins about its centre uniformly.

Ans. (*c*): If all the parts of body moves with same velocity in same straight line, then motion is called uniform motion or uniform translatory motion.

Q5.2. A meter scale is moving with uniform velocity. This implies

(*a*) The force acting on the scale is zero, but a torque about the centre of mass can act on the scale.

(*b*) The force acting on the scale is zero, and the torque acting about the centre of mass of the scale is also zero.

(*c*) The total force acting on it need not be zero but the torque on it is zero.

(*d*) Neither the force nor the torque need to be zero.

Ans. (*b*): As the meter scale is moving with uniform velocity.

$\therefore$ No change in its velocity i.e., acceleration of it zero by Newton's second law $\vec{F} = m \times 0 = 0$

$\therefore$ Hence net or resultant force must act on body zero.

$\because$ $\vec{\tau} = \vec{r} \times \vec{F}$. So torque must be zero.

Hence, for uniform motion force and torque both must be zero. It verifies the option (*b*).

Q5.3. A cricket ball of mass 150 g has the initial velocity $\vec{u} = (3\hat{i} + 4\hat{j})$ ms^{-1} and final velocity $\vec{v} = -(3\hat{i} + 4\hat{j})$ ms^{-1} after being hit. The change in momentum (final momentum – initial momentum) is (kg ms^{-1})

(*a*) zero

(*b*) $-(0.45\hat{i} + 0.6\hat{j})$

(*c*) $-(0.9\hat{i} + 1.2\hat{j})$

(*d*) $-5(\hat{i} + \hat{j})$

Ans. (*c*): $m = 150 \text{ g} = 0.15 \text{ kg}$

$\vec{u} = (3\hat{i} + 4\hat{j})$ ms^{-1} and $\vec{v} = (3\hat{i} + 4\hat{j})$ ms^{-1}

Change in momentum $\Delta\vec{p}$ = Final momentum – Initial momentum

$= m\vec{v} - m\vec{u}$

$$= m[\vec{v} - \vec{u}] = 0.15\left[-(3\hat{i} + 4\hat{j}) - (3\hat{i} + 4\hat{j})\right]$$
$$= 0.15\left[-3\hat{i} - 4\hat{j} - 3\hat{i} - 4\hat{j}\right] = 0.15\left[-6\hat{i} - 8\hat{j}\right]$$
$$\Delta\vec{p} = -0.9\hat{i} - 1.2\hat{j} = -\left[0.9\hat{i} + 1.2\hat{j}\right]$$

Hence, verifies the option (*c*).

Q5.4. In a previous problem 5.3, the magnitude of momentum transferred during the hit is:

(*a*) zero (*b*) 0.75 kg ms^{-1} (*c*) 1.5 kg ms^{-1} (*d*) 1.4 kg ms^{-1}

Ans. (*c*): From Q. 5.3 $\Delta\vec{p} = -0.9\hat{i} - 1.2\hat{j}$

$$\text{Magnitude of } |\Delta\vec{p}| = \sqrt{(-0.9)^2 + (1.2)^2} = \sqrt{.81 + 1.44} = \sqrt{2.25}$$
$$= 1.5 \text{ kg ms}^{-1}$$

Verifies the option (*c*).

Q5.5. Conservation of momentum in a collision between particles can be understood from

(*a*) Conservation of energy
(*b*) Newton's first law only
(*c*) Newton's second law only
(*d*) Both Newton's second and third law.

Ans. (*d*): (*i*) By Newton's second law $\dfrac{d\vec{p}}{dt} = \vec{F}_{ext}$

As $\vec{F}_{ext}$ in law of conservation of momentum is zero.

i.e., $$\vec{F}_{ext} = 0$$
$$\frac{d\vec{p}}{dt} = 0$$

$\Rightarrow$ $\vec{p}$ is constant.

(*ii*) By Newton's third law action force is equal to reaction force in magnitude but in opposite direction.

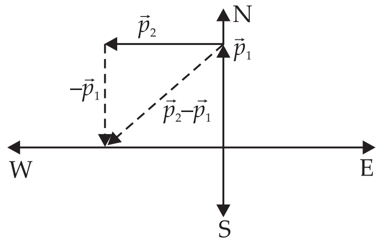
$\therefore$ $$\vec{F}_{12} = -\vec{F}_{21} \qquad (\vec{F}_{ext} = 0)$$
$$\frac{d\vec{p}_{12}}{dt} = \frac{-d\vec{p}_{21}}{dt} \quad \text{or} \quad d\vec{p}_{12} = -d\vec{p}_{21}$$
$$d\vec{p}_{12} + d\vec{p}_{21} = 0.$$

So proves the law of conservation of momentum and verifies the option (*d*).

Q5.6. A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is:

(*a*) frictional force along westward
(*b*) muscle force along southward
(*c*) friction force along south-west
(*d*) muscle force acting south-west.

Ans. (*c*): The force on player will be due to rate of change of momentum. The direction of force acting on player will be the same as the direction of change in momentum $\vec{p}_2 - \vec{p}_1$. It is clear from figure. The direction of $\vec{p}_2 - \vec{p}_1$ is towards south-west. It will be the direction of force on player. Hence, verifies option (*c*).



Q5.7. A body of mass 2 kg travels according to the law $x(t) = pt + qt^2 + rt^3$, where $p = 3$ m/s, $q = 4$ ms^{-2} and $r = 5$ ms^{-3}. The force acting on the body at $t = 2$ seconds is:

(*a*) 136 N (*b*) 134 N (*c*) 158 N (*d*) 68 N

Ans. (*a*): $\vec{F} = m\vec{a} = m.\dfrac{d^2x}{dt^2}$

$\because$ $x(t) = pt + qt^2 + rt^3$ $p = 3$ ms^{-1}

$x(t) = 3t + 4t^2 + 5t^3$ $q = 4$ ms^{-2}

$\dfrac{dx(t)}{dt} = 3 + 8t + 15t^2$ $r = 5$ ms^{-3}

$$\frac{d^2x(t)}{dt^2} = 0 + 8 + 30t$$

$$\left[\frac{d^2x(t)}{dt^2}\right]_{t=2} = 8 + 30 \times 2 = 68 \text{ ms}^{-2}$$

$\therefore$ $\vec{F} = 2 \times 68 = 136$ N. Verifies the option (*a*).

Q5.8. A body of mass 5 kg is acted upon by a force $\vec{F} = (-3\hat{i} + 4\hat{j})$ N. If its initial velocity at $t = 0$ is $\vec{v} = (6\hat{i} - 12\hat{j})$ ms^{-1}, the time at which it will just have a velocity along the y-axis is:

(*a*) never (*b*) 10 s (*c*) 2 s (*d*) 15 s

Ans. (*b*): $u = (6\hat{i} - 12\hat{j})$ ms^{-1}

$F = (-3\hat{i} + 4\hat{j})$ N

$m = 5$ kg $\quad \vec{a} = \dfrac{\vec{F}}{m} = \left(\dfrac{-3}{5}\hat{i} + \dfrac{4}{5}\hat{j}\right)$ ms^{-2}

As the final velocity he has only Y component and X component is zero.

$$v_x = u_x + a_x t$$

$$0 = 6 + \frac{-3}{5}t \quad \Rightarrow \quad \frac{3}{5}t = 6$$

$t = 10$ sec. Verifies the option (*b*).

Q5.9. A car of mass m starts from rest and acquires a velocity along east $\vec{v} = v\hat{i}$ $(v > 0)$ is two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is:

(*a*) $\frac{mv}{2}$ eastward and is exerted by the car engine

(*b*) $\frac{mv}{2}$ eastward and is due to friction on tyres exerted by the road.

(*c*) more than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road.

(*d*) $\frac{mv}{2}$ exerted by engine.

Ans. (*b*): $u = 0$, $\vec{v} = v\hat{i}$, $t = 2$, $m = m$

$$v = u + at$$
$$\vec{v}\hat{i} = 0 + \vec{a} \times 2$$
$$\boxed{\vec{a} = \frac{\vec{v}\hat{i}}{2}}$$
$$\vec{F} = m\vec{a} = \frac{m\vec{v}}{2}\hat{i}$$

Force by engine is internal force.

Hence, force $\frac{m\vec{v}}{2}\hat{i}$ acting on car is due to force of friction is $\frac{m\vec{v}}{2}\hat{i}$ towards east, which moves the car in eastward direction.

MULTIPLE CHOICE QUESTIONS-II

Q5.10. The motion of a particle of mass m is given by $x = 0$ for $t < 0$ s, $x(t) = A \sin 4\pi t$ for $0 < t < \left(\frac{1}{4}\right)$ s. (A > 0), and $x = 0$ for $t > \left(\frac{1}{4}\right)$ s. Which of the following statements is true?

(*a*) The force at $t = \left(\frac{1}{8}\right)$ s on the particle is $-16\pi^2 Am$

(*b*) The particle is acted upon by on impulse of magnitude $4\pi^2 Am$ at $t = 0$ s and $t = \left(\frac{1}{4}\right)$ s

(*c*) The particle is not acted upon by any force.

(*d*) The particle is not acted upon by a constant force.

(*e*) There is no impulse acting on the particle.

Ans. (*a, b, d*): $x(t) = 0$ for $0 < t$ $m = m$

$$x(t) = A \sin 4\pi t \quad \text{for } 0 < t < \frac{1}{4}\text{ s}$$
$$x(t) = 0 \quad \text{for } t > \frac{1}{4}\text{ s}$$

For $0 < t < \frac{1}{4}$ s

$$x(t) = A \sin 4\pi t$$
$$v = \frac{dx(t)}{dt} = 4A\pi \cos 4\pi t$$
$$a = \frac{dv}{dt} = -16\pi^2 A \sin 4\pi t$$
$$F(t) = ma(t) = -16\pi^2 mA \sin 4\pi t \quad \text{...(i)}$$

As the force is a function of time as in above equation, so force acting on particle is not constant verifies option (*d*).

(*a*) At $t = \frac{1}{8}$ s $\qquad a(t) = -16\pi^2 A \sin 4\pi \frac{1}{8}$

$$a(t) = -16\pi^2 A \sin \frac{\pi}{2}$$

$$a(t) = -16\pi^2 A$$

Force at, $t = \frac{1}{8}$ sec, $\qquad F = ma = m(-16\pi^2 A)$

$$F = -16\pi^2 Am \text{ N. Verifies the option } (a).$$

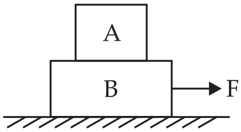
(*b*) Impulse = Change in momentum between $t = 0$ s and $\frac{1}{4}$ sec from (*i*) F(t) varies from zero at $t = 0$ the minimum value to maximum value $F(t) = -16\pi^2 mA$ as maximum value of $\sin 4\pi t = 1$ is at $\frac{1}{8}$ sec which lies between 0 to $\frac{1}{4}$ sec.

Hence by $\vec{I} = \vec{F}t$ at $t = \frac{1}{4}$ s

we get $\qquad \vec{I} = -16\pi^2 mA \times \frac{1}{4} = -4\pi^2 mA$

Verified the option (*b*).

Q5.11. In the given figure, the coefficient of friction between the floor and the body B is 0.1. The coefficient of friction between bodies B and A is 0.2. A force $\vec{F}$ is applied as shown on B. The mass of A is $\frac{m}{2}$ and of B is m. Which of the following statements are true?

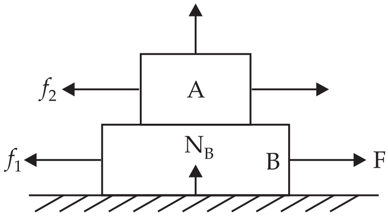


(*a*) The bodies will move together if $\vec{F} = 0.25\ mg$.
(*b*) The body A will slip with respect to B if $\vec{F} = 0.5\ mg$.
(*c*) The bodies will move together if $\vec{F} = 0.5\ mg$.
(*d*) The bodies will be at rest if $\vec{F} = 0.1\ mg$.
(*e*) The maximum value of $\vec{F}$ for which the two bodies will move together is 0.45 mg.

Ans. (*a, b, d, e*): Move together is 0.45 mg Newton

$m_A = \frac{m}{2}; \quad m_B = m$

Let acceleration in body A and B is 'a'.



Body A will move along with body B by force F till the force of friction between surface of A and B is larger or = 0 then force acting on A

$$a = \frac{F - f_1}{m_A + m_B} = \frac{F - f_1}{\frac{m}{2} + m} = \frac{2(F - f_1)}{3m}$$

$$\text{Force on A} = m_A a = \frac{m}{2}\frac{2(F - f_1)}{3m}$$

So force on A $\quad F_{AB} = \frac{(F - f_1)}{3}$

If F_{AB} is equal or smaller than f_2 then body A will move along with body B.

$$\therefore \quad f_2 = F_{AB}$$

$$\mu N = \frac{F - f_1}{3}$$

$$0.2 \times m_A g = \frac{F - f_1}{3} \qquad ...(i)$$

N = Reaction force by B on A

$$f_1 = \mu N_B = \mu(m_A + m_B)g$$

[N_B = Normal reaction on B along with A by surface]

$$= 0.1 \times (m_A + m_B)g$$

$$f_1 = 0.1 \times \frac{3}{2} mg = 0.15mg \qquad ...(ii)$$

From (*i*)

$$F - f_1 = 3 \times 0.2\, m_A g$$

$$F - 0.15mg = 0.6 \times \frac{m}{2} g$$

$$F_{max} = 0.3mg + 0.15mg = 0.45mg \qquad ...(iii)$$

$F = 0.45mg$ Newton is maximum force on B. So that A and B can move together.

It verifies option (*e*).

Both bodies can move together if F is less than or equal to $0.45mg$ Newton.

So verifies the options (*a*) and (*b*) and rejects the option (*c*) as $0.5mg > 0.45mg$.

For option (*d*): Minimum force which can move A and B together

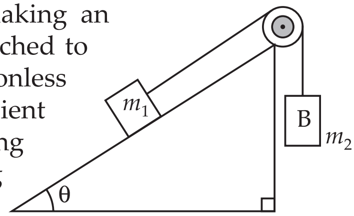
$$F_{min} \geq f_1 + f_2$$

$$\geq 0.15mg + 0.2 \times \frac{m}{2} g \qquad \text{[from (i), (ii)]}$$

$$F_{min} \geq 0.25mg \text{ Newton}$$

Given force in option (*d*) 0.1 mg Newton < $0.25mg$ Newton. So body A and B will not move. i.e., Bodies A and B will remains in rest verifies option (*d*).

Q5.12. Mass m_1 moves on a slope making an angle θ with the horizontal and is attached to mass m_2 by a string passing over a frictionless pulley as shown in figure. The co-efficient of friction between m_1 and the sloping surface is μ. Which of the following statements are true?



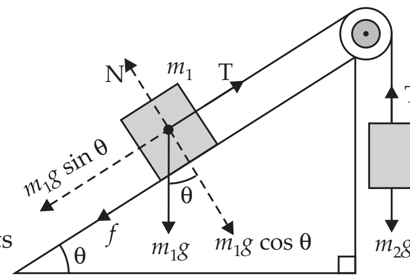
(*a*) If $m_2 > m_1 \sin\theta$, the body will move up the plane.
(*b*) If $m_2 > m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane.
(*c*) If $m_2 < m_1 (\sin\theta + \mu\cos\theta)$, the body will move up the plane.
(*d*) If $m_2 < m_1 (\sin\theta - \mu\cos\theta)$, the body will move down the plane.

Ans. (*b, d*): **Case I** Normal reaction $N = m_1g\cos\theta$

$$f = \mu N = \mu m_1 g \cos\theta$$

∴ Above equation becomes

$T - m_1g\sin\theta - m_1g\cos\theta = m_1a$



m_1 will up and m_2 down when

$m_2g - (m_1g\cos\theta + f) > 0$

$m_2g - m_1g\sin\theta - \mu m_1g\cos\theta > 0$

$m_2g > m_1g(\sin\theta + \mu\cos\theta)$

or $m_2 > m_1 (\sin\theta + \mu\cos\theta)$

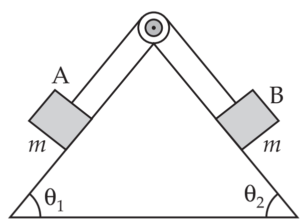
Verifies option (*b*) and rejects option (*a*).

Case II: If body m_1 moves down and m_2 up then, direct of *f* becomes upward (opp. to motion).

$$-f + m_1g\sin\theta > m_2g$$
$$-\mu m_1g\cos\theta + m_1g\sin\theta > m_2g$$
$$m_1(-\mu\cos\theta + \sin\theta) > m_2$$
$$m_2 < m_1(\sin\theta - \mu\cos\theta)$$

Verifies option (*d*) and rejects option (*c*).

Q5.13. In given figure, a body A of mass *m* slides on plane inclined at an angle θ_1 to the horizontal and μ_1 is the coefficient of friction between A and the plane. A is connected by a light string passing over a frictionless pulley to another body B, also of mass *m*, sliding on a frictionless plane inclined at angle θ_2 to the horizontal. Which of the following statements are true?

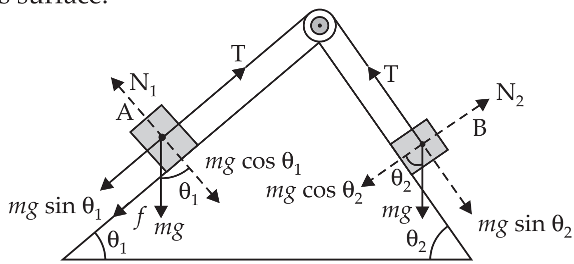


(*a*) A will never move up the plane.
(*b*) A will just start moving up the plane when

$$\mu = \frac{\sin\theta_2 - \sin\theta_1}{\cos\theta_1}.$$

(*c*) For A to move up the plane, θ_2 must always be greater than θ_1.
(*d*) B will always slide down with constant speed.

Ans. (*b, c*): Plane below A has co-efficient of friction μ but B is on frictionless surface.



(*i*) When A just to start

$$f = \mu N_1 = \mu mg \cos \theta_1$$

$$mg \sin \theta_1 + f = mg \sin \theta_2$$

Body A moves up and B down the plane.

$$mg \sin \theta_1 + \mu mg \cos \theta_1 = mg \sin \theta_2$$

$$\mu \cos \theta_1 = \sin \theta_2 - \sin \theta_1$$

$$\mu = \frac{\sin \theta_2 - \sin \theta_1}{\cos \theta_1}$$. Verifies option (*b*).

(*ii*) When A moves upwards and B downward.

$$mg \sin \theta_2 - mg \sin \theta_1 > 0$$

$$\sin \theta_2 - \sin \theta_1 > 0$$

$$\sin \theta_2 > \sin \theta_1$$

$$\theta_2 > \theta_1$$

Verifies the option (*c*) and rejects option (*a*).

If B moves up and A downward then

$$mg \sin \theta_1 - f - mg \sin \theta_2 > 0$$

$$mg \sin \theta_1 - \mu mg \cos \theta_1 - mg \sin \theta_2 > 0$$

$$\sin \theta_1 - \sin \theta_2 > \mu \cos \theta_1$$

As θ_1 increases $\sin \theta$ also increases and θ_1 increases $\cos \theta$ decreases. θ_1 and θ_2 are acute angles so $\theta_1 > \theta_2$ and $\sin \theta_1 - \sin \theta_2 > \mu \cos \theta_1$ may true.

So the body B can moves up rejects option (*d*).

Q5.14. Two billiard balls A and B, each of mass 50g and moving in opposite directions with speed of 5 ms^{-1} each, collide and rebound with the same speed. If the collision lasts for 10^{-3}s, which of the following statements are true?

(*a*) The impulse imparted to each ball is 0.25 kg ms^{-1} and the force on each ball is 250 N.
(*b*) The impulse imparted to each ball is 0.25 kg ms^{-1} and the force exerted on each ball is 25×10^{-5} N.

(*c*) The impulse imparted to each ball is 0.5 Ns.

(*d*) Initial and final momentum on each ball are equal in magnitude and opposite in direction.

Ans. (*c*, *d*): Mass of each ball $m = 0.05$ kg

Speed of each ball $v = 5$ m/s

$\therefore$ Initial momentum of each ball $\vec{p}_i = m\vec{v}$

$$\vec{p}_i = (0.05)\,(5) = 0.25 \text{ kg ms}^{-1}$$
$$= 0.25 \text{ N-s} \qquad (1)$$

As after the collision, the direction of velocity of each ball is reversed on rebounding.

$\therefore$ Final momentum of each ball $\vec{p}_f = m(-\vec{v})$

$$\vec{p}_f = 0.05 \times (-5) = -0.25 \text{ kg ms}^{-1}$$
$$= -0.25 \text{ N-s} \qquad (2)$$

Eqns (1) and (2) verifies option (*d*).

$\therefore$ Impulse imparted to each ball = Change in momentum of each ball

$$= p_f - p_i$$
$$= -0.25 - (0.25) = -0.50 \text{ kg ms}^{-1}$$
$$= -0.50 \text{ N-s}$$

Equation (3) verifies option (*c*).

i.e., magnitude of impulse imparted by one ball due to collision with the other ball = 0.50 kg ms^{-1}. These two impulse are opposite to each other.

Q5.15. A body of mass 10 kg is acted upon by two perpendicular forces, 6 N and 8 N. The resultant acceleration of the body is

(*a*) 1 ms^{-2} at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w.r.t. 6 N force.

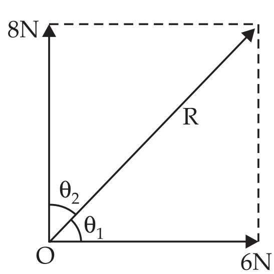
(*b*) 0.2 ms^{-2} at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w.r.t. 6 N force.

(*c*) 1 ms^{-2} at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w.r.t. 8 N force.

(*d*) 0.2 ms^{-2} at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w.r.t. 8 N force.

Ans. (*a*, *c*): Given $m = 10$ kg $\qquad F_2 = 8$ N $\qquad F_1 = 6$ N

$$R = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100}$$
$$R = 10 \text{ N}$$
$$F = ma \Rightarrow a = \frac{F}{m} = \frac{R}{m} = \frac{10}{10} = 1 \text{ ms}^{-2} \qquad ...(i)$$
$$\tan\theta_1 = \frac{8}{6} = \frac{4}{3} \Rightarrow \theta_1 = \tan^{-1}\left(\frac{4}{3}\right) \qquad ...(ii)$$
$$\tan\theta_2 = \frac{6}{8} = \frac{3}{4} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{3}{4}\right) \qquad ...(iii)$$



(*i*), (*ii*) verifies option (*a*) and (*i*), (*iii*) verifies option (*c*).

Acceleration $a \neq 0.2$ ms^{-2}, rejects the option (*b*) and (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q5.16. A girl riding a bicycle along a straight road with a speed of 5 m/s throws a stone of 0.5 kg which has a speed of 15 ms^{-1} with respect to the ground along her direction of motion. The mass of girl and bicycle is 50 kg. Does the speed of the bicycle changes after the stone is thrown? What is the change in speed, if so?

Ans.

Girl and cycle	Body
$m_1 = 50$ kg	$m_2 = 0.5$ kg
$u_1 = 5$ m/s forward	$u_2 = 5$ m/s forward
$v_1 = ?$	$v_2 = 15$ m/s forward

According to law of conservation of momentum.

Initial momentum (Girl, cycle, body) = Final momentum (cycle + Girl) and body

$$(m_1 + m_2)u_1 = m_1v_1 + m_2v_2$$
$$(50 + 0.5) \times 5 = 50 \times v_1 + 0.5 \times 15$$
$$50.5 \times 5 - 7.5 = 50v_1$$
$$50v_1 = 252.5 - 7.5 = 245.0$$
$$v_1 = \frac{245.0}{50} = 4.9 \text{ m/s}$$

Hence, the speed of cycle and girl decreased by 5-4.9 = 0.1 m/s.

Q5.17. A person of 50 kg stands on a weighing scale on a lift. If the lift is descending with downward acceleration of 9 m/s^2. What would be the reading of weighing scale? ($g = 10$ ms^{-2})

Ans. When lift is descending with acceleration *a*, the apparent weight decreases on weighing scale

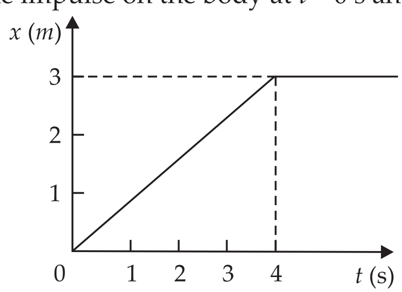
$$\therefore \quad W' = R = (mg - ma) = m(g - a)$$

Apparent weight due to reaction force by lift on weighing scale.

$$\therefore \quad W' = 50(10 - 9) = 50 \text{ N}$$

Reading of weighing scale = $\dfrac{R}{g} = \dfrac{50}{10} = 5$ kg.

Q5.18. The position time graph of a body of mass 2 kg is as given in figure. What is the impulse on the body at $t = 0$ s and $t = 4$ s?



Ans. Mass of body (m) = 2 kg at $t = 0$. Initial velocity (v_1) is zero, $v_1 = 0$ from $t \geq 0$ to $t \leq 4$, (x-t) graph is straight line. So the velocity (v) of body is constant.

$$v_2 = \tan\theta = \frac{3}{4} = 0.75 \text{ m/s}$$

At $t \geq 4$ the slope of graph is zero so velocity $v_3 = 0$

$$\text{Impulse} = \vec{F}.t = \frac{d\vec{p}}{dt}.dt = d\vec{p}$$

$$\text{Impulse} = \text{Change in momentum}$$

At $t = 0$ The Impulse $= m\vec{v}_2 - m\vec{v}_1 = m(\vec{v}_2 - \vec{v}_1)$

$$\text{Impulse at } t = 0 = 2[0.75 - 0] = 1.50 \text{ kg ms}^{-1} \text{ (increased)}$$

$$\text{Impulse at } t = 4 = m(v_3 - v_2) = 2[0 - 0.75]$$

$$\text{Impulse at } t = 4 = -1.50 \text{ kg ms}^{-1}$$

So impulse at $t = 0$ increases by + 1.5 kg ms^{-1} and at $t = 4$ it decreased by (– 1.5 kg ms^{-1}).

Q5.19. A person driving a car suddenly applies the brakes on seeing a child on the road ahead. If he is not wearing seat belt, he falls forward and hits his head against the steering wheel. Why?

Ans. When a person applies breaks suddenly, the lower part of person slows rapidly with car, but upper part of driver continue to move with same speed in same direction due to inertia of motion and his head can hit with steering.

Q5.20. The velocity of a body of mass 2 kg as a function of t is given by $\vec{v}(t) = 2t\hat{i} + t^2\hat{j}$. Find the momentum and the force acting on it, at time $t = 2$ s.

Ans. $m = 2$ kg

$$\vec{v}(t) = 2t\hat{i} + t^2\hat{j}$$

$\vec{v}$ at 2 sec, $\vec{v}(2) = 2(2)\hat{i} + (2)^2\hat{j}$, $\vec{v}(2) = 4\hat{i} + 4\hat{j}$

Momentum $\vec{p}(2) = m\vec{v}(2)$

$$\vec{p}(2) = 2[4\hat{i} + 4\hat{j}],\ p(2) = 8\hat{i} + 8\hat{j} \text{ kg ms}^{-1}$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}(2) = m\vec{a}(2)$$

$$\vec{v}(t) = 2t\hat{i} + t^2\hat{j}$$

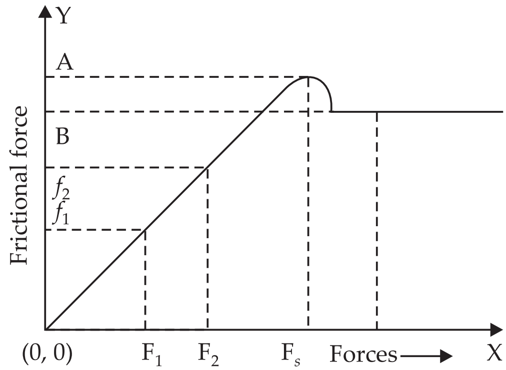
$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = 2\hat{i} + 2t\hat{j}$$

$$\vec{a}(2) = 2\hat{i} + 2(2)\hat{j} = 2\hat{i} + 4\hat{j}$$

$\therefore$ $\vec{F}(2) = 2(2\hat{i} + 4\hat{j}) = 4\hat{i} + 8\hat{j}$ N

Q5.21. A block placed on a rough horizontal surface is pulled by a horizontal force F. Let f be the force applied by the rough surface on the block. Plot the graph of f versus F.

Ans. When a small force F_1 is applied on a heavier box, it does not move. At this state force of friction f_1 is equal to F_1. On increasing force box does not move till $F = F_s$ the maximum static frictional or limiting force. Its corresponding frictional force f_s on Y-axis.



After force F_s, the frictional force decrease i.e., less force $F_k < F_s$ is applied on body and it starts to move with less friction $f_k < f_s$.

A = limiting frictional force and at B = kinetic frictional force.

Q5.22. Why are porcelain objects wrapped in paper or straw before packing for transportation?

Ans. Porcelain (or glass) objects are brittle in nature and can crack even small jerk on it. During transportation sudden jerks or even fall takes place.

When objects are packed in paper or straw etc. the objects takes more time to stop or change velocity during jerks (due to breaks, or uneven road) so acceleration $\frac{(v-u)}{t}$ decreased. So force on objects will be smaller and objects becomes more safe.

Q5.23. Why does a child feel more pain when she falls down on a hard cement floor, than when she falls on the soft muddy ground in the garden?

Ans. The effect of force $F = ma$. i.e., if mass is constant for a system to decrease force, the 'a' should be decreased $a = \frac{v-u}{t}$ initial and final velocity of falling body on a surface are u and zero. So it cannot be change. If time during hitting is increased, the acceleration decreased and force will decreased.

On cemented hard floor the time to stop after fall on it is very-very small. But when she falls on soft ground of garden she sinks in ground and takes more time to stop hence smaller force or pain acts on her.

Q5.24. A woman throws an object of mass 500 g with speed of 25 ms^{-1}.

(*a*) What is the impulse imparted to the object?

(*b*) If the object hits a wall and rebounds with half the original speed, what is the change in momentum of the object?

Ans. (*a*) Mass of object $m = 500$ g $= 0.5$ kg

$u = 0, \quad v = 25$ m/s

$$\text{Impulse} = \vec{F}.dt = \frac{d\vec{p}}{dt}.dt = d\vec{p} = m\vec{v} - m\vec{u}$$

$$I = \Delta\vec{p} = m(\vec{v} - \vec{u}) = 0.5\ (25 - 0) = 12.5 \text{ N-s}$$

(*b*) $m = 0.5$ kg $\qquad u = +25$ ms^{-1} (forward)

$$v = \frac{-25}{2}\text{ ms}^{-1} \quad \text{(as backward)}$$

$$\therefore \quad \Delta p = m(v - u) = 0.5\left[\frac{-25}{2} - 25\right]$$

$$= 0.5[-12.5 - 25] = 0.5 \times (-37.5)$$

$$\Delta p = -18.75 \text{ kg ms}^{-1} \text{ or N-s}$$

Hence, the Δp or $\frac{\Delta p}{\Delta t}$ or force is opposite to the initial velocity of ball.

Q5.25. Why the mountain roads generally made winding upward rather than going straight up?

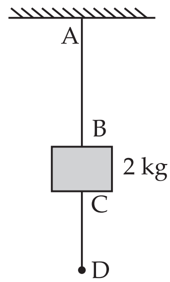
Ans. On an inclined plane force of friction on a body going upward is $f_s = \mu N \cos\theta$ downward where θ is angle of inclination of plane with horizontal as θ increases cos θ decreases so f_s decrease and vehicle can skid in backward direction.

If the f is smaller, forward reaction by road to move vehicle become smaller.

Hence, roads are made winding up to decrease θ, the angle of inclination of road with horizontal, to avoid skidding and to get larger reaction force by road.

SHORT ANSWER TYPE QUESTIONS

Q5.26. A mass of 2 kg is suspended with thread AB (figure). Thread CD of the same type is attached to the other end of 2 kg mass. Lower thread is pulled gradually, harder and harder in the downward direction so as to apply force on AB. Which of the threads will break and why?

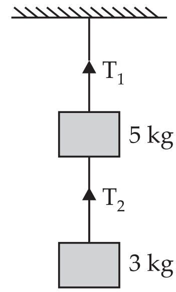


Ans. Thread AB will break. Force on CD is equal to the force (F) applied at D downward, but force on thread AB is equal to the force F along with force due to mass 2 kg downward. So force on AB is 2g Newton more than applied force at D. Hence the thread AB will breakup.

Q5.27. In the above given problem if the lower thread is pulled with a jerk, what happens? (Fig. in Q. 5.26)

Ans. Thread CD will break up if CD is pulled with jerk, because pull on thread CD is not transmitted from CD to AB by mass 2 kg due to its **inertia of rest**.

Q5.28. Two masses of 5 kg and 3 kg are suspended with the help of massless inextensible strings as shown in figure. Calculate T_1 and T_2 when whole system is going upwards with acceleration = 2 ms^{-2} (use g = 9.8 ms^{-2})



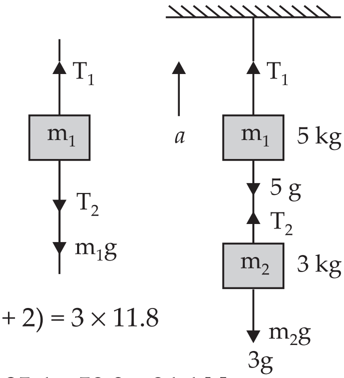
Ans. As the whole system is going up with acceleration = a = 2 ms^{-2}

m_1 = 5 kg $\qquad$ m_2 = 3 kg $\qquad$ g = 9.8 m/s^2

Tension a string is equal and opposite in all parts of a string.

Forces on mass m_1

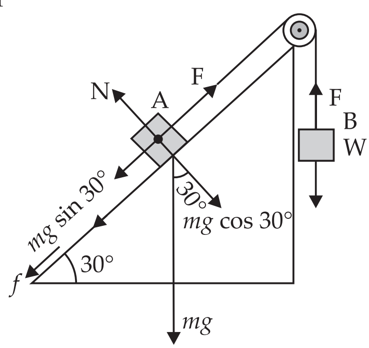
$$T_1 - T_2 - m_1 g = m_1 a$$
$$T_1 - T_2 - 5g = 5a$$
$$T_1 - T_2 = 5g + 5a$$
$$T_1 - T_2 = 5(9.8 + 2)$$
$$= 5 \times 11.8$$
$$T_1 - T_2 = 59.0 \text{ N}$$



Forces on mass m_2

$$T_2 - m_2 g = m_2 a$$
$$T_2 = m_2(g + a) = 3(9.8 + 2) = 3 \times 11.8$$
$$T_2 = 35.4$$
$$T_1 = T_2 + 59.0 \Rightarrow T_1 = 35.4 + 59.0 = 94.4 \text{ N}$$

Q5.29. Block A of weight 100 N rests on a frictionless inclined plane of slope angle 30° (figure). A flexible cord attached to A passes over a frictionless pulley and is connected to block B of weight W. Find the weight W for which the system is in equilibrium.



Main concept used: On balanced condition i.e., no motion then no frictional force or f = 0.

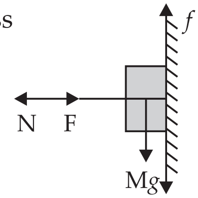
Ans. During equilibrium of A or B

$$mg \sin 30° = F$$
$$\frac{1}{2} mg = F \qquad [\because mg = 100 \text{ N}]$$
$$\therefore \qquad F = \frac{1}{2} \times 100 = 50$$

For B is at rest $\quad$ W = F = 50 N.

Q5.30. A block of mass M is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is μ and the acceleration due to gravity g, calculate the minimum force required to be applied by the finger to hold the block against the wall?

Ans. Let F force is applied by finger on a body of mass M to hold the block rest against the wall.



Under the balanced condition

$$F = N$$

and $$f = Mg$$

$$\mu N = Mg$$

$$\mu F = Mg \quad \text{or} \quad \boxed{F = \frac{Mg}{\mu}}$$ is the minimum force to hold the block against the wall at rest.

Q5.31. A 100 kg gun fires a ball of 1 kg horizontally from a cliff of height 500 m. It falls on the ground at a distance of 400 m from the bottom of the cliff. Find the recoil velocity of the gun (acceleration due to gravity = 10 ms^{-2}).

Main concept used: Speed of recoil of gun can be find out by the velocity of bullet by projectiles formulae.

Ans. Let the horizontal speed of bullet u ms^{-1} its vertical component will be zero.

Consider the motion of bullet vertically downward

$u = 0 \qquad s = h = 500$ m $\qquad g = 10$ ms^{-2}

$$s = ut + \frac{1}{2}at^2$$

$$500 = 0 \times t + \frac{1}{2} \times 10t^2 \Rightarrow t^2 = \frac{500}{5} = 100$$

$$t = \sqrt{100} = 10 \text{ sec}$$

$$\text{Horizontal range} = u \times 10$$

$$400 = u \times 10 \Rightarrow u = 40 \text{ m/s}$$

By the law of conservation of momentum

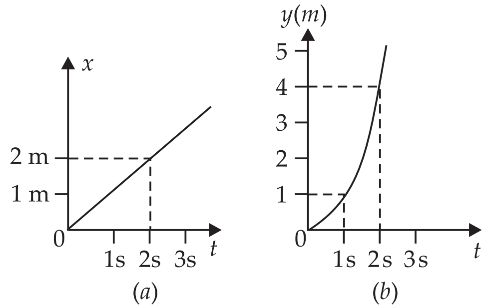
$$m_b u_b + M_G u_g = m_b v_b + M_G v_G$$

$$m_b \times 0 + M_G \times 0 = 1 \times 40 + 100 v_G$$

$$100\, v_G = -40$$

Recoil velocity of Gun = $\frac{-40}{100}$ ms^{-1} = $\frac{-2}{5}$ ms^{-1} opposite to the speed of ball.

Q5.32. Figure (a) and (b), shows (x, t), (y, t) diagram of a particle moving in 2 dimensions. If the particle has a mass of 500 g, find the force (direction and magnitude) acting on the particle.



Ans. From graph (a)

$$v_x = \frac{dx}{dt} = \frac{2}{2} = 1 \text{ ms}^{-1}$$

$$a_x = \frac{d^2x}{dt^2} = \frac{dv_x}{dt} = 0$$

From figure (*b*) $\quad y = t^2$

$$v_y = \frac{dy}{dt} = 2t$$

$$a_y = \frac{dv_y}{dt} = 2$$

$\therefore \quad F_y = ma_y \qquad m = 500\ g = .5\ kg$

$F_y = .5 \times 2 = 1$ N toward Y-axis

$F_x = .5 \times 0 = 0$ N

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{0^2 + 1^2} = \sqrt{1}$$

F = 1 N toward Y-axis.

Q5.33. A person in an elevator accelerating upwards with an acceleration of 2 ms^{-2}, tosses a coin vertically upwards with a speed of 20 ms^{-1}. After how much time will the coin fall back into his hand? ($g = 10\ ms^{-2}$)

Main concept used: Effective acceleration (a') when, elevator is going upward with acceleration (a)

$$a' = (a + g)$$

Ans. Upward acceleration of elevator (a) = 2 m/s^2

Acceleration due to gravity (g) = 10 ms^{-2}

$\therefore$ Net effective acceleration $a' = (a + g) = (2 + 10)$

$$a' = 12\ ms^{-2}$$

Consider the effective motion of coin

$v = 0 \qquad t$ = time of coin to achieve maximum height

$u = 20\ ms^{-1} \qquad a' = 12\ ms^{-2}$

$\therefore \quad v = u + at \qquad$ here $a = a'$

$0 = 20 - 12t \qquad$ (upward motion)

$$t = \frac{20}{12}\ s = \frac{5}{3}\ s$$

Time of ascent is equal to time of decent.

$\therefore$ Total time to return in hand after achieving maximum height

$$= \frac{5}{3} + \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}\ \text{sec.}$$

LONG ANSWER TYPE QUESTIONS

Q5.34. There are three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ acting on a body, all acting on a point P on the body. The body is found to move with uniform speed.

(*a*) Show that the forces are coplanar.

(*b*) Show that the torque acting on the body about any point due to these three forces is zero.

Ans. (*a*) As the body is moving with uniform speed after the action of three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ on a point on body, so the acceleration of body is zero (as no circular motion) $\because$ F = ma, so the resultant force due to $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$$\vec{F}_1 + \vec{F}_2 = -\vec{F}_3 \quad \text{or} \quad \vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$$

Consider the forces $\vec{F}_1$ and $\vec{F}_2$ are in the plane of paper, the resultant of $\vec{F}_1$ and $\vec{F}_2$ will also be in the same plane of paper, in $[-(\vec{F}_1 + \vec{F}_2)]$ only direction is reverse on the same plane. As $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$. So $\vec{F}_3$ will be in the same plane i.e., $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are coplanar.

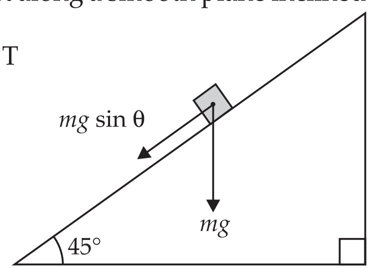
(*b*) As the resultant of $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ is zero and

$$\text{Torque} = r \times \vec{F} = 0 \qquad \text{as } \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

So torque acting on body at any point will be always zero.

Q5.35. When a body slides down from rest along a smooth inclined plane making an angle of 45° with the horizontal, it takes time T. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it is seen to take time pT, where p is some number greater than 1. Calculate the co-efficient of friction between the body and the rough plane.

Ans. As the body slides down from rest along a smooth plane inclined at angle 45° in Time T



$u = 0 \qquad s = s \qquad t = \text{T}$

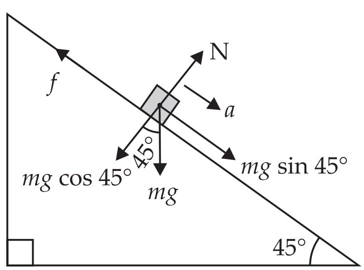
$$a = g \sin 45° = \frac{g}{\sqrt{2}}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}\frac{g}{\sqrt{2}}\text{T}^2$$

$$s = \frac{g\text{T}^2}{2\sqrt{2}}$$

Motion of body along rough inclined plane



$$u = 0, \qquad s = \frac{g\text{T}^2}{2\sqrt{2}} \quad ...(i)$$

$$ma = mg \sin 45° - f$$

$$= mg\frac{1}{\sqrt{2}} - \mu\text{N}$$

$$= \frac{mg}{\sqrt{2}} - \mu\, mg \cos 45° = mg\left[\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right]$$

$$ma = \frac{mg}{\sqrt{2}}\left[1 - \mu\right] \Rightarrow a = \frac{g}{\sqrt{2}}(1 - \mu)$$

$$t = p\text{T}, \quad s = s,\ a = \frac{g}{\sqrt{2}}(1 - \mu)$$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}.\frac{g}{\sqrt{2}}(1-\mu)(p\mathrm{T})^2$$

$$s = \frac{g}{2\sqrt{2}}(1-\mu)\,p^2\mathrm{T}^2 \qquad ...(ii)$$

Distances in both cases are equal (given)

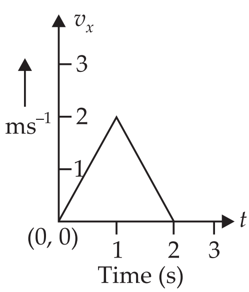
$$\frac{g\mathrm{T}^2}{2\sqrt{2}} = \frac{g}{2\sqrt{2}}(1-\mu)\,p^2\mathrm{T}^2$$

$$1 = (1-\mu)p^2 \quad \Rightarrow \quad 1 = p^2 - \mu p^2$$

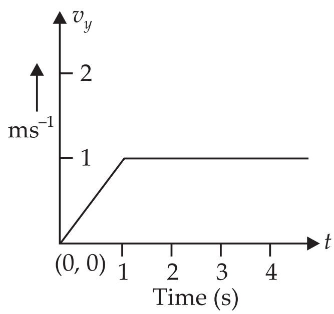
$$\mu p^2 = p^2 - 1$$

$$\mu = \left[1 - \frac{1}{p^2}\right]$$

Q5.36. Figure (*a*) and (*b*) shows ($v_x - t$) and ($v_y - t$) diagrams for a body of unit mass. Find the force, as a function of time.



(*a*)



(*b*)

Ans. Consider Figure (*a*)

$$v_x = 2t \qquad \text{for } 0 < t < 1 \text{ s}$$

$$a_x = 2(2-t) \qquad \text{for } 1 < t < 2 \text{ s}$$

$$a_x = \frac{2}{1} = 2 \qquad \text{for } 0 < t < 1 \text{ s}$$

$$a_x = \frac{-2}{1} = -2 \qquad \text{for } 1 < t < 2 \text{ s}$$

$$\therefore \quad \mathrm{F}_x = ma \qquad \because m = 1 \text{ unit}$$

$$\mathrm{F}_x = 1 \times 2 = 2 \text{ unit} \qquad \text{for } 0 < t < 1 \text{ s}$$

$$\mathrm{F}_x = 1 \times (-2) = -2 \text{ units} \qquad \text{for } 1 < t < 2 \text{ s}$$

From Figure (*b*)

$$a_y = \frac{1}{1} = 1 \text{ ms}^{-2} \qquad 0 < t < 1$$

$$\mathrm{F}_y = ma = 1.1 = 1 \text{ unit} \qquad \text{for } 0 < t < 1$$

$$a_y = 0 \quad \text{for} \quad 1 < t$$

$$\vec{\mathrm{F}}_y = 1 \times 0 = 0 \text{ units} \qquad \text{for } 1 < t < 2 \text{ s}$$

$$\vec{\mathrm{F}} = \vec{\mathrm{F}}_x \hat{i} + \vec{\mathrm{F}}_y \hat{j}$$

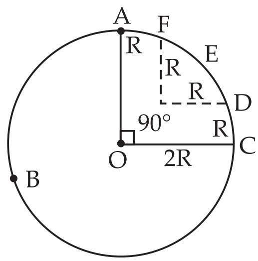
$$\boxed{\vec{F} = 2\hat{i} + 1\hat{j}} \qquad \text{for } 0 < t < 1 \text{ s}$$

$$\vec{F} = -2\hat{i} + 0\hat{j} \qquad \text{for } 1 < t < 2 \text{ s}$$

$$\boxed{\vec{F} = -2\hat{i}}$$

For more than 2 sec $\left.\begin{array}{l} a_y = 0 \\ a_x = 0 \end{array}\right\}$ $\because \vec{F} = 0$

Q5.37. A racing car travels on a track (without banking) ABCDEFA (figure). ABC is a circular arc of radius 2R. CD and FA are straight paths of length R and DEF is a circular arc of radius R = 100 m. The co-efficient of friction on the road is μ = 0.1. The maximum speed of the car is 50 ms^{-1}. Find the minimum time for completing one round.



Main concept used: The centripetal force to keep car in circular motion is provided by frictional force inward to centre O.

Ans. (*i*) **Time taken from A → B → C**

$$s_1 = \text{length of path} = \frac{3}{4} 2\pi\,(2R) = \frac{3}{4} \times 4\pi \times 100 = 300\pi \text{ m}$$

v_1 = maximum speed of car along the circular path

$$= \sqrt{\mu r g} = \sqrt{0.1 \times 2R \times g}$$

$$v_1 = \sqrt{0.1 \times 2 \times 100 \times 10} = \sqrt{200} = 10 \times \sqrt{2} \text{ m/s}$$

$$\therefore \quad t_1 = \frac{s_1}{v_1} = \frac{300\pi}{10\sqrt{2}} \text{ s} = \frac{30\pi}{\sqrt{2}} \text{ s} = \frac{30 \times 3.14\sqrt{2}}{2}$$

$$= 30 \times 1.57 \times 1.4 = 65.94 \text{ s}$$

(*ii*) **Time from C → D and F → A**

$$s_2 = CD + FA = R + R = 100 + 100 = 200 \text{ m}$$

As path CD and FA are in straight so car will travel with it's maximum speed v_2 = 50 m/s

$$\therefore \quad t_2 = \frac{s_2}{v_2} = \frac{200}{50} = 4 \text{ sec}$$

(*iii*) **Time for path D → E → F is**

$$s_3 = \frac{1}{4} 2\pi R = \frac{1}{4} \times 2\pi \times 100 = 50\pi$$

$$v_3 = \sqrt{\mu r g} = \sqrt{0.1 \times R \times g} = \sqrt{0.1 \times 100 \times 10} = 10 \text{ m/s}$$

$$t_3 = \frac{s_3}{v_3} = \frac{50\pi}{10} = 5\pi \text{ sec} = 5 \times 3.14 = 15.70 \text{ s}$$

Total time taken by car = $t_1 + t_2 + t_3$

$$t = 65.94 + 4 + 15.70 = 85.6 \text{ s.}$$

Q5.38. The displacement vector of a particle of mass m is given by

$$\vec{r}(t) = \hat{i}\,\text{A}\cos\omega t + \hat{j}\,\text{B}\sin\omega t$$

(*a*) Show that the trajectory is an ellipse

(*b*) Show that $\vec{\text{F}} = -m\omega^2\vec{r}$

Main concept used: To plot the graph $(r - t)$ or trajectory we relate x and y co-ordinates.

Ans. (*a*) $\vec{r}(t) = \hat{i}\,\text{A}\cos\omega t + \hat{j}\,\text{B}\sin\omega t$

$x = \text{A}\cos\omega t$ and $y = \text{B}\sin\omega t$

$\dfrac{x}{\text{A}} = \cos\omega t$ ...(*i*) $\qquad \dfrac{y}{\text{B}} = \sin\omega t$ ...(*ii*)

Squaring and adding (*i*), (*ii*)

$$\frac{x^2}{\text{A}^2} + \frac{y^2}{\text{B}^2} = \cos^2\omega t + \sin^2\omega t$$

$\boxed{\dfrac{x^2}{\text{A}^2} + \dfrac{y^2}{\text{B}^2} = 1}$ it is the equation of ellipse. So the trajectory is ellipse.

(*b*)

$x = \text{A}\cos\omega t$

$v_x = \dfrac{dx}{dt} = -\text{A}\omega\sin\omega t$

$a_x = \dfrac{dv_x}{dt} = -\text{A}\omega^2\cos\omega t$

$y = \text{B}\sin\omega t$

$v_y = \dfrac{dy}{dt} = \text{B}\omega\cos\omega t$

$a_y = \dfrac{dv_y}{dt} = -\text{B}\omega^2\sin\omega t$

$$\begin{aligned} a &= a_x\hat{i} + a_y\hat{j} \\ &= -\hat{i}\text{A}\omega^2\cos\omega t - \hat{j}\text{B}\omega^2\sin\omega t \\ &= -\omega^2\left[\hat{i}\text{A}\cos\omega t + \hat{j}\text{B}\sin\omega t\right] \\ a &= -\omega^2 r(t) \end{aligned}$$

$\therefore$ Force acting on particle $= ma = -m\omega^2\vec{r}(t)$

Q5.39. A cricket bowler releases the ball in two different ways:

(*a*) giving it only horizontal velocity, and

(*b*) giving it horizontal velocity and a small downward velocity. The speed v_s at the time of release is the same. Both are released at a height H from the ground. Which one will have greater speed when the ball hits the ground? Neglect air resistance.

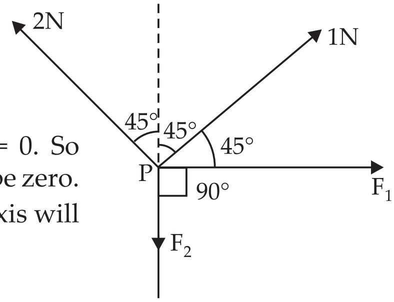
Ans. For (*a*) $\qquad v_z^2 = 2g\text{H} \Rightarrow v_z = \sqrt{2gH}$

$$\text{Speed at ground} = \sqrt{v_s^2 + v_z^2} = \sqrt{v_s^2 + 2g\text{H}}$$

For (*b*) also $\left[\dfrac{1}{2}mv_s^2 + mg\text{H}\right]$ is the total energy of the ball when it hits the ground.

So the speed would be the same for both (*a*) and (*b*).

Q5.40. There are four forces acting at a point P produced by strings as shown in figure, which is at rest. Find the forces F_1 and F_2.



Ans. As the particle is rest or $a = 0$. So resultant force due to all forces will be zero.

$\therefore$ Net components along X and Y-axis will be zero.

Resolving all forces along X-axis

$$F_x = 0$$

$$F_1 + 1 \cos 45° - 2 \cos 45° = 0 \quad \text{or} \quad F_1 - 1 \cos 45° = 0$$

$$F_1 = \cos 45° = \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.414}{2} = 0.707 \text{ N Ans.}$$

Resolving all forces along Y-axis

$$F_y = 0$$

$$-F_2 + 1 \cos 45° + 2 \cos 45° = 0$$

$$-F_2 = -3 \cos 45°$$

$$F_2 = 3.\frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = \frac{3 \times 1.414}{2} = 3 \times 0.707 = 2.121 \text{ N}.$$

Q5.41. A rectangular box lies on a rough inclined plane. The co-efficient of friction between surface and box is μ. Let mass of the box is *m*.

(*a*) At what angle of inclination θ of the plane to the horizontal will the box just start to slide down the plane?

(*b*) What is the force acting on the box down the plane, if the angle of inclination of the plane is increased to α > θ?

(*c*) What is the force needed to be applied upwards along the plane to make the box either remain stationary or just move up with uniform speed?

(*d*) What is the force needed to be applied upwards along the plane to make the box move up the plane with acceleration *a*?

Ans. (*a*) As the box just start to slide down the plane then μ = tan θ (by angle of repose) $\boxed{\theta = \tan^{-1}(\mu)}$

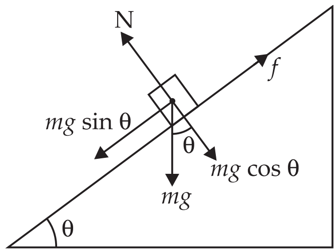
(*b*) If angle α > θ, the angle of inclination of plane with horizontal it will **slide down** (*f* upward) as θ is angle of repose. So net force downward

$$F_1 = mg \sin \alpha - f = mg \sin \alpha - \mu N$$

$$= mg \sin \alpha - \mu\, mg \cos \alpha$$

$$F_1 = mg [\sin \alpha - \mu \cos \alpha]$$

(*c*) To keep the box either stationary or just move it up with uniform velocity ($a = 0$) upward (f downward)



$$F_2 - mg \sin \alpha - f = ma$$

or $\quad F_2 - mg \sin \alpha - \mu N = 0 \ (\because a = 0)$

$$F_2 = mg \sin \alpha - \mu\, mg \cos \alpha$$
$$= mg (\sin \alpha - \mu \cos \alpha)$$

(*d*) The force applied F_3 to move the box upward with acceleration a, $F_3 - mg \sin \alpha - \mu\, mg \cos \alpha = ma$

$\therefore$ $F_3 = mg (\sin \alpha + \mu \cos \alpha) + ma$

Q5.42. A helicopter of mass 2000 kg rises with a vertical acceleration of 15 ms^{-2}. The total mass of the crew and passengers is 500 kg. Give the magnitude and direction of the ($g = 10\ ms^{-2}$).

(*a*) force on the floor of the helicopter by the crew and passengers.

(*b*) action of the rotor of the helicopter on the surrounding air.

(*c*) force on the helicopter due to the surrounding air.

Ans. Mass (M) of helicopter = M = 2000 kg

Mass of the crew and passengers = m = 500 kg.

Acceleration of helicopter alongwith crew and passengers = 15 ms^{-2}

(*a*) Force on floor of helicopter by crew and passenger will be equal to apparent weight (left)

$$= m(g + a) = 500\ (10 + 15)$$
$$F_1 = 500 \times 25 = 12500 \text{ N downward}$$

(*b*) Action of the rotor of helicopter on surrounding air will be equal to the reaction force by Newton's third law due to which helicopter along with crew and passenger rises up with acceleration on 15 ms^{-2} = (M + m) (g + a). So the action by rotor on surrounding air

$$= (2000 + 500)\ (g + a)$$
$$F_2 = 2500 \times (10 + 15) = 2500 \times 25 = 62500 \text{ N downward.}$$

(*c*) Force (F_3) acting on the helicopter by the reaction force by surrounding air.

So $\quad F_3$ = – Action force (by Newton's third law)

= – 62500 N downward

or $\quad F_3$ = + 62500 N upward.

❑❑❑

6 Work, Energy and Power

MULTIPLE CHOICE QUESTIONS-I

Q6.1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because:

(*a*) the two magnetic forces are equal and opposite, so they produce no net effect.

(*b*) the magnetic forces do, no work on each particle.

(*c*) the magnetic forces do equal and opposite (but non-zero) work on each particle.

(*d*) the magnetic forces are necessarily negligible.

Main concept used: The direction of force acting on a charge particle is perpendicular to the direction of motion.

Ans. (*b*): By Flemings left hand rule the direction of force acting on a harge particle is perpendicular to the direction of motion and magnetic field.

$$\text{W.D.} = \text{F}.s \cos\theta = \text{F}.s \cos 90° = 0$$

So, magnetic forces do no work on moving charge particle verifies option (*b*).

Q6.2. A proton is kept at rest. A positively charged particle is released from rest at a distance *d* in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time (*t*), the work done on the two moving charged particles is

(*a*) same as the same force law is involved in the two experiments.

(*b*) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.

(*c*) more for the case of a positron, as the positron moves away a larger distance.

(*d*) same as the work done by charged particle on the stationary proton.

Main concept used: W.D. = $\vec{\text{F}}.\vec{s}$ and $\text{F}_e = \dfrac{kq_1q_2}{r^2}$

Ans. (*c*): As the charges on proton and positron are equal and mass of proton is 1836 times larger of positron.

So force by Coulomb's law on positron and proton in both experiment will be same. As positron is too much lighter so it will move faster than proton by same force. Hence, the distance covered by positron is larger than proton. So WD on positron will be larger than proton verifies the option (*c*).

Q6.3. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is:

(*a*) constant and equal to mg in magnitude.

(*b*) constant and greater than mg in magnitude.

(*c*) variable but always greater than mg.

(*d*) at first greater than mg, and later becomes equal to mg.

Ans. (*d*): When the man squatting on the ground, he is tilted somewhat, hence he also has to apply force of friction besides his weight.

$$\text{Reaction force (R)} = mg + \text{frictional force } (f)$$

$$R > mg$$

When the man does not squat and standby then no frictional force acts at this situation $f = 0$ and $R = mg$

Hence, the reaction force R is larger when squatting and become equal to mg when no squatting verifies option (*d*).

Q6.4. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200 N and is directly opposite to the motion. The work done by the cycle on the road is:

(*a*) + 2000 J (*b*) – 200 J (*c*) Zero (*d*) –20,000 J

Ans. (*c*): W.D. = $\vec{F} \cdot \vec{s}$

W.D. by the bicycle on the road W.D. = Force × 0 = 0.

As cycle cannot displaced road. So W.D. is zero verifies the option (*c*).

Q6.5. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?

(*a*) Kinetic energy (*b*) Potential energy

(*c*) Total mechanical energy (*d*) Total linear momentum

Main concept used: Law of conservation of energy and force of friction.

Ans. (*c*): As the motion is under gravity, and no external force acts on body in vacuum, so law of conservation of energy hold good i.e., during free fall sum of KE and PE remains constant as KE increases as much as PE decreases, so verifies the option (*c*).

Q6.6. During inelastic collision between two bodies, which of the following quantities always remains conserved?

(*a*) Total kinetic energy (*b*) Total mechanical energy

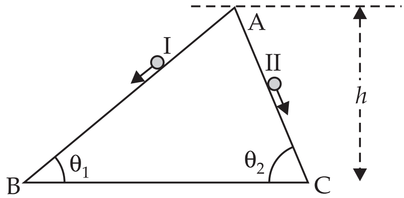
(*c*) Total linear momentum (*d*) Speed of each body

Main concept used: In an inelastic collision some energy lost in the form of heat and sound. As no external force on system so momentum remains conserved.

Ans. (*c*): In an inelastic collision KE does not conserved but linear momentum remains conserved verifies option (*c*).

Q6.7. Two inclined frictionless tracks, one gradual and other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.

Which of the following statement is correct?



(*a*) Both the stones reach the bottom at the same time but not with the same speed.

(*b*) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.

(*c*) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.

(*d*) Both the stones reach the bottom at different times and with different speed.

Ans. (*c*): As the inclined surfaces are frictionless, and no external force acts on system. So law of conservation of mechanical energy holds good.
KE + PE of stone I at top = KE + PE at bottom of I

$$0 + mgh = \frac{1}{2}mv_1^2 + 0$$

Hence, $v_1 = \sqrt{2gh}$ similarly $v_2 = \sqrt{2gh}$

As the $a_1 = g \sin \theta_1$

$a_2 = g \sin \theta_2$ as $\theta_1 < \theta_2$

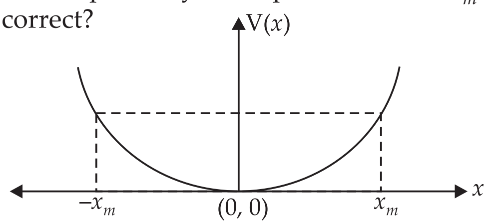
$\therefore$ $\sin \theta_1 < \sin \theta_2$

or $a_1 < a_2$

As the a_2 is larger so stone II reach earlier than I.
As PE of both stones at A are equal as masses are equal. So their KE at the bottom will be same.
Hence, verifies the option (*c*).

Q6.8. The potential energy function for a particle executing linear S.H.M. is given by:

$V(x) = \frac{1}{2}kx^2$ where k is the force constant of the oscillator (figure). For $k = 0.5$ N/m, the graph of V(x) versus x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and k indicate the P.E. and K.E. respectively of the particle at $x = +x_m$, then which of the following correct?



(*a*) V = 0, K = E

(*b*) V = E, K = 0

(*c*) V < E, K = 0

(*d*) V = 0, K < E

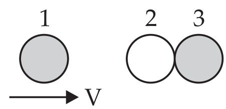
Ans. (*b*, *a*): In S.H.M.

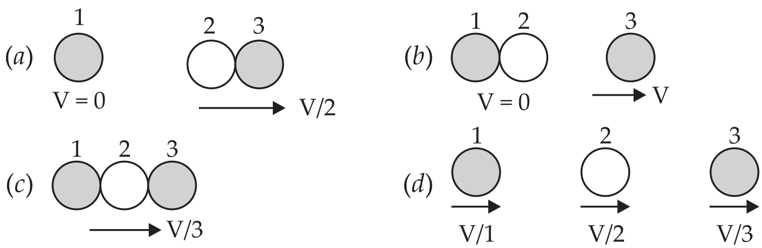
Mechanical energy E = KE + PE at any instant

$$E = K + V \qquad ...(i)$$

At extreme position particle return back and its velocity become zero for an instant. So at $x = x_m$. The K = 0. So V = E from (*i*) verifies the option (*b*).
At its mean position at origin V(x) = 0.
$\therefore$ K = E from (*i*), verifies option (*a*).

Q6.9. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V as shown in figure here. If the collision is elastic, which of the following figure (below) is a possible result after collision?





Ans. (*b*): As the collision is elastic, and balls 1, 2, 3 are identical in all respect, during motion of balls there is no force of friction.
When ball 1 collide it transfer it's own all momentum $p = mv$ to ball 2 and itself become $p_1 = 0 \Rightarrow v_1 = 0$ and $p_2 = m\text{V}$. So $v_2 = \text{V}$.
Now ball 2 strike to ball 3 and it transfer it's momentum $p_2 = m\text{V}$ to ball 3 and itself comes in rest
$\therefore\ p_2 = 0 \quad \Rightarrow \quad v_2 = 0 \quad$ and $\quad p_3 = m\text{V}$. So $v_3 = \text{V}$
So ball 1 and ball 2, become in rest and ball 3, move with velocity V in forward direction. So verifies option (*b*).

Q6.10. A body of mass 0.5 kg travels in the straight line with velocity $v = ax^{3/2}$ where $a = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is:

(*a*) 1.5 J (*b*) 50 J (*c*) 10 J (*d*) 100 J

Ans. (*b*): $$\text{W.D.} = \int_{s_1}^{s_2} \vec{\text{F}}.\overrightarrow{ds} = \int_{s_1}^{s_2} m\vec{a}.\overrightarrow{ds}$$

$m = 0.5$ kg, $\quad a = m^{-1/2}\ \text{s}^{-1}$

$$v = ax^{3/2} \quad \text{or} \quad v = 5x^{3/2} \qquad ...(i)$$

$$a = \frac{dv}{dt} = \frac{dv}{dt}.\frac{dx}{dx} = \frac{dx}{dt}.\frac{dv}{dx} = v.\frac{d}{dx}5x^{3/2}$$

$$a = 5x^{3/2}.5 \times \frac{3}{2}x^{1/2} = \frac{75}{2}x^2 \qquad \text{[using } (i)\text{]}$$

$$\text{F} = ma = 0.5 \times \frac{75}{2}x^2 = \frac{75}{4}x^2\ \text{N}$$

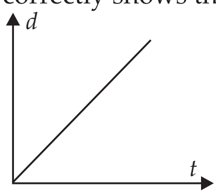
$$\text{W.D.} = \int_{x=0}^{x=2} F.dx = \int_{x=0}^{x=2} \frac{75}{4}x^2\, dx = \left[\frac{75}{4} \times \frac{x^3}{3}\right]_{x=0}^{x=2}$$

$$\text{W.D.} = \frac{75}{4 \times 3}\left[(2)^3 - 0^3\right] = \frac{25}{4} \times 8 = 50 \text{ J}$$

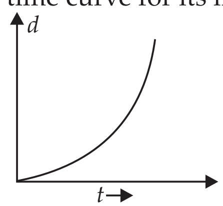
Verifies the option (*b*).

Q6.11. A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in figure correctly shows the displacement-time curve for its motion?

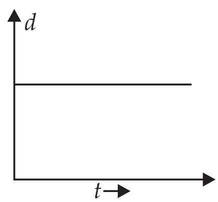
(*a*)



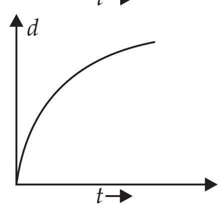
(*b*)



(*c*)



(*d*)



Ans. (*b*): $$P = \frac{dW}{dt} = \frac{d}{dt}\vec{F}.\overrightarrow{dx} = F.\frac{dx}{dt}$$

As the body is moving unidirectionally

$$\vec{F}.\overrightarrow{dx} = F\, dx \cos 0° = F\, dx \qquad [\because \cos 0° = 1]$$

$$P = F.\frac{dx}{dt}$$

$$\therefore \quad F.\frac{dx}{dt} = \text{constant} \qquad [\because P = \text{constant}]$$

$$\Rightarrow \quad F.v = 0$$

$$[F]\,[v] = \text{constant} \qquad \text{(Dimensional formula)}$$

$$[MLT^{-2}]\,[LT^{-1}] = \text{constant}$$

$$[ML^2T^{-3}] = \text{constant}$$

$$L^2 = \frac{T^3}{M}$$

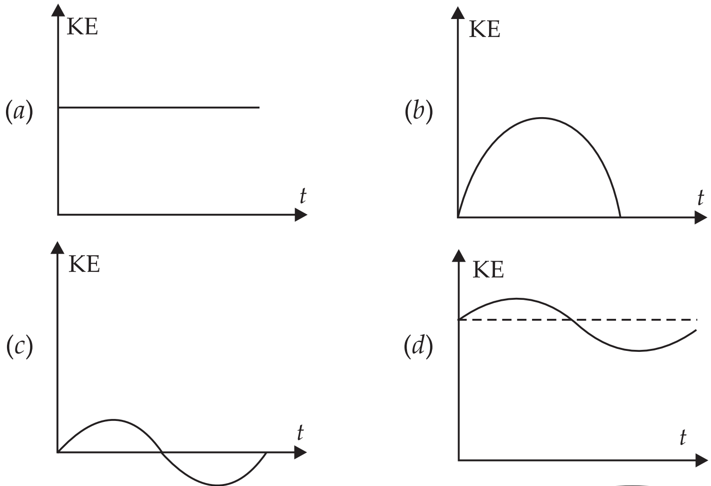
As mass of body constant

$$\therefore \quad L^2 \propto T^3$$

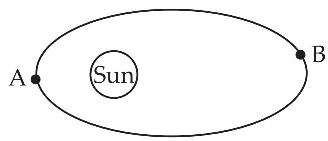
$$L \propto T^{3/2} \quad \text{or } d \propto T^{3/2}$$

Verifies the graph (curve) (*b*).

Q6.12. Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?

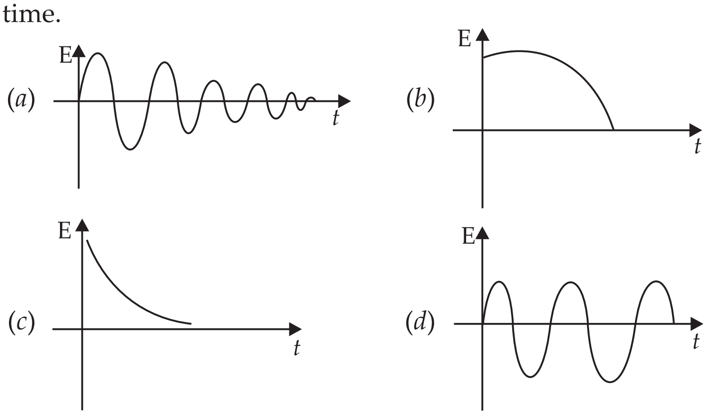


Ans. (*d*): The speed of earth around the sun can never be zero or negative so the kinetic energy of earth cannot be zero and negative so option (*b*) and (*c*) rejected.



As the speed of earth around the sun is minimum when farthest from sun at B and maximum when nearest at A to the sun.
Hence, the KE is not constant so rejected option (*a*). KE of earth increases B to A and then decreases so verifies option (*d*).

Q6.13. Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as function of time.



Ans. (*c*): Due to air resistance the force of friction acts between bob of pendulum and air, so amplitude of vibration decreases so, the total energy (KE + PE) decreases continuously and finally becomes zero. sum of KE and PE can never be negative, so option (*a*) and (*d*) rejected.

As the velocity increases the force of friction increases and velocity decreases with amplitude i.e., sum of KE and PE decreases rapidly initially and slowly afterward. It verifies option (*c*) and rejects option (*b*).

Q6.14. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves 300 revolution per minute its KE will be:

(*a*) $250\pi^2$ J (*b*) $100\pi^2$ J (*c*) $5\pi^2$ J (*d*) 0 J

Ans. (*a*): $m = 5$ kg, $r = 1$ m

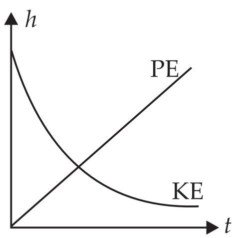
$$\omega = \frac{2\pi n}{t} = \frac{2\pi \times 300}{60} = 10\pi \text{ rad/s}$$

$$v = \omega r = 10\pi \times 1 = 10\pi \text{ m/s.}$$

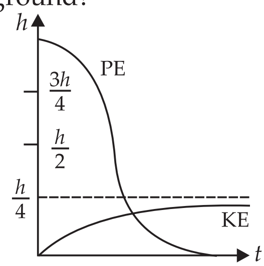
$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 10\pi \times 10\pi = 250\pi^2 \text{ J}.$$ So verifies the option (*a*).

Q6.15. A raindrop falling from a height h above the ground, attains a near terminal velocity when it has fallen through a height $\frac{3}{4}h$. Which of the diagrams shown in figure correctly shows the change in KE and PE of the drop during it's fall up to the ground?

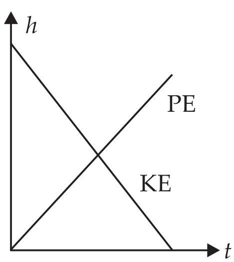
(*a*)



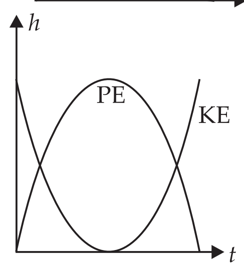
(*b*)



(*c*)



(*d*)



Ans. (*b*): PE is maximum when drop start falling at $t = 0$ as it fall is PE decreases gradually to zero. Rejects the graph (*a*), (*c*) and (*d*).

KE at $t = 0$ is zero as drop falls with zero velocity it increases gradually till its velocity become equal to terminal (constant) velocity, after this velocity of drop remains constant i.e., KE remain constant it happens when it falls $\frac{3}{4}$ height or remains at $\frac{h}{4}$ from ground. Hence verifies the option (*b*).

Q6.16. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1 m s^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 ms^{-2}, the KE of the shotput when it just reaches the ground will be:

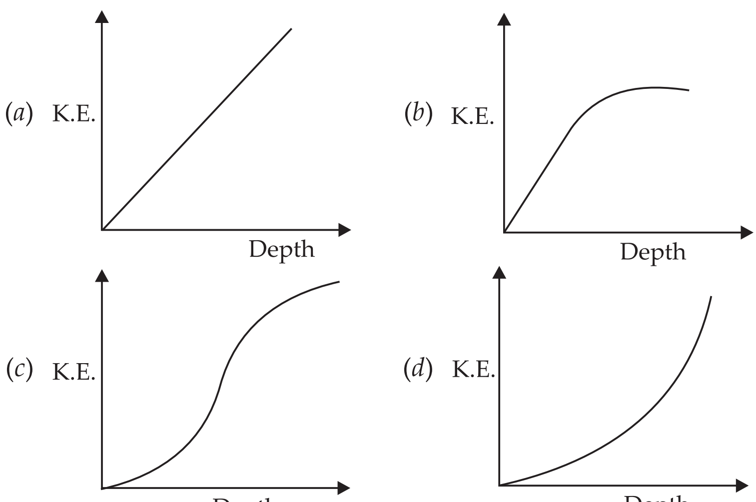
(*a*) 2.5 J (*b*) 5.0 J (*c*) 52.5 J (*d*) 155.0 J

Ans. (*d*): By the law of conservation of energy as no force acts on shotput after thrown.

$m = 10$ kg $h = 1.5$ m $v = 1$ m/s $g = 10$ ms^{-2}

$$\begin{aligned} \text{Initial KE + PE} &= \text{Final KE + PE} \\ \frac{1}{2}mv_i^2 + mgh_i &= \text{KE} + 0 \\ \frac{1}{2}\times 10\times 1\times 1 + 10\times 10\times 1.5 &= \text{Total energy when it reaches ground} \\ 5 + 150 &= \text{Total E} \\ \text{E} &= 155 \text{ J. Verifies option } (d). \end{aligned}$$

Q6.17. Which of the diagrams in figure correctly shows the change in kinetic energy of an iron sphere falling freely in the lake having sufficient depth to impart it a terminal velocity?



Ans. (*b*): When iron sphere is dropped into the lake its velocity increase from zero to $v = \sqrt{2gh}$ after sometime its velocity become constant due to resistance of sphere and water. Verifies the option (*b*).

Q6.18. A cricket ball of mass 150 g moving with a speed of 126 km/hr hits at the middle of bat, held firmly at its position by the batsman. The ball moves straight back to bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001s, the force that the batsman had to apply to hold the bat firmly at its place would be:

(*a*) 10.5 N (*b*) 21 N (*c*) 1.05×10^4 N (*d*) 2.1×10^4 N

Ans. (*c*): $m = 150$ g $= 0.15$ kg $\quad u = 126 \times \frac{5}{18} = 35$ m/s

$t = 0.001$ s $\quad v = -35$ m/s final velocity is opp. to initial force applied by batsman

$$\frac{dp}{dt} = \frac{mv - mu}{t} = \frac{m(v-u)}{t}$$

$$= \frac{0.15[-35-35]}{0.001} = 0.15\,(-70) \times 10^{+3}\ \text{N}$$

$$F = -10.50 \times 10^{3} = -1.05 \times 10^{4}\ \text{N}$$

Negative sign shows that direction of force is opposite to initial velocity which taken positive direction. Hence, verifies the option (*c*).

MULTIPLE CHOICE QUESTIONS-II

Q6.19. A man, of mass *m*, standing at the bottom of the staircase, of height L climbs it and stands at its top.

(*a*) Work done by all forces on man is equal to the rise in potential energy *mgL*.
(*b*) Work done by *all* forces on man is zero.
(*c*) Work done by gravitational force on man is *mg*L.
(*d*) The reaction force from a step does not do work because the point of application of the force does not move while the force exists.

Ans. (*b*, *d*): WD by gravitation force on man is ($-mg$L) as gravitational force is downward and displacement L is upward. The WD by man to lift him up by muscular force will be ($+mg$L) as force applied by muscles is in the direction of displacement. So net WD $= -mg\text{L} + mg\text{L} = 0$. Verifies option (*b*).

As there is no displacement point where the reaction acts so, WD by reaction force is zero. As the velocity of person atmost zero at top. So KE = 0. Hence, WD by reaction force is zero. Verifies the option (*d*).

Q6.20. A bullet of mass *m* fired at 30° to the horizontal leaves the barrel of the gun with a velocity *v*. The bullet hits a soft target at a height *h* above the ground while it is moving downward and emerges out with half the kinetic energy, it had before hitting the target.

Which of the following statements are correct in respect of bullet after it emerges out of the target?

(*a*) The velocity of the bullet will be reduced to half its initial value.
(*b*) The velocity of the bullet will be more than half of its earlier velocity.
(*c*) The bullet will continue to move along the same parabolic path.
(*d*) The bullet will move in a different parabolic path.
(*e*) The bullet will fall vertically downward after hitting the target.
(*f*) The internal energy of the particles of the target will increase.

Ans. (*b*), (*d*), (*f*): (*a*) Let KE_2, KE_1 are the kinetic energy of bullet before and after hitting the target

$$1\ KE_2 = \frac{1}{2} KE_1$$

$$\frac{1}{2}mv_2^2 = \frac{1}{2}.\frac{1}{2}mv_1^2$$

$$v_2^2 = \frac{1}{2}v_1^2 = \left(\frac{v_1}{\sqrt{2}}\right)^2 = \left(\frac{v_1\sqrt{2}}{2}\right)^2 = (.707v_1)^2$$

$v_2 = 0.707v_1$. Hence, the velocity of bullet after target is not reduce to half. It rejects option (*a*).

(*b*) $v_2 = 0.707v_1$. So velocity of bullet after target is more than half of its earlier velocity verified option (*b*).

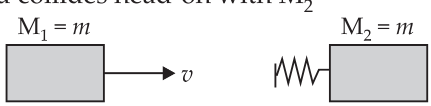
(*c*) Bullet has horizontal velocity so its path will be parabolic but with new parabola as both components v_x and v_y changes after emerging out from target. So rejects the option (*c*).

(*d*) As above discussed path of bullet after target will be of new parabola, verifies the option (*d*).

(*e*) As bullet has horizontal and vertical components so has new parabola of range smaller than previous. So rejects the option (*e*).

(*f*) As some parts of kinetic energy of bullet converted into heat so internal energy of target increased. Verifies option (*f*).

Q6.21. Two blocks M_1 and M_2 having equal mass are free to move on a horizontal frictionless surface. M_2 is attached to a massless spring as shown in figure. Initially M_2 is at rest and M_1 is moving towards M_2 with speed v and collides head-on with M_2



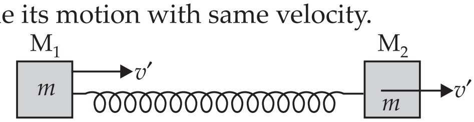
(*a*) While spring is fully compressed all the KE of M_1 is stored as PE of spring.

(*b*) While spring is fully compressed the system momentum is not conserved, though final momentum is equal to initial momentum.

(*c*) If the spring is massless, the final state of the M_1 is state of rest.

(*d*) If the surface on which blocks are moving has friction, then collision cannot be elastic.

Ans. (*c*): Consider the figure here when M_1 comes in contact with spring of M_2, then velocity of M_1 decreases and M_2 start moving with increasing velocity. When the velocities of both become equal, then M_1M_2 continue its motion with same velocity.



(*a*) All KE of M_1 does not transferred to PE of spring. So rejects option (*a*).

(*b*) As surfaces are frictionless. So law of conservation of mass hold good so rejects option (*b*).

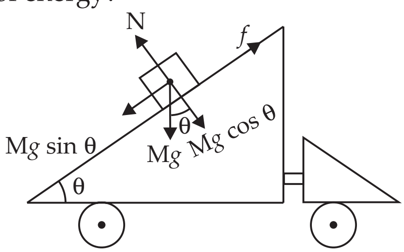
(*c*) If the spring is massless then whole KE of M_1 transferred to M_2, then M_2 moves with velocity v and M_1 becomes at rest. Verifies option (*c*).

(*d*) Collision is inelastic even if the force friction is not involve rejects the option (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q6.22. A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy?

Ans. As block is at rest on inclined plane as shown in figure.



$\therefore \quad f = Mg \sin \theta$

Force of friction on body is due to the tendency of block M to slide down over the inclined plane. As there is no displacement in block so work done by f and block is zero. As there is no work, so no dissipation of energy takes place.

Q6.23. Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?

Ans. When elevator is descending then it is not its free fall under gravity if decent with uniform speed. Power is require to increase the velocity due to free fall.

Power of motor or system of an elevator is constant and a limited or specified power can stop the speed of free falling of passenger along with elevator.

Q6.24. A body is being raised to a height h from the surface of earth. What is the sign of work done by: (*a*) applied force (*b*) gravitational force?

Ans. To raised a body up to height h a person will apply the force upward in the direction of displacement

(*a*) Work done by force is positive

$$WD = \vec{F}d\vec{s} = Fds \cos \theta \qquad \theta = 0°$$

$$\Rightarrow Fds \cos 0° = Fds \text{ is +ive.}$$

(*b*) Gravitational force is always downward, but displacement is upward here. So $\theta = 180°$

$$\therefore \quad WD = Fds \cos 180° = -Fds$$

Hence, WD by gravitational force is negative.

Q6.25. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m.

Ans. $\because$ $\quad$ WD = Fs cos θ

As the angle between horizontal distance 2 m and gravity vertically downward is 90°. So WD

$$WD = Fs \cos 90° = 0.$$

So work done by car against the gravity is zero.

Q6.26. A body falls towards earth in air. Will its total mechanical energy be conserved during the fall? Justify.

Ans. For a body falling freely under gravity. The mechanical energy is not conserved because some part of mechanical energy utilised against force of friction of air molecules which is non conservative force.

But if a body is falling freely under gravity in vacuum, the total mechanical energy remain conserved.

Q6.27. A body is moved along the closed loop. Is the work done in moving the body necessarily zero? If not, state the condition under which work done over a closed path is always zero.

Ans. Work done by a body moving along closed loop can be zero if only conservative force acting on the body during motion.

Work done by a body moving along a loop is not zero if any non-conservative force, *i.e.,* frictional, electrostatic, magnetic force are acting on body.

Q6.28. In an elastic collision of two billiard balls, which of the following quantities remain conserved during the short time of collision of the balls (i.e., when they are in contact).

(*a*) Kinetic energy (*b*) Total linear momentum.

Give the reason for your answer in each case.

Ans. When two billiard balls collide each other then their linear momentum and kinetic energy remains conserved. Because here it is considered that there is not any non-conservative force (like air resistance/friction on surface etc.) and speed of ball is not so high so that they deformed on collision.

Q6.29. Calculate the power of a crane in Watts, which lifts a mass 100 kg to a height of 10 m in 20 seconds.

Ans.
$$P = \frac{\text{W.D.}}{\text{time}} = \frac{F.s \cos\theta}{t} = \frac{mg.h\cos\theta}{t}$$

As the direction of displacement (height) and force applied by crane are same. So θ = 0°

$F = mg = 100 \times 10$ $\qquad h = 10$ m $\qquad t = 20$ s

$\therefore$
$$P = \frac{100 \times 10 \times 10 \cos 0°}{20} = 500 \text{ Watts.}$$

Q6.30. The average work done by human heart while it beats once is 0.5 J. Calculate the power used by heart if it beats 72 times in a minute.

Ans. $$P = \frac{\text{W.D.}}{\text{time}}$$

WD in 1 beat by heart = 0.5 J

WD in 72 beats by heart = $0.5 \times 72 = 36$ J

Time = 1 minute = 60 s

$$\therefore \qquad P = \frac{\text{W.D.}}{t} = \frac{36}{60} = \frac{6}{10} = 0.6 \text{ W}$$

Hence, power used by heart is 0.6 W.

Q6.31. Give an example of a situation in which an applied force does not result in a change in kinetic energy.

Ans. $$\text{WD} = F.s \cos\theta$$

WD in situation will be zero if the angle between force and displacement are 90°. Such as:

(*i*) If a body is moving horizontally with uniform motion the WD by gravity is zero.

(*ii*) WD during circular motion.

(*iii*) When the direction of motion of a charge particle is perpendicular to the magnetic field the direction of force due to the magnetic field by Fleming's left hand rule is perpendicular to both direction of motion of charge and magnetic field. So no work is done on charge particle by magnetic field.

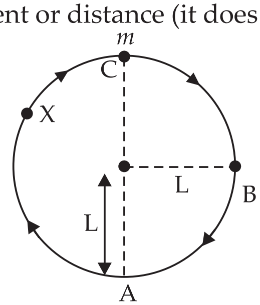
Q6.32. Two bodies of unequal mass are moving in the same direction with equal kinetic energy. The two bodies are brought to rest by applying retarding force of the same magnitude. How would the distance moved by them before coming to rest compare?

Ans. By work energy theorem change in KE is equal to work done by body. Hence KE = WD

$$KE_1 = KE_2 \qquad \text{(Given)}$$
$$WD_1 = WD_2$$
$$F_1 s_1 = F_2 s_2$$
$$\because \qquad F_1 = F_2 \quad \text{(Given)}$$
$$\therefore \qquad s_1 = s_2$$

Hence, both bodies will travel equal displacement or distance (it does not depend on mass of bodies.)

Q6.33. A bob of mass m suspended by a light string of length L is whirled into a vertical circle as shown in figure. What will be the trajectory of the particle, if the string is cut at:



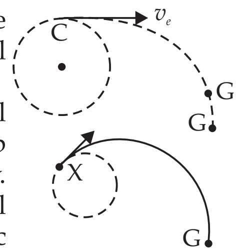
(*a*) Point B?

(*b*) Point C?

(*c*) Point X?

Ans. When the bob is whirled into circular path the required centripetal force is provided by string (towards the centre of circular path) due to tension and it balanced by centrifugal force provided by tangential velocity of particle.

Hence, when string is cut off, the centripetal force become zero and body move with tangential velocity under gravity.



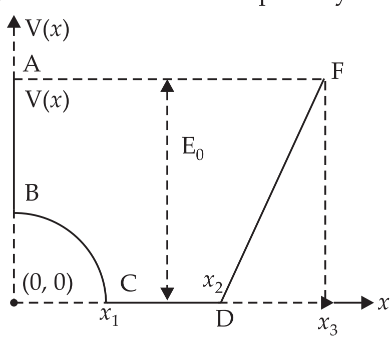
(*a*) When string is cut off at B, it's tangential velocity will be vertically downward, so bob will move along vertical path under gravity.

(*b*) When string is cut off at C. bob has horizontal velocity, so bob will move in half parabolic path as shown in figure.

(*c*) When string is cut off at X then the velocity of bob is makes some angle θ with horizontal so it moves parabolic path reaches at higher height then again parabolic.

SHORT ANSWER TYPE QUESTIONS

Q6.34. A graph of potential energy V(*x*) versus *x* is shown in figure. A particle of Energy E_0 is executing motion in it. Draw graph of velocity and kinetic energy versus *x* for one complete cycle AFA.



Ans. (*i*) KE versus *x* graph: By the law of conservation of energy.

Total Mechanical

$$E = KE + PE$$

$$E_0 = KE + V(x)$$

$$\boxed{K.E. = E_0 - V(x)}$$

At point A

PE is maximum,

PE = E_0 ∴ KE = $E_0 - E_0 = 0$

At B, $x = 0$ PE = V_B (let)

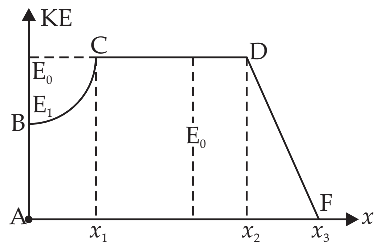
KE = $E_0 - V_0$ KE = $E_1 > 0$

At C, $x = x_1$ PE = 0

∴ KE = $E_0 - V(x) = E_0 - 0 = E_0$

At D, $x = x_2$ KE = E_0
At E, $x = x_3$ PE = E_0
$\therefore$ KE = 0
Figure shows graph KE versus x.

x	KE	Point
0	0	A
0	E_1	B
x_1	E_0	C
x_2	E_0	D
x_3	0	F



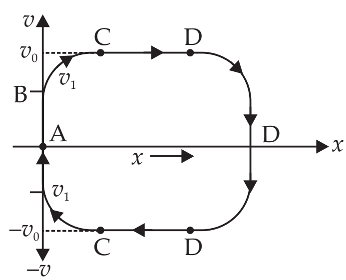
(*ii*) **Velocity (v) versus (x):**

$$KE = -mv$$

$$v = \sqrt{\frac{2K}{m}}$$

$$v = \sqrt{K}$$

Point	v	x
A	0	0
B	$\pm v_1$	0
C	$\pm v_0$	x_1
D	$\pm v_0$	x_2
E	0	x_3



At point A and F

$$KE = 0$$

and $x_A = 0$, $x_D = x_3$
At C and D, $x_C = x_1$, $x_D = x_2$

$$KE = E_0$$

$\therefore$ $v_{max} = \pm\sqrt{E_0} = v_0$
$\therefore$ At B, $x = 0$ KE = E_1
$\therefore$ $v_B = \sqrt{E_1} = \pm v$
($v - x$) graph is shown in given figure here.

Q6.35. A ball of mass 'm' moving with a speed $2v_0$, collides inelastically ($e > 0$) with an identical ball at rest. Show that

(*a*) For head-on collision, both the balls move forward.

(*b*) For a general collision, the angle between the two velocities of scattered balls is less than 90°.

Ans. (*a*) Let the v_1, v_2 are the velocities of the two balls after the collision. Now by the principle of law of conservation of momentum

$$mv_0 + mv_0 = mv_1 + mv_2$$

$$2v_0 = v_1 + v_2 \quad ...(i)$$

$$e = \frac{v_2 - v_1}{v_0 + v_0} \Rightarrow v_2 - v_1 = 2ev_0$$

$$v_2 = v_1 + 2ev_0 \quad ...(ii)$$

From (*i*) $\quad v_1 = -v_2 + 2v_0$

$$v_1 = -v_1 - 2ev_0 + 2v_0$$

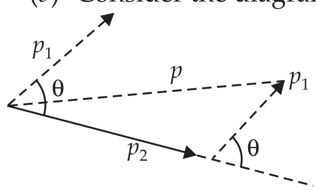
$$2v_1 = 2v_0 - 2ev_0$$

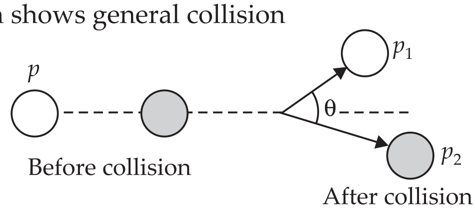
$$v_1 = v_0(1 - e)$$

$\because \quad e < 1$

$\therefore$ v_0 is positive so the direction of v_1 is same as v_0, or v_1 is in forward direction. Hence proved.

(*b*) Consider the diagram shows general collision





Let angle between the p_1 and p_2 is θ.

By the law of conservation of momentum.

$$\vec{p} = \vec{p}_1 + \vec{p}_2$$

In inelastic collision some part of KE lost in the form of heat, deshaping etc.

$\therefore \quad KE_i > KE_1 + KE_2$

$$\frac{1}{2}mv^2 + \frac{1}{2}m(0)^2 > \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\frac{p^2}{2m} > \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$$

$\therefore \quad \vec{p}^2 > \vec{p}_1^2 + \vec{p}_2^2 \quad ...(iii)$

$\Rightarrow$ If $p^2 = p_1^2 + p_2^2$ then angle between p_1 and p_2 is 90°.

So equation (*iii*) is true when angle between p_1, p_2 less than 90° or acute as shown in figure also.

Q6.36. Consider a one-dimensional motion of a particle with total energy E. There are four region A, B, C and D in which the relation between potential energy (V), kinetic energy (K) and total energy E is as given below:

Region A : V > E $\qquad$ Region B : V < E

Region C : K > E $\qquad$ Region D : V > K

State with reason in each case whether a particle can be found in the given region or not.

Ans. (*i*) For region A : V > E

$$E = V + K$$
$$K = E - V$$

$\because$ V > E. So K < 0 or KE is negative, which is not possible.

(*ii*) In region B : V < E

$$K = E - V$$
$$K > 0$$

This case is possible. $\because$ Both energies are greater than zero.

(*iii*) Region C : K > 0

$$V = E - K$$

$\Rightarrow$ $V < 0$ PE is negative.

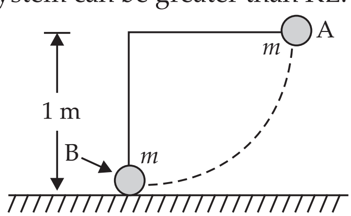
This is also possible because PE can be negative.

(*iv*) Region D : V > K

$$K = E - V$$

This is also possible as PE for a system can be greater than KE.

Q6.37. The bob A of pendulum released from horizontal to the vertical hits another bob B of a same mass at rest on a table as shown in figure. If the length of pendulum is 1 m, calculate



(*a*) the height to which the bob A will rise after collision.

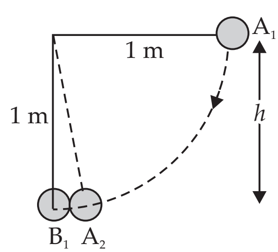
(*b*) the speed with which bob B starts moving.

Neglect the size of the bob and assume the collision to be elastic.

Ans. (*a*) When a moving ball strike to an identical ball at rest then one ball transfers it momentum to other and itself become at rest.

When ball A reaches at the position of ball B. The PE of A converts into KE Where ball A transfer its momentum to ball B and ball A becomes itself in rest after elastic collision. So ball will no rise.

(*b*) For speed of ball B



$$\text{P.E.}_{A_1} = \text{K.E.}_{A_2} = \text{K.E.}_{B_1}$$

$$\frac{1}{2}mv_B^2 = mgh$$

$$v_B = \sqrt{2g \times 1}$$

$$= \sqrt{2 \times 9.8} = \sqrt{\frac{2}{10} \times 2 \times 49}$$

$$= 2 \times 7 \frac{1}{\sqrt{10}} = \frac{14 \times \sqrt{10}}{10}$$

$$= 1.4 \times 3.16$$

$$v_B = 4.42 \text{ m/s.}$$

Q6.38. A raindrop of mass 1.00 g falling from a height of 1 km hits the ground with a speed of 50 m/s. Calculate.

(*a*) the loss of PE of the drop.

(*b*) the gain in KE of the drop.
(*c*) Is the gain in KE equal to loss of PE? If not why?

(Take $g = 10\ m/s^2$)

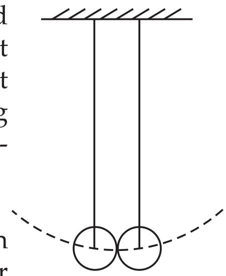
Ans. Drop $m = .001$ kg, $h = 1$ km $= 1000$ m
Speed of $v = 50$ m/s $u = 0$

(*a*) PE at highest point of drop $= mgh = .001 \times 10 \times 1000 = 10$ J
So loss of PE = 10 J.

(*b*)
$$\text{Gain in KE} = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 0.001 \times 50 \times 50 = 1.250$$
$$\text{Gain in KE} = 1.250 \text{ J.}$$

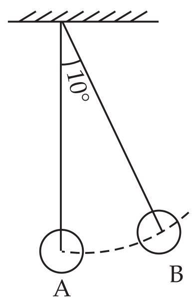
(*c*) Gain in KE is not equal to the loss in PE. It is due to the loss of PE or KE against resistance or dragging force of air.

Q6.39. Two pendulums with identical bobs and lengths are suspended from a common support such that in rest position the two bobs are in contact (figure). One of the bobs is released after being displaced by 10° so that it collides elastically head-on with the other bob.



(*a*) Describe the motion of two bobs.
(*b*) Draw a graph showing the variation in energy of either pendulum with time, for $0 \le t \le 2T$, where T is the period of each pendulum.

Ans. (*a*) Let at $t = 0$ A is at lowest position and B is at its height position at 10°.



$PE_A = 0$ $PE_B = E$, $KE_A = KE_B = 0$
K.E. of both are zero. Now bob is released.

$t = \frac{T}{4}$ B reaches to A and collide elastically as both bobs are identical then

$KE_A = 0$ $KE_B = E$, $PE_A = 0$,
$PE_B = 0$

At $t = \frac{2T}{4}$ A reaches at it's maximum height and B remains at it's lowest position.

$KE_A = 0$ $KE_B = 0$, $PE_A = E$, $PE_B = 0$

At $t = \frac{3T}{4}$ Bob A hits the bob B which was at rest elastically and ball A becomes at rest B moves upward.

$KE_A = 0$ $KE_B = E$, $PE_A = 0$, $PE_B = 0$
$E_A = 0$ $E_B = E$

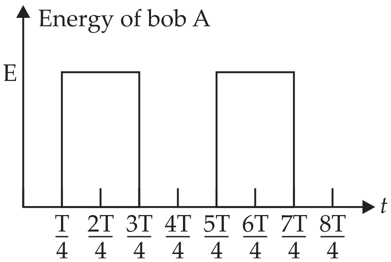
At $t = \frac{4T}{4}$ Bob B is at its maximum height and A is at lowest height.

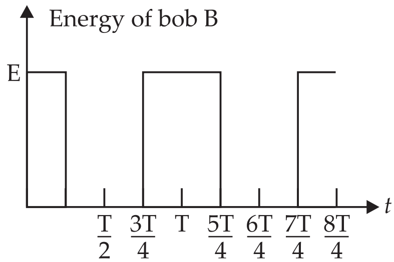
So, $KE_A = 0$ $KE_B = 0$, $PE_A = 0$, $PE_B = E$

The entire process is repeated.

(*b*)

Time t	KE_A	PE_A	E_A	KE_B	PE_B	E_B
0	0	0	0	0	E	E
T/4	E	0	E	0	0	0
2T/4	0	0	0	0	E	E
3T/4	E	0	E	0	0	0
4T/4 = T	0	0	0	0	E	E





Q6.40. Suppose the average mass of raindrops is 3×10^{-5} kg and their average terminal velocity is 9 ms^{-1}. Calculate the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year.

Ans. Energy transfer by rain to surface of earth is kinetic energy

$$= \frac{1}{2}mv^2$$

The velocity of rain or water is 9 m/s

$$\begin{aligned} \text{For mass } m &= \text{Volume} \times \text{density} \\ &= \text{Area of base} \times \text{height} \times \rho \\ &= 1\text{ m}^2 \times 1\text{ m} \times 1000 \\ &= 1000\text{ kg} \end{aligned}$$

$$\begin{aligned} \text{So energy transfer by 100 cm rainfall} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 1000 \times 9 \times 9 = 500 \times 81 \\ &= 40500\text{ J} = 4.05 \times 10^4\text{ J.} \end{aligned}$$

Q6.41. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass 50,000 kg is moving with a speed of 36 km/h when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, calculate the spring constant.

Ans. $\text{KE} = \frac{1}{2}mv^2$ $m = 50{,}000$ kg

$v = 36 \times \frac{5}{18}$ m/s $= 10$ m/s

$$\text{KE} = \frac{1}{2} \times 50{,}000 \times 10 \times 10 = 2500000 \text{ J}$$

90% of KE of wagon lost due to friction by breaks only 10% are passed to spring.

KE of Spring = 10% of KE wagon

$$\frac{1}{2}kx^2 = \frac{10}{100} \times 2500000$$

$x = 1$ m $\frac{1}{2}k \times 1 \times 1 = 250000$

$$k = 500000 = 5 \times 10^5 \text{ N/m.}$$

Q6.42. An adult weighing 600 N raises the centre of gravity of his body by 0.25 m while taking each step of 1 m length in jogging. If he jogs for 6 km, calculate the energy utilized by him in jogging assuming that there is no energy loss due to friction of ground and air. Assuming that the body of the adult is capable of converting 10% of energy intake in the form of food, calculate the energy equivalents of food that would be required to compensate energy utilised for jogging.

Ans. Energy used up by raising the centre of gravity by 0.25 m by jogger in one step = mgh

$mg = 600$ N $g = 10$ m s^{-2} $h = 0.25$ m

$\therefore$ Number of steps in 6 km $= \frac{6000 \text{ m}}{1 \text{ m}} = 6000$ steps

Energy utilised in 6000 m $= 6000 \times 600 \times 0.25$ J

Since 10% of energy utilised in jogging.

$\therefore$ Energy utilised in jogging $= \frac{10}{100} \times 600 \times 6000 \times 0.25$

$= 360000 \times 0.25$

$= 90000 \text{ J} = 9 \times 10^4$ J.

Q6.43. On complete combustion a litre of petrol gives off heat equivalent to 3×10^7 J. In a test drive a car weighing 1200 kg, including the mass of driver, runs 15 km per litre while moving with a uniform speed on a straight track. Assuming that friction offered by the road surface and air to be uniform, calculate the force of friction acting on the car during the test drive, if the efficiency of car engine were 0.5.

Ans. Efficiency of car engine = 0.5

$\therefore$ Energy give by car by 1 litre of petrol $= 0.5 \times 3 \times 10^7$

$= 1.5 \times 10^7$

Work done by car is 15 km $= \text{F}.s$

$= \text{F} \times 15000$ J

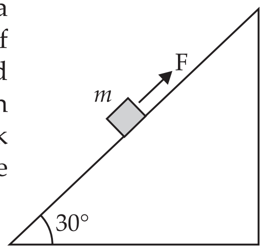
This work done by car is only against force of friction (f) as car is going horizontally only.

$$\therefore \quad f = F \text{ and } f \times 15000 = 1.5 \times 10^7$$

$$f = \frac{1.5 \times 10^7}{15000 \times 10} = 10^3 \text{ N.}$$

LONG ANSWER TYPE QUESTIONS

Q6.44. A block of mass 1.0 kg is pushed up a surface, inclined to horizontal at an angle of 30° by a force of 10 N parallel to the inclined plane surface. (figure) The coefficient of friction between block and the incline is 0.1. If the block is pushed up by 10 m along the incline, calculate

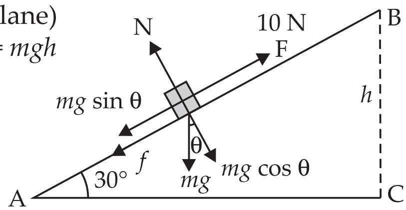


(*a*) Work done against gravity.
(*b*) Work done against force of friction.
(*c*) Increase in potential energy.
(*d*) Increase in kinetic energy.
(*e*) Work done by applied force.

Ans. $m = 1$ kg $\quad \theta = 30°$

$F = 10$ N $\quad \mu = 0.1$

Distance $d = 10$ m (on inclined plane)



(*a*) Work done against gravity = mgh

$$\sin 30° = \frac{h}{10}$$

$$h = 10 \sin 30° = 10 \times \frac{1}{2} = 5 \text{ m}$$

$\therefore$ WD against gravity

$$= 1 \times 10 \times 5 = 50 \text{ J.}$$

(*b*) WD against force of friction (f) = WD = $f.s$

WD against force of friction = $\mu N.s$

$$= \mu \,.\, mg \cos \theta \; s$$

$$= 0.1 \times 1 \times 10 \cos 30° \times 10$$

$$= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ J .}$$

(*c*) Increase in PE = WD against gravity = 50 J [from part (*a*)]

(*d*) Increase in KE (ΔK) by work energy theorem is equal to the WD by system = ΔK = WD (all)

$$\Delta K = -mgh - fs + Fs$$

$$= -50 - 5\sqrt{3} + 10 \times 10 = 50 - 5\sqrt{3}$$

$$= 5[10 - 1.732] = 5[8.268]$$

$$\Delta K = 41.340 \text{ J}$$

(*e*) WD by applied force = $F.s = 10 \times 10 = 100$ J.

Q6.45. A curved surface is shown in figure. The portion BCD is free of friction. There are three spherical balls of identical radii and masses. Balls are released from rest one by one from A which is at a slightly greater height than C.

With the surface AB, ball 1 has large enough friction to cause rolling down without slipping; ball 2 has a small friction and ball 3 has a negligible friction.

(*a*) For which balls is total mechanical energy conserved?

(*b*) Which ball(s) can reach D?

(*c*) For balls which do not reach D, which of the balls can reach back A?

Ans. (*a*) Ball (1) is rolling down without slipping, so zero (0) force of friction acts or no loss of energy, hence, the total mechanical energy is conserved.

Ball 3 has negligible friction hence there is no loss of energy, so total mechanical energy is conserved. Hence, mechanical energy is conserved for ball (1) and (3).

(*b*) Ball (1) acquires rotational energy (due to friction) ball (2) has small friction, so loses energy by friction. So it cannot reach at C. Ball (1) will slip due to rotation on frictionless surface hence, does not reach at C.

(*c*) Ball (1) and (2) cannot reach at C as discussed in part (*b*), so ball (3) having negligible friction and A is above C, so crosses C and can reach at D. Ball (1) cannot reach at A due to rotational motion in forward direction. Ball (2) loses its energy. Ball 3 crosses C so no ball can reach at A.

Q6.46. A rocket accelerate straight up by ejecting gas downwards. In a small time interval Δt, it ejects a gas of mass Δm at a relative speed u. Calculate KE of entire system at $(t + \Delta t)$ and t and show that the device that ejects gas does work $= \frac{1}{2}\Delta m u^2$ in this time interval (neglect gravity).

Ans. Let mass of rocket at any time $t = \text{M}$

Velocity of rocket at any time $t = v$

Δm is mass of gas ejected in time interval Δt

$$(\text{KE})_{t+\Delta t} = \frac{1}{2}(\text{M} - \Delta m)(v + \Delta v)^2 + \frac{1}{2}\Delta m\,(v - u)^2$$

$$= \frac{1}{2}\left[(\text{M} - \Delta m)(v^2 + \Delta v^2 + 2v\Delta v) + \Delta m(v^2 + u^2 - 2uv)\right]$$

$$(\text{KE})_{t+\Delta t} = \frac{1}{2}\left[\text{M}v^2 + \text{M}\Delta v^2 + 2\text{M}v\Delta v - \Delta m v^2 - \Delta m \Delta v^2 - 2v\Delta m\Delta v + \Delta m v^2 + \Delta m u^2 - 2uv\Delta m\right]$$

$$(\text{KE})_{t+\Delta t} = \frac{1}{2}\text{M}v^2 + Mv\Delta v + \frac{1}{2}\Delta m u^2 - uv\Delta m$$

[neglecting the very small terms $M\Delta v^2$, $\Delta m\Delta v^2$, $2v\Delta m\Delta v$ contains Δv^2 and $\Delta m\Delta v$]

$$(KE)_t = \frac{1}{2}Mv^2$$

$$(KE)_{t+\Delta t} - (KE)_t = \frac{1}{2}Mv^2 + (M\Delta v - u\Delta m)v + \frac{1}{2}u^2\Delta m - \frac{1}{2}Mv^2$$

$$\Delta K = (M\Delta v - u\Delta m)v + \frac{1}{2}u^2\Delta m \qquad ...(i)$$

By Newton's third law,

Reaction force on Rocket (upward) = Action force by burnt gas (downward)

$$M\frac{dv}{dt} = \frac{dm}{dt}|u| \qquad (\because F = ma)$$

or $\quad M\Delta v = \Delta mu \quad \Rightarrow \quad M\Delta v - u\Delta m = 0$

Substitute this value in (i)

$$K = \frac{1}{2}u^2\Delta m$$

By work energy theorem Δ (KE) = WD

or $$W = \Delta K = \frac{1}{2}\Delta mu^2.$$

Q6.47. Two identical steel cubes (masses 50 g, side 1 cm) collide head-on face to face with a speed of 10 cm/s each. Find the maximum compression of each. Young's modulus for steel $Y = 2 \times 10^{11}$ N/m^2.

Ans. When two identical cubes collide head-on collision. Then KE of cubes converts into PE and compresses the faces of cube by ΔL. By Hooks Law stress $\propto$ strain

or $$Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow Y = \frac{F.L}{A\Delta L}$$

or $$F = AY\frac{\Delta L}{L} \text{ or } F = L^2Y\frac{\Delta L}{L} = LY\Delta L$$

$$WD = F.\Delta L = LY\Delta L^2$$

$$\text{KE of both cubes} = 2\left(\frac{1}{2}mv^2\right) = 0.05 \times .1 \times .1 = 5 \times 10^{-4} \text{ J}$$

$$W.D. = K.E.$$

$$LY\Delta L^2 = 5 \times 10^{-4}$$

$$\Delta L^2 = \frac{5 \times 10^{-4}}{0.01 \times 2 \times 10^{11}} = \frac{5}{2} \times 10^{-4+2-11} = 2.5 \times 10^{-13}$$

$$\Delta L = \sqrt{25 \times 10^{-14}} = 5 \times 10^{-7} \text{ m.}$$

Q6.48. A balloon filled with helium rises against gravity increasing its potential energy. The speed of the balloon also increases at it rises. How do you reconcile this with the law of conservation of mechanical energy? You can neglect viscous drag of air and assume that density of air is constant.

Ans. As the dragging viscous force of air on balloon is neglected so there is Net Buoyant Force = $V\rho\, g$

$$= \text{Vol. of air displaced} \cdot (\text{net density upward}) \cdot g$$
$$= V(\rho_{air} - \rho_{He})\, g \quad (\text{upward})$$

Let a be the upward acceleration of balloon then

$$ma = V(\rho_{air} - \rho_{He})\, g \qquad ...(i)$$

where m = mass of balloon

V = Volume of air displaced by balloon = Volume of balloon

ρ_{air} = density of air

ρ_{He} = density of helium

$$m\frac{dv}{dt} = V.(\rho_{air} - \rho_{He})\, g$$
$$m\, dv = V.(\rho_{air} - \rho_{He})\, g\, .dt$$

Integrating both sides

$$mv = V.(\rho_{air} - \rho_{He})\, gt$$
$$v = \frac{V}{m}(\rho_{air} - \rho_{He})\, gt$$
$$\text{KE of balloon} = \frac{1}{2}mv^2$$
$$\frac{1}{2}mv^2 = \frac{1}{2}m\frac{V^2}{m^2}(\rho_{air} - \rho_{He})^2 g^2t^2 = \frac{V^2}{2m}(\rho_{air} - \rho_{He})^2 g^2t^2 \qquad ...(ii)$$

If the balloon rises to a height h, from (i)

$$a = \frac{V}{m}(\rho_{air} - \rho_{He})\, g$$
$$h = ut + \frac{1}{2}at^2 = 0.t + \frac{1}{2}\left[\frac{V}{m}(\rho_{air} - \rho_{He})g\right]t^2$$
$$\therefore \qquad h = \frac{V}{2m}(\rho_{air} - \rho_{He})gt^2 \qquad ...(iii)$$

From (ii) and (iii) rearranging the terms of (ii) according to h in (iii)

$$\frac{1}{2}mv^2 = \left(\frac{V}{2m}(\rho_{air} - \rho_{He})\, gt^2\right).V(\rho_{air} - \rho_{He})\, g$$
$$\frac{1}{2}mv^2 = (h)\, V.(\rho_{air} - \rho_{He})\, g$$
$$\frac{1}{2}mv^2 = V.(\rho_{air} - \rho_{He})\, gh$$
$$\frac{1}{2}mv^2 = V\,\rho_{air}\, gh - V\,\rho_{He}\, gh$$
$$\frac{1}{2}mv^2 + \rho_{He}Vgh = \rho_{air}\, V\, gh$$

$\text{KE}_{balloon} + \text{PE}_{balloon}$ = Change in PE of air.

So, as the balloon goes up, an equal volume of air comes down, increases in PE and KE of the balloon is at the cost of PE of air (which comes down).

❑❑❑

7 System of Particles and Rotational Motion

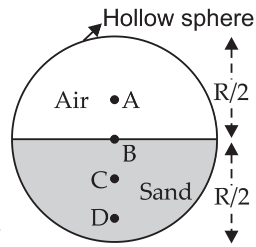
MULTIPLE CHOICE QUESTIONS-I

Q7.1. For which of the following does the centre of mass lies outside the body

(*a*) A pencil (*b*) A shot put (*c*) A dice (*d*) A bangle

Ans. (*d*): A bangle is a ring like shape and the centre of mass of ring lies at its centre which is outside the ring or bangle. Hence verifies option (*d*).

Q7.2. Which of the following points is likely the position of centre of mass of the system shown in given figure:



(*a*) A (*b*) B

(*c*) C (*d*) D

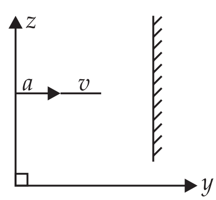
Main concept used: Centre of mass of system of particles is closer to the heavier mass or masses.

Ans. (*c*): As the air and sand is half the volume of sphere; so the volume of sand is equal to the volume of air. The $\rho_{air} \ll \rho_{sand}$

$\therefore$ $M_{sand} \gg M_{air}$ inside the sphere.

As mass of sand is larger than air so centre of mass will shift towards sand from centre of sphere B i.e., centre mass of the system is at C. Verifies the option (*c*).

Q7.3. A particle of mass m is moving in y-z plane with a uniform velocity v with its trajectory running parallel to +ve y-axis and intersecting z-axis at $z = a$ as in figure. The change in its angular momentum about the origin as it bounces elastically from a wall at y = constant is:



(*a*) $mv\,a\,\hat{e}_x$ (*b*) $2mv\,a\,\hat{e}_x$ (*c*) $y\,mv\,\hat{e}_x$ (*d*) $2y\,mv\,\hat{e}_x$

Main concept used: $\vec{L} = \vec{r} \times m\vec{v}$ and direction of L by RHTR (Right Hand Thumb Rule)

Ans. (*b*): The trajectory is at constant distance (a) on z-axis and as particle moves its y component changes along y-axis. So the position particle moving along y-axis $\vec{r} = y\,\hat{e}_y + a\,\hat{e}_z$

$v_i = v\,\hat{e}_y$ $\quad$ $v_f = -v\,\hat{e}_y$

$$\vec{L} = \vec{r} \times \vec{p}$$

As the $\vec{r}$, $\vec{v}$ and $\vec{p}$ are in plane of y-z.

So the L will be in the plane of $+x$

$$L = (y\,\hat{e}_y + a\,\hat{e}_z) \times m(v_f - v_i)$$

$$= (y\,\hat{e}_y + a\,\hat{e}_z) \times m[-\,v\,\hat{e}_y - v\,\hat{e}_y]$$

$$L = (y\,\hat{e}_y + a\,\hat{e}_z) \times mv(-\,2\,\hat{e}_y)$$

$\because$ $$y\hat{e}_y \times \hat{e}_y = y \sin 0° = 0$$

$$a\hat{e}_z \times \hat{e}_y = a \sin 90°(-\hat{e}_x) = -\,a\,\hat{e}_x$$

$\therefore$ $$L = -\,a\,\hat{e}_x\;mv(-\,2) = +\,2a\,mv\,\hat{e}_x$$

Hence, verifies the option (*b*).

Q7.4. When a disc rotates with uniform angular velocity, which of the following is not true?

(*a*) The sense of rotation remains same.
(*b*) The orientation of the axis of rotation remains same.
(*c*) The speed of rotation is non-zero and remains same.
(*d*) The angular acceleration is not non-zero and remains same.

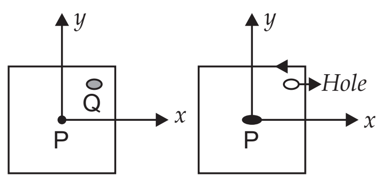
Ans. (*d*): $\because$ ω is uniform or constant

$\therefore$ $$\alpha = \frac{d\omega}{dt} \Rightarrow \alpha = 0$$

Hence, angular acceleration is zero, not non zero.

Hence, verifies option (*d*).

Q7.5. A uniform square plate has a smaller piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind (figure). The moment of inertia about the *z*-axis is then



(*a*) increased (*b*) decreased
(*c*) the same (*d*) changed in unpredicted manner

Ans. (*b*): After removing the matter from Q it is stick at P through axis of rotation passes, but axis of rotation does not passes through Q. So the gap at Q will **decrease** the moment of inertia as mass (removed) comes closer to the axis of rotation.

Q7.6. In Q. 7.5 the C.M. of the plate is now in the following quadrant of X-Y plane.

(*a*) I (*b*) II (*c*) III (*d*) IV

Ans. (*c*): As the mass at Q is decreased and placed at centre of mass so new C.M. will shift towards other side of Q on the line joining QP. So new C.M. will lie in III quadrant.

Q7.7. The density of a non-uniform rod of length 1 m is given by:

$$\rho(x) = a(1 + bx^2).$$

where a and b are constants and $0 \le x \le 1$. The centre of mass of the rod will be at:

(*a*) $\dfrac{3(2+b)}{4(3+b)}$ (*b*) $\dfrac{4(2+b)}{3(3+b)}$ (*c*) $\dfrac{3(3+b)}{4(2+b)}$ (*d*) $\dfrac{4(3+b)}{3(2+b)}$

Ans. (*a*): $$\rho(x) = a(1 + bx^2)$$

At $b = 0$, $\rho(x) = a$ i.e., ρ is constant in this case C.M. must lies at mid-point of 1 m i.e., at $x = 0.5$ m, by substituting value of $b = 0$ in option (*a*), (*b*), (*c*), (*d*).

We observe that the C.M. lies at 0.5 m at $b = 0$.

(*a*) $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2} = .5$ m (*b*) $\frac{4}{3} \times \frac{2}{3} \neq .5$ m

(*c*) $\frac{3}{4} \times \frac{3}{2} \neq .5$ m (*d*) $\frac{4}{3} \times \frac{3}{2} \neq .5$ m

Hence, option (*a*) verifies.

Q7.8. A Merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round afterward is:

(*a*) 2ω (*b*) ω (*c*) $\frac{\omega}{2}$ (*d*) 0

Ans. (*a*): As there is external torque acting on system

$$I_1\omega_1 = I_2\omega_2$$

As the mass of person and merry-go-round is M. So the initial mass of system

$m_1 = 2M$ $m_2 = M$ $\omega_1 = \omega$ $\omega_2 = ?$

$r_1 = R$ $r_2 = R$

As person jumps down tangentially i.e., from periphery

$$\therefore \quad m_1 r_1^2 \omega_1 = m_2 r_2^2 \omega_2$$

$$2MR^2\omega = MR^2\omega_2$$

$$\omega_2 = 2\omega.$$

Verifies the option (*a*).

MULTIPLE CHOICE QUESTIONS-II

Q7.9. Choose the correct alternatives:

(*a*) For a general rotational motion, angular momentum $\vec{L}$ and angular velocity ω need not be parallel.

(*b*) For a rotational motion about a fixed axis, angular momentum $\vec{L}$ and angular velocity ω are always parallel.

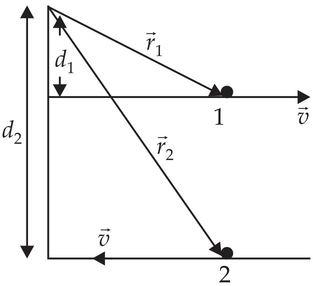
(*c*) For a general translational motion, momentum $\vec{p}$ and velocity $\vec{v}$ are always parallel.

(*d*) For a general translational motion, acceleration $\vec{a}$ and velocity $\vec{v}$ are always parallel.

Ans. (*a, c*): For general rotational motion, where axis of rotation is not symmetric. Angular momentum and angular velocities are not parallel verifies option (*a*).

For general translation motion momentum $\vec{p} = m\vec{v}$ and hence $\vec{p}$ and $\vec{v}$ are always in same direction i.e., parallel, verifies the option (*c*).

Q7.10. Figure shows two identical particles 1 and 2, each of mass m, moving in opposite directions with same speed $\vec{v}$ along parallel lines. At particular instant, $\vec{r}_1$ and $\vec{r}_2$ are their respective position vectors drawn from point A which is in the plane of the parallel lines. Choose the correct options:



(*a*) Angular momentum $\vec{l}_1$ of particle 1 about A is $\vec{l}_1 = mv(d_1) \odot$

(*b*) Angular momentum $\vec{l}_2$ of particle 2, about A is $\vec{l}_2 = mv\vec{r}_2 \odot$

(*c*) Total angular momentum of the system about A is
$$\vec{l} = mv\,(\vec{r}_1 + \vec{r}_2) \odot$$

(*d*) Total angular momentum of the system about A is
$$\vec{l} = mv\,(d_2 - d_1) \otimes$$

$\odot$ represents a unit vector coming out of the page.

$\otimes$ represents a unit vector going into the page.

Main concept used: In $\vec{L} = \vec{r} \times \vec{p}$, direction of L is perpendicular to plane of $\vec{r}$ and $\vec{p}$, can be find out by Right Hand Grip Thumb Rule.

Ans. (*a, d*): For particle 1, $\vec{L}_1 = \vec{r}_1 \times \vec{p} \odot$
$$\vec{L}_1 = \vec{r}_1 \times m\vec{v} \odot = m\vec{v}\, d_1 \odot$$
$$\vec{L}_2 = \vec{r} \times m(-\vec{v}) \otimes = -m\vec{v} d_2 \otimes$$
Hence, option (*a*) is verified and (*b*) rejected.

Total Angular Momentum $\vec{L} = \vec{L}_1 + \vec{L}_2$
$$\vec{L} = m\vec{v}\, d_1 \odot - m\vec{v}\, d_2 \otimes$$
As $\quad d_2 > d_1 \quad \therefore \; |\vec{L}_2| > |\vec{L}_1|$

(–) sign shows (only) the direction of L.
$$\therefore \quad \vec{L} = m\vec{v}\,(d_2 - d_1) \otimes.$$
Verifies option (*d*).

Q7.11. The net external torque on a system of particles about an axis is zero. Which of the following are compatible with it?

(*a*) The forces may be acting radially from a point on the axis.

(*b*) The forces may be acting on the axis of rotation.

(*c*) The forces may be acting parallel to the axis of rotation.

(*d*) The torque caused by some forces may be equal and opposite to that caused by other forces.

Ans. (*a, b, c, d*): We know that $\vec{\tau} = \vec{r} \times \vec{F}$
$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta\, \hat{n}$$
where θ is angle between $\vec{r}$ and $\vec{F}$, and $\hat{n}$ is unit vector by right hand thumb rule is perpendicular to the plane of $\vec{r}$ and $\vec{F}$.

(*a*) As the forces are radially in the direction $\vec{r}$.
Hence, $\theta = 0$

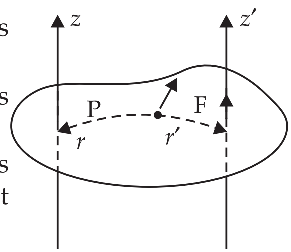
$\therefore \quad \tau = r\text{F} \sin 0° = 0$ verifies (*a*).

(*b*) If forces acting on axis of rotation then $\theta = 0°$.
Hence, $\tau = 0$ also verifies option (*b*).

(*c*) When forces (F) are parallel to axis of rotation then its component in plane of $\vec{r}$ and $\vec{\text{F}}$ is $\text{F} \cos 90° = 0$. Hence, as $\text{F} = 0 \Rightarrow \tau = 0$ verifies the option (*c*).

(*d*) When torques are equal and opposite then net torque = $\tau_1 - \tau_2 = 0$. Hence, verifies the option (*d*).

Q7.12. Figure shows a lamina in *x-y* plane. Two axes *z* and *z′* pass perpendicular to its plane. A force $\vec{\text{F}}$ acts in the plane of lamina at point P as shown. Which of the following are true? (The point P is closer to *z′*-axis than the *z*-axis)



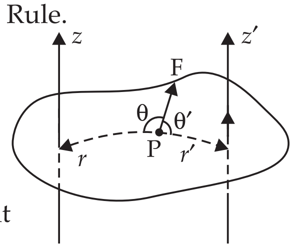
(*a*) Torque τ caused by $\vec{\text{F}}$ about *z*-axis is along $(-\hat{k})$.

(*b*) Torque τ' caused by $\vec{\text{F}}$ about *z′*-axis is along $-\hat{k}$.

(*c*) Torque τ caused by $\vec{\text{F}}$ about *z′*-axis is greater in magnitude than that about *z*-axis.

(*d*) Total torque is given be $\tau = \tau + \tau'$.

Main concept used: $\vec{\tau} = \vec{r} \times \vec{\text{F}}$ direction of τ is perpendicular to the plane of $\vec{r}$ and $\vec{\text{F}}$ by Right Hand Grip Thumb Rule.



Ans. (*b*, *c*): (*a*) $\quad r' < r$ (given)

$$\vec{\tau}_{z'} = \vec{r}' \times \vec{\text{F}}$$

$$\tau_{z'} = r\text{F} \sin \theta' (+\hat{k})$$

As $\vec{r}$ and $\vec{\text{F}}$ are *x-y* plane.

So $\vec{r} \times \vec{\text{F}}$ will be $+\hat{k}$ direction by Right Hand Grip Thumb Rule.

(*b*) $$\vec{\tau}_z = \vec{r} \times \vec{\text{F}} = r\text{F} \sin \theta (-\hat{k})$$

reason same as in part (*a*) verifies the option (*b*).

(*c*) $\because \quad r > r'$ and $\theta > \theta'$

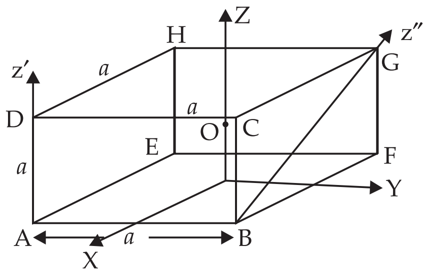
$$\sin \theta > \sin \theta'$$

$$r\text{F} \sin \theta > r'\text{F} \sin \theta'$$

$\tau_z > \tau_{z'}$. Hence, verifies the option (*c*).

(*d*) $\tau = \tau_z + \tau_{z'}$ will be true if τ_z and $\tau_{z'}$ are along the same axis but here the axis are different *z* and *z′*. So $\tau \neq \tau_z + \tau_{z'}$ rejects the option (*d*).

Q7.13. With reference to figure of a cube of edge a and mass m, state whether the following are true or false. (O is the centre of the cube).



(*a*) The moment of inertia of cube about z-axis is

$$I_z = I_x + I_y$$

(*b*) The moment of inertia of cube about z' is

$$I_{z'} = I_z + \frac{ma^2}{2}$$

(*c*) The moment of inertia of cube about z'' is $I_z + \dfrac{ma^2}{2}$

(*d*) $I_x = I_y$

Ans. (*a, b, d*): (*a*) z-axis is perpendicular to both x and y-axis so by perpendicular axis theorem $I_z = I_x + I_y$ verifies the option (*a*).

I_x = moment of inertia of cube about x-axis

I_y = M.I. of cube about y-axis both are non zero.

(*b*) By parallel axis theorem: z and z'-axis are parallel and the distance z and z'-axis is equal to

$$\frac{d}{2} = \frac{DG}{2} = \frac{1}{2}\sqrt{a^2 + a^2} = \frac{\sqrt{2a^2}}{2}$$

$\therefore$ Distance between z and z'-axis = $\dfrac{a}{\sqrt{2}}$.

$$\text{By theorem } I_{z'} = I_z + m\left(\frac{a}{\sqrt{2}}\right)^2 = I_z + \frac{ma^2}{2}$$

Hence, verifies the option (*b*).

(*c*) Axis BG i.e., z'' and Z-axis are not parallel to each other so parallel axis theorem cannot be applied, it rejects the option (*c*).

(*d*) As z-axis passes through centre 'O' of cube so the x and y-axis are symmetric.

Hence $I_x = I_y$, verifies the option (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q7.14. The centre of gravity of a body on the earth coincides with its centre of mass for a 'small' object whereas for an 'extended' object it may not. What is the qualitative meaning of 'small' and 'extended' in this regard?

For which of the following the two coincides? A building, a pond, a lake, a mountain?

Ans. Main concept used: Centre of gravity is the centre of it's Geometry but centre of mass is the point where the whole mass of body can be considered.

When the vertical height or Geometric centre of object is very near to surface of earth the object is called small. If it is larger then it is called extended objects.

(*i*) Building (high), pond are small objects.

(*ii*) Mountain and lake are big objects so their geometrical centre will be above and below the surface of earth respectively, with appreciable distances, so called extended objects.

Q7.15. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of the same mass and radius, about an axis passing through their axes of symmetry?

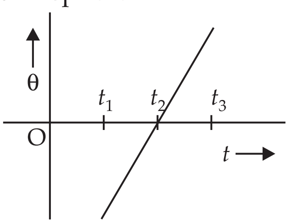
Ans. $$I = \sum_{i=1}^{n} m_i r_i^2$$

Moment of inertia is directly proportional to the square of distance of mass from the axis of rotation.

In solid sphere whole mass is distributed from centre to radius of sphere R. But in hollow sphere whole mass is concentrated near the periphery or surface of the sphere so average value of r_i becomes larger in hollow sphere as compared to solid sphere.

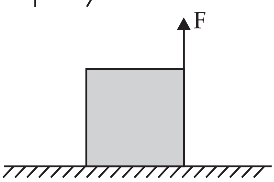
So MI of hollow sphere become larger than solid sphere.

Q7.16. The variation of angular position θ, of a point on a rotating rigid body, with time t is shown in figure. Is the body rotating clockwise or anti-clockwise?



Ans. As the θ-t graph has +ive slope so $\frac{d\theta}{dt} = \omega$ is +ive so the rotation is clockwise.

Q7.17. A uniform cube of mass m and side a is placed on a frictionless horizontal surface. A vertical force $\vec{F}$ is applied to the edge as shown in figure. Match the following (most appropriate choice):



(*a*)	$mg/4 < F < mg/2$	(*i*)	Cube will move up
(*b*)	$F > mg/2$	(*ii*)	Cube will not exhibit motion
(*c*)	$F > mg$	(*iii*)	Cube will begin to rotate and slip at A
(*d*)	$F = mg/4$	(*iv*)	Normal reaction effectively at $a/3$ from A, no motion.

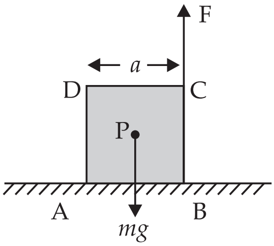
Ans. Moment of force due to F at A is anti-clockwise

$$\tau_1 = \overrightarrow{AB} \times \vec{F} = a \times \vec{F}$$

The moment of force due to mg at A is clockwise

$$\tau_2 = mg\frac{a}{2}$$

(*i*) Cube will not move if $\tau_1 = \tau_2$



$$a \times \vec{F} = mg\frac{a}{2}$$

$$\vec{F} = \frac{mg}{2}, \text{ so } (a) \to (ii)$$

(*ii*) Cube will rotate if $\tau_1 > \tau_2$ [Anti-clockwise]

$$Fa > mg\frac{a}{2} \text{ or } F > \frac{mg}{2} \text{ so } (b) \to (iii)$$

(*iii*) If normal reaction effectively at $\frac{a}{3}$ from A

Torque due to $mg = \tau_2 = mg\frac{a}{3}$

Torque due to F is $\tau_1 = F.a$

$$\tau_1 = \tau_2 \Rightarrow F.a = mg\frac{a}{3}$$

$$F = \frac{mg}{3} > \frac{mg}{4}, \text{ so due to } F = \frac{mg}{4}$$

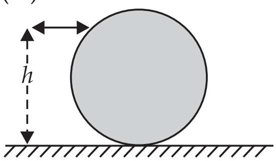
Block will not move $(d) \to (iv)$.

From part (*i*) cube move up if $F = \frac{mg}{2} < mg$

So if $F > mg$ block move upside $(c) \to (i)$

i.e., $(a) \to (ii)$, $(b) \to (iii)$, $(c) \to (i)$, $(d) \to (iv)$.

Q7.18. A uniform sphere of mass *m* and radius R is placed on a rough horizontal surface (figure). The sphere is struck horizontally at a height *h* from the floor. Match the following:

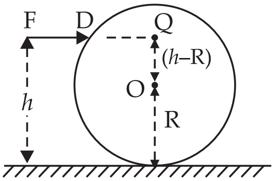


(*a*)	$h = \frac{R}{2}$	(*i*)	The sphere rolls without slipping with a constant velocity and no loss of energy.
(*b*)	$h = R$	(*ii*)	Sphere spins clockwise, loses energy by friction.
(*c*)	$h = \frac{3R}{2}$	(*iii*)	Sphere spins anti-clockwise loses energy by friction.
(*d*)	$h = \frac{7R}{5}$	(*iv*)	Sphere has only a translational motion looses energy by friction.

Ans. Consider the figure in which a force F is applied at P, height *h* above the ground. R = Radius of sphere.

The sphere will roll without slipping when $\boxed{\omega = \frac{v}{r}}$

v = linear velocity, ω = angular velocity
r = radius of sphere



Angular momentum of sphere about centre of mass 'O'. Let the sphere moves with velocity (linear) v after applying F then by law of conservation of angular momentum.

$$mv(h - R) = I_s\omega = \frac{2}{5}mR^2\,\frac{v}{R} \text{ (for no slipping)}$$

$$h - R = \frac{2}{5}R, \quad h = \frac{2}{5}R + R = \frac{7}{5}R$$

So the sphere will roll without slipping with constant velocity v and hence no loss of energy, so (*d*) → (*i*).

(*ii*) Torque due to force F = $\tau = (h - R) \times F$
If $\tau = 0$, $F \neq 0$ so $h - R = 0$ or $h = R$
So F is along 'O' and sphere will have only translational motion and slips against force of friction and loss of energy takes place by friction matches (*b*) → (*iv*).

(*iii*) Sphere will spin clockwise if $\tau > 0$

$$(h - R) \times F > 0 \text{ or } h > R.$$

Matches (*c*) → (*ii*).

(*iv*) The sphere will spin clockwise if $\tau < 0$

$$(h - R)F < 0$$

$$+h < +R \quad \because F \not< 0$$

So matches (*a*) → (*iii*)
Hence, option (*a*) → (*iii*), (*b*) → (*iv*), (*c*) → (*ii*), (*d*) → (*i*).

SHORT ANSWER TYPE QUESTIONS

Q7.19. The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point?

Ans. The vector sum of all torques due to forces at a point is zero. It does not mean that the resultant of forces are zero. e.g., Torque on sea-saw of a boy and child can be equal (can be balance). If the point of support of sea-saw changes without changing their position, the torques will not balance the sea-saw. So it is not necessary that, if the sum of all torques due to different forces at a point is zero, it will may not be zero for other arbitrary point.

$$G_i \sum_{i=1}^{n} F_i \neq 0$$

τ about a point P(let)

$$\therefore \qquad \tau = \tau_1 + \tau_2 + \ldots + \tau_n = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i = 0 \text{ (given)}$$

τ about any other point Q1 (say) $\vec{r}_i$ will be different forces

$$\sum_{i=1}^{n}(\vec{r}_i - a)\times F_i = \sum_{i=1}^{n}\vec{r}\times F_i - a\sum_{i=1}^{n} F_i$$

As a and $\sum F_i$ are not zero. So sum of all the torques about any arbitrary point need not be zero necessarily.

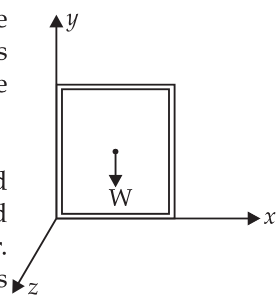
Q7.20. A wheel in uniform motion about an axis passing through its centre and perpendicular to its plane is considered to be in mechanical (translational plus rotational) equilibrium because no net external force or torque is required to sustain its motion. However, the particles that constitute the wheel do experience a centripetal acceleration directed towards the centre. How do you reconcile this fact with the wheel being in equilibrium?

How would you set a half-wheel into uniform motion about an axis passing through the centre of mass of the wheel and perpendicular to its plane? Will you require external forces to sustain the motion?

Ans. Wheel is a rigid elastic body. It is in uniform motion about axis passing through it's centre and perpendicular to the plane of wheel. Each particles of wheel which constitute the wheel are in circular motion about above axis and each particle will experience a centripetal acceleration directed towards axis of rotation due to elastic forces which are in pairs.

In a half wheel the distribution of mass of half wheel is not symmetric about the axis of wheel. Therefore the direction of angular momentum and angular velocity does not coincide. Hence the external torque is required to maintain the motion in half wheel.

Q7.21. A door is hinged at one end and is free to rotate about a vertical axis (figure). Does its weight cause any torque about this axis? Give reason for your answer.



Ans. We know that $\vec{\tau} = \vec{r}\times\vec{F}$

Here axis of rotation of door is along Y-axis and door is in x-y plane and force F can be applied along $\pm z$-axis, the torque is experience by door. So a force can produce torque only along axis in the direction normal to force. Force due to gravity of door is parallel to the axis of rotation. So cannot produce torque along y-axis. Gravity due to door is along $-y$-axis. So it can rotate the door in axis along $\pm z$-axis.

Hence the weight of door cannot rotate the door along y-axis.

Q7.22. $(n - 1)$ equal point masses each of mass m are placed at the vertices of a regular n-polygon. The vacant vertex has a position vector

$\vec{a}$ with respect to the centre of the polygon. Find the position vector of centre of mass.

Ans. The centre of mass of a regular n-polygon lies at it's geometric centre.

Let $\vec{b}$ is the position vector of the centre of mass of a regular n-polygon. $(n-1)$ equal point mass are placed at $(n-1)$ vertices of n-polygon then r_{cm} when mass m is placed at nth vertex.

$$r_{cm} = \frac{(n-1)mb + ma}{(n-1)m + m}$$

If mass m is placed at nth remaining vertex then

$$r_{cm} = 0$$

$$\frac{(n-1)mb + ma}{(n-1)m + m} = 0$$

$$(n-1)\, mb + ma = 0$$

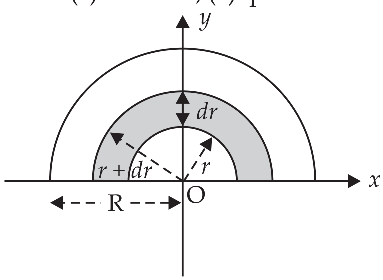
$$\vec{b} = \frac{-m\vec{a}}{(n-1)m} = \frac{-\vec{a}}{(n-1)}$$

(–) sign shows that c.m. lies other side from nth vertex geometrical centre of n-polygon i.e., $\vec{b}$ is opposite to the vector $\vec{a}$ (from centre to nth vertex).

LONG ANSWER TYPE QUESTIONS

Q7.23. Find the centre of mass of a uniform (*a*) half disc, (*b*) quarter disc.

Ans. Let mass of half disc is M.



$$\text{Area of half disc} = \frac{\pi R^2}{2}$$

$$\text{Mass per unit area } m = \frac{2M}{\pi R^2}$$

(*a*) The half disc can be divided into a large number of semi circular strips. Whose radii varies from $0 \to R$.

$$\text{Surface area of a semicircular strip} = \frac{\pi}{2}\left[(r+dr)^2 - r^2\right]$$

$$= \frac{\pi}{2}\left[r^2 + dr^2 + 2rdr - r^2\right]$$

$$= \pi r\, dr$$

$$\therefore \text{ Mass of strip} \quad dm = \frac{2M}{\pi R^2}.\pi r\, dr$$

$$dm = \frac{2M}{R^2}.r\, dr$$

Let (x, y) are the co-ordinates of c.m. of this strip

$$(x, y) = \left(0, \frac{2r}{\pi}\right)$$

$$x = x_{cm} = \frac{1}{M}\int_0^R x\, dm = \int_0^R 0\, dm = 0$$

$$y_{cm} = \frac{1}{M}\int_0^R y\, dm = \frac{1}{M}\int_0^R \frac{2r}{\pi} \times \frac{2M}{R^2}.rdr$$

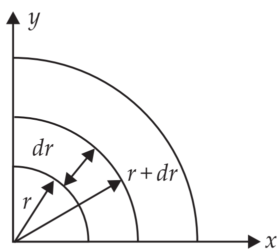
$$= \frac{1}{M}.\frac{4M}{\pi R^2}\int_0^R r^2 dr = \frac{4}{\pi R^2}\left[\frac{r^3}{3}\right]_0^R = \frac{4}{3\pi R^2}.R^3$$

$$y_{cm} = \frac{4R}{3\pi}$$

So centre of mass of circular half disc = $\left(0, \frac{4R}{3\pi}\right)$.

(*b*) Mass per unit area of quarter disc centre of mass of a uniform quarter disc

$$= \frac{M}{\frac{\pi R^2}{4}} = \frac{4M}{\pi R^2}.$$



Using symmetry

For a half disc along y-axis c.m. will be at $y = \frac{4R}{3\pi}$

For a half disc along x-axis c.m. will be at $x = \frac{4R}{3\pi}$

So the c.m. of quarter disc = $\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$.

Q7.24. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(*a*) Does the law of conservation of angular momentum apply to the situation? Why?

(*b*) Find the angular speed of two-disc system.

(*c*) Calculate the loss in kinetic energy of the system in the process.

(*d*) Account for this loss.

Ans. (*a*) The law of conservation of angular momentum can be apply to this situation because there is no net external torque on the system. Gravitational and its normal reaction are external forces but their net torque is zero, hence will not produce any effect.

(*b*) By the law of conservation of angular momentum

$$L_f = L_i \Rightarrow I\omega = I_1\omega_1 + I_2\omega_2$$

where I and ω are the moment of inertia and angular speed of combined system.

$$\therefore \qquad \omega = \frac{I_1\omega_1 + I_2\omega_2}{I} \qquad \because I = I_1 + I_2$$

$$\therefore \qquad \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(*c*) Final kinetic energy = (rotational + translational) Kinetic energy

$$K_f = KE_R + KT_T$$

As there is no translational energy $\therefore KE_T = 0$

$$\therefore \qquad K_f = KE_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(I_1 + I_2)\left[\frac{(I_1\omega_1 + I_2\omega_2)}{I_1 + I_2}\right]^2$$

$$K_f = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)}$$

$$K_i = KE_{1R} + KE_{2R} + KE_{1T} + KE_{2T}$$

As no translational motion in the discs so KE_{1T} and KE_{2T} are zero.

$$\therefore K_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$$

$$\therefore \Delta K = K_f - K_i = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2} - \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2)$$

$$= \frac{1}{2}\left[\frac{I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\omega_1\omega_2 - [(I_1+I_2)(I_1\omega_1^2 + I_2\omega_2^2)]}{(I_1 + I_2)}\right]$$

$$= \frac{\left[I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\omega_1\omega_2\right] - \left[I_1^2\omega_1^2 + I_1I_2\omega_2^2 + I_1I_2\omega_1^2 + I_2^2\omega_2^2\right]}{2(I_1 + I_2)}$$

$$= \frac{\left[I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\omega_1\omega_2 - I_1^2\omega_1^2 - I_1I_2\omega_2^2 - I_1I_2\omega_1^2 - I_2^2\omega_2^2\right]}{2(I_1 + I_2)}$$

$$= \frac{-I_1I_2}{2(I_1 + I_2)}(-2\omega_1\omega_2 + \omega_2^2 + \omega_1^2)$$

$$\Delta K = \frac{-I_1I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 < 0$$

(*d*) Negative sign shows that $K_f < K_i$ as the energy is lost during friction between the moving surfaces of discs.

Q7.25. A disc of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_k.

(*a*) What was the velocity of its centre of mass before being brought in contact with the table?

(*b*) What happens to the linear velocity of a point on its rim when placed in contact with the table?

(*c*) What happens to the linear speed of the centre of mass when disc is placed in contact with the table?

(*d*) Which force is responsible for the effects in (*b*) and (*c*).

(*e*) What condition should be satisfied for rolling to begin?

(*f*) Calculate the time taken for the rolling to begin.

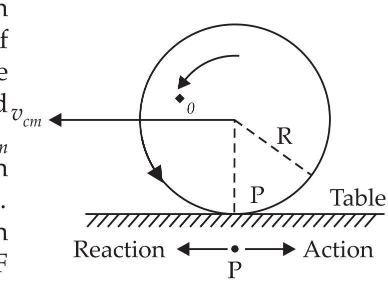
Ans. (*a*) Before being brought in contact with table the disc was in only rotational motion about its axis passing through centre. $\therefore v_{cm} = 0$ as the point on axis are considered at rest.

(*b*) When rotating disc is placed in contact with surface of table, linear velocity of a point on rim will decrease due to force of friction with table.

(*c*) When rotating disc is placed the contact with surface of table, linear velocity gives action force (change in momentum) on table in the direction of rotation by Newtons third law, reaction force is applied on disc due to which it moves in the direction of reaction force so centre of mass of rotating disc acquires linear velocity. (due to reaction force by force of friction).

(*d*) As discussed above in (*b*) and (*c*) part force of friction is responsible for the effect.

(*e*) When rolling of disc starts on table then velocity of centre of mass v_{cm} is due to reaction force due to rotation angular speed ω_0 of disc of radius R. Hence v_{cm} at the start when just comes in contact with table is $v_{cm} = \omega_0 R$.



(*f*) Acceleration (*a*) produced in C.M. due to reaction force F due to frictional force in disc of mass *m* is:

$$F = ma \qquad \Rightarrow \qquad a = \frac{F}{m} = \frac{\mu_k mg}{m} = \mu_k g$$

Angular retardation (α) produce by torque produced due to frictional reaction

$$\tau = I\alpha \qquad \Rightarrow \alpha = \frac{\tau}{I} = \frac{r \times F}{I} = \frac{\vec{R} \times \mu_k\ mg}{I} = R\mu_k mg \sin\theta$$

As the angle between $\vec{R}$ and F is 90°

$$\therefore \qquad \alpha = \frac{-\mu_k\ mgR}{I}$$

$$v_{cm} = u_{cm} + a_{cm}t \qquad \text{(for linear velocity)}$$

$$v_{cm} = 0 + \mu_k\, gt = \mu_k\, gt \qquad ...(i)$$

$$\omega = \omega_0 + \alpha t \qquad \text{(for rotational motion)}$$

$$\omega = \omega_0 - \frac{\mu_k \, mgR}{I} t$$

Condition for rolling without slipping

$$v_{cm} = \omega R \quad \Rightarrow \quad \omega = \frac{v_{cm}}{R}$$

$$\therefore \quad \frac{v_{cm}}{R} = \omega_0 - \frac{\mu_k mg\,R\,t}{I}$$

$$\frac{\mu_k gt}{R} = \omega_0 - \frac{\mu_k mg\,Rt}{I}$$

$$\frac{\mu_k gt}{R} + \frac{\mu_k mg\,Rt}{I} = \omega_0$$

$$\mu_k gt\left[\frac{1}{R} + \frac{mR}{I}\right] = \omega_0$$

$$\frac{\mu_k gt}{R}\left[1 + \frac{mR^2}{I}\right] = \omega_0 \text{ or } \mu_k gt\left[1 + \frac{mR^2}{I}\right] = R\omega_0$$

$$\therefore \quad t = \frac{R\omega_0}{\mu_k g\left[1 + \frac{mR^2}{I}\right]}$$

Here, frictional force help in pure rolling motion without slipping.

Q7.26. Two cylindrical hollow drums of radii R and 2R, and of a common height h, are rotating with angular velocities ω (anti-clockwise) and ω (clockwise), respectively. Their axes, fixed are parallel and in a horizontal plane separated by $(3R + \delta)$. They are now brought in contact $(\delta \to 0)$.

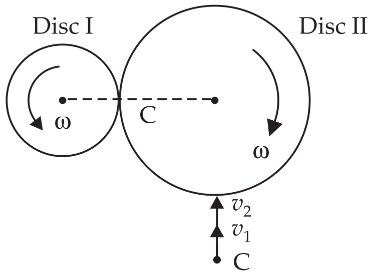
(*a*) Show the frictional forces just after contact.

(*b*) Identify forces and torques external to the system just after contact.

(*c*) What should be the ratio of final angular velocities when friction ceases?

Ans. (*a*) $\because$ $v_1 = \omega R$

$v_2 = \omega.2R = 2\omega R$



The direction of v_1 and v_2 at point of contact C are tangentially upward.

Frictional force (f) acts due to difference in velocities of disc 1 and, f on 1 due to 2 is f_{12} = upward and f_{21} = downward it will be equal and opposite by Newtons Third Law

$$f_{12} = -f_{21}$$

(*b*) External forces acting on system are f_{12} and f_{21} which are equal and opposite so net force acting on system $f_{12} = -f_{21}$

or $f_{12} + f_{21} = 0$

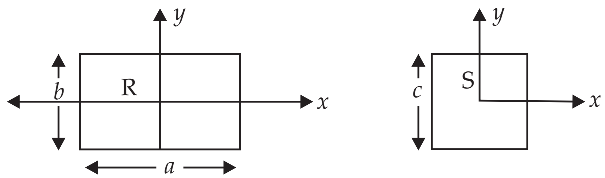
$$|f_{12}| = |-f_{21}| = F$$

$\therefore$ External torque = $F \times 3R$ (anti-clockwise)

As velocity of drum 2 is double i.e., $v_2 = 2v_1$ as in part (*a*).

(*c*) Let ω_1 (anti clockwise) and ω_2 (clockwise) are angular velocities of drum 1 and 2 respectively. Finally when their velocities become equal no force of friction will act due to no slipping at this stage $v_1 = v_2$ or $\omega_1 R = 2\omega_2 R$ or $\dfrac{\omega_1}{\omega_2} = \dfrac{2}{1}$.

Q7.27. A uniform square plate S (side *c*) and a uniform rectangular plate R (sides *b*, *a*) have identical areas and masses (figure).



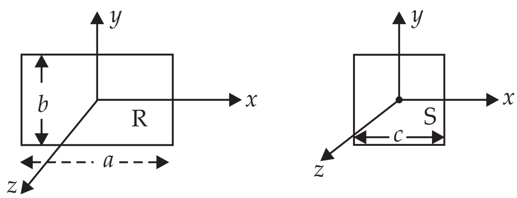
Show that

(*a*) $\dfrac{I_{xR}}{I_{xS}} < 1$ (*b*) $\dfrac{I_{yR}}{I_{yS}} > 1$ (*c*) $\dfrac{I_{zR}}{I_{zS}} > 1$

Ans. $m_R = m_S = m$

Area of square = Area of rectangle

$$c^2 = ab \quad ...(i)$$



(*a*) $\boxed{\because I = mr^2}$

$$\frac{I_{xR}}{I_{xS}} = \frac{m.\left(\frac{b}{2}\right)^2}{m\left(\frac{c}{2}\right)^2} = \frac{b^2}{4}\frac{4}{c^2} = \frac{b^2}{c^2}$$

$\because$ $c > b$ [from (*i*)]

or $c^2 > b^2$

$$1 > \frac{b^2}{c^2} \quad \therefore \frac{I_{xR}}{I_{xS}} < 1 \text{ Hence proved.}$$

(*b*) $$\frac{I_{yR}}{I_{yS}} = \frac{m\left(\frac{a}{2}\right)^2}{m\left(\frac{c}{2}\right)^2} = \frac{a^2}{4}.\frac{4}{c^2} = \frac{a^2}{c^2}$$

$$\because \qquad a > c \quad \Rightarrow \quad \frac{a^2}{c^2} > 1$$

$$\frac{I_{yR}}{I_{yS}} > 1$$

(*c*) $$I_{zR} - I_{zS} = m\left(\frac{d_R}{2}\right)^2 - m\left(\frac{d_S}{2}\right)^2$$

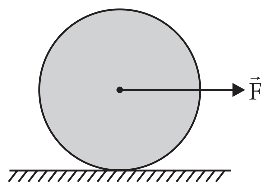
$$I_{zR} - I_{zS} = \frac{m}{4}\left[d_R^2 - d_S^2\right] = \frac{m}{4}\left[a^2 + b^2 - 2c^2\right]$$

$$\therefore \quad I_{zR} - I_{zS} = \frac{m}{4}(a^2 + b^2 - 2ab) = \frac{m}{4}(a-b)^2 \qquad (c^2 = ab)$$

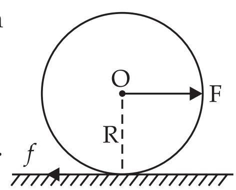
$$\therefore \quad I_{zR} - I_{zS} > 0 \quad \because \frac{m}{4}(a-b)^2 > 0$$

$$\Rightarrow \quad \frac{I_{zR}}{I_{zS}} > 1$$ Hence proved.

Q7.28. A uniform disc of radius R, is resting on its rim. The coefficient of friction between disc and table is μ (figure). Now the disc is pulled with a force $\vec{F}$ as shown in the figure. What is the maximum value of $\vec{F}$ for which the disc rolls without slipping?



Ans. Let '*a*', α be the linear and angular acceleration respectively. For linear motion



$$F - f = Ma \qquad ...(i)$$

where M is mass of disc.

Force of friction (f) applies torque about centre O.

But torque due to F is zero as F is along 'O'.

$\therefore$ Torque on disc $\tau = I_D\alpha$

$\because$ M.I. of disc is $I_D = \frac{1}{2}MR^2$

$$f.R = \frac{1}{2}MR^2.\frac{a}{R} \qquad \because a = R\alpha$$

$$fR = \frac{1}{2}MRa \quad \Rightarrow \quad Ma = 2f \qquad ...(ii)$$

$$F - f = 2f$$

$$3f = F \quad \text{or} \quad f = \frac{F}{3} \qquad \because N = Mg$$

$$\mu.N = \frac{F}{3} \quad \text{or} \quad \mu\, Mg = \frac{F}{3}$$

F = 3μ Mg is the maximum force applied on disc to roll on surface without slipping.

❑❑❑

8 Gravitation

MULTIPLE CHOICE QUESTIONS-I

Q8.1. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity.

(*a*) Will be directed towards the centre but not the same everywhere.
(*b*) Will have the same value everywhere but not directed towards the centre.
(*c*) Will be same everywhere in magnitude directed towards the centre.
(*d*) Cannot be zero at any point.

Ans. (*d*): If the density of earth is non uniform then '*g*' at different point will be different $\left(\because g = \frac{4}{3}\pi\rho GR\right)$. So the $g = 0$ cannot be any point. Verifies option (*d*).

Q8.2. As observed from earth, the sun appears to move in an approximate circular orbit. For the motion of another planets like mercury as observed from the earth. This would

(*a*) be similarly true.
(*b*) not be true because the force between earth and mercury is not inverse square law.
(*c*) not be true because the major gravitational force on mercury is due to sun.
(*d*) not be true because mercury is influenced by forces other than gravitational forces.

Ans. (*c*): Force of attraction between any two objects obeys the inverse square law as its universal law. The relative motion between earth, mercury as observed from earth will not be circular as the force on mercury due to sun is very large than due to earth and due to the relative motion to sun and earth with mercury.

Q8.3. Different points in earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the c.m. (centre of mass) causing translational and a net torque at the c.m. causing rotation around an axis through the c.m. For the earth-sun system (approximating the earth as a uniform density sphere)

(*a*) the torque is zero
(*b*) the torque causes the earth to spin

(*c*) the rigid body result is not applicable since the earth is not even approximately a rigid body.
(*d*) the torque causes the earth to move around the sun.

Ans. (*a*): The torque on earth due to gravitational attractive force on earth is zero, because the direction of force (F) (gravitational) and line joining $(\vec{r})$ the point of application of force (which also at c.m. of earth) is along same line so angle between $\vec{r}$, and $\vec{F}$ is zero so by $\vec{\tau} = \vec{r} \times \vec{F}$ $= |\vec{r}|\,|\vec{F}| \sin 0° = 0$ verifies the answer (*a*).

Q8.4. Satellites orbiting the earth have a finite life and sometimes debris of satellites fall to the earth. This is because,
(*a*) the solar cells and batteries in satellites run out.
(*b*) the laws of gravitation predict a trajectory spiralling inwards.
(*c*) of viscous forces causing the speed of satellite and hence height to gradually decrease.
(*d*) of collisions with other satellites.

Ans. (*c*): The P.E. of satellite orbiting in orbit of radius *r* due to earth of mass M is $\left(\frac{-\,GM}{2r}\right)$ negative sign shows force of attraction between satellite and earth. Energy (P.E.) is consumed against the viscous force due to air so P.E. of satellite causes decrease in radius. So ultimates it comes on earth verifies the option (*c*).

Q8.5. Both earth and moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon
(*a*) will be elliptical.
(*b*) will not be strictly elliptical because the total gravitational force on it is not central.
(*c*) is not elliptical but will necessarily be a closed curve.
(*d*) deviate considerably from being elliptical due to influence of planets other than earth.

Ans. (*b*): The major force acting on moon is due to gravitational force of attraction by sun and earth and moon is not always in the line of joining sun and earth. So the two forces have different lines of action or not central force, so its motion will not be strictly elliptical. Verifies the option (*b*).

Q8.6. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They
(*a*) will not move around the sun since they have very small masses compared to sun
(*b*) will move is an irregular way because of their small masses and will drift away into outer space.
(*c*) will move around the sun in closed orbits, but not obey Kepler's law.
(*d*) will move in orbits like planets and obey Kepler's laws.

Ans. (*d*): Asteroids will move in orbits like planets and obey Kepler's law because they are also being acted upon by central gravitational forces.

Q8.7. Choose the wrong option.

(*a*) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass.

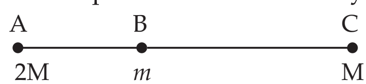
(*b*) That the gravitational mass and inertial mass are equal is an experimental result.

(*c*) That the acceleration due to gravity on earth is the same for all bodies is due to the equality of gravitational mass and inertial mass.

(*d*) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot.

Ans. (*d*): Gravitational mass of proton is equivalent to its inertial mass and is independent of presence of neighbouring heavy objects, so verifies the option (*d*).

Q8.8. Particles of masses 2M, m and M are respectively at points A, B and C with AB = $\frac{1}{2}$(BC). m is much-much smaller than M and at time $t = 0$, they are all at rest in figure. At subsequent times before any collision takes place:



(*a*) m will remain at rest.

(*b*) m will move towards M.

(*c*) m will move towards 2M.

(*d*) m will have oscillatory motion.

Ans. (*c*): $$F_{BC} = \frac{G.m.M}{(BC)^2} \text{ towards B to C}$$

Let AB = x then $x = \frac{1}{2}BC \Rightarrow BC = 2x$

$$\therefore \quad F_{BC} = \frac{GMm}{(2x)^2} = \frac{GMm}{4x^2} = \frac{1}{4}F_g \quad \text{(Let)}$$

$$F_{BA} = \frac{G2Mm}{(x)^2} = \frac{2GMm}{x^2} = 2F_g \text{ towards B to A}$$

As $F_{BA} > F_{BC}$. Hence m will move towards A i.e., towards 2M, and verifies option (*c*).

MULTIPLE CHOICE QUESTIONS-II

Q8.9. Which of the following options are correct?

(*a*) Acceleration due to gravity decreases with increasing altitude.

(*b*) Acceleration due to gravity increases with increasing depth (assume the earth to be a sphere of uniform density).

(*c*) Acceleration due to gravity increases with increasing latitude.

(*d*) Acceleration due to gravity is independent of the mass of the earth.

Ans. (*a*, *c*): $\because$ $g_h = g\left[1-\frac{2h}{R}\right]$ and $g_d = g\left[1-\frac{d}{R}\right]$

So on increasing altitude, and depth inside earth the g_h and g_d both decreases.

So verifies option (*a*) and rejects the option (*b*).

If λ is latitude on earth then

$$g_\lambda = g - \omega^2 R \cos^2 \lambda$$

As $\cos \lambda$ decrease from 0° to 90° from 1 to 0. So acceleration due to gravity increases from equator ($\lambda = 0°$) to pole ($\lambda = 90°$). So verifies the option (*c*).

$\because$ Acceleration due to gravity on surface of earth is $g = \frac{GM_e}{R_e^2}$

So g on earth depends on mass of earth and rejects the option (*d*).

Q8.10. If the law of gravitation, instead of being inverse-square law, becomes an inverse-cube law:

(*a*) Planets will not have elliptic orbits.

(*b*) Circular orbits of planets is not possible.

(*c*) Projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic.

(*d*) There will be no gravitational force inside a spherical shell of uniform density.

Ans. (*a*, *c*): Force of gravitation = Centrifugal force for a body in circular motion. So it becomes

$$\frac{GM_s m_p}{r^3} = \frac{m_p v_0^2}{r} \Rightarrow v_0^2 = \frac{GM_s}{r^2}$$

r = distance between planet and sun.

m_p = mass of planet, M_s = mass of sun

Orbital velocity of planet $v_0 = \frac{\sqrt{GM_s}}{r} \Rightarrow v_0 \propto \frac{1}{r}$

Time period of planet around sun = $\frac{2\pi r}{v_0}$

$$T = \frac{2\pi r \times r}{\sqrt{GM_s}} = \frac{2\pi r^2}{\sqrt{GM_s}} \quad \text{or} \quad T \propto r^2 \qquad ...(i)$$

or $T^2 \propto r^4$ for elliptical orbit the condition is $T^2 \propto r^3$ by Kepler's law. Hence, the orbit of planetary motion will not be elliptical. Verifies option (*a*).

$$F = \frac{Gm_p}{R^3} = mg'$$

here R = radius of earth

m = mass of a body on earth

g' = new acceleration due to gravity

$$\therefore \qquad g' = \frac{Gm_p}{R^3}$$

g' is again constant for a planet or earth so path of a projectile will be approximately parabolic T $\propto r^2$ [from (i)]. Verifies the option(c) and rejects (b).

Gravitational force is universal force acts every where so rejects the option (d).

Q8.11. If the mass of sun were ten times smaller and gravitational constant G were ten times larger in magnitudes.

(a) walking on ground would became more difficult.

(b) the acceleration due to gravity on the earth will not change.

(c) raindrops will fall much faster.

(d) airplanes will have to travel much faster.

Ans. (a, c, d): $\quad G' = 10\,G$

$$M_s' = \frac{1}{10}M_s \Rightarrow 10M_s' = M_s$$

Weight of body (m) on earth = $\dfrac{G' m_e m}{R^2}$

$$mg' = \frac{10G\,m_e m}{R^2}$$

$$g' = \frac{10G\,m_e}{R^2} = 10g$$

Weight of person becomes 10 times larger so it will be more difficult to walk verifies option (a).

As $g' = 10g$ so the acceleration due to gravity changes so rejects the option (b).

As the terminal velocity $v_T \propto g$ and g' becomes 10 g so terminal velocity increased by 10 times so the rain drops becomes faster 10 times verifies option (c). As the g' becomes 10 times of g so the (centripetal) force on aeroplane increased, to balance it speed of aeroplane increased. Hence verifies the option (d).

Q8.12. If the sun and the planets carried huge amounts of opposite charges,

(a) All three of Kepler's laws would still be valid.

(b) Only the third law will be valid.

(c) The second law will not change.

(d) The first law will still be valid.

Ans. (a, c, d): As the forces between +ive and (–)ive charge and gravitational forces are attractive. So force of attraction becomes larger (between sun and planet).

Hence, all the three laws of Kepler's will hold good. Verifies option (a, c, d).

Q8.13. There have been suggestions that the value of the gravitation constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,

(a) Nothing will change.

(b) We will become hotter after billions of years.

(c) We will be going around but not strictly in closed orbits.

(d) After sufficiently long time we will leave the solar system.

Ans. (*c, d*): We know that centripetal force is provided by gravitational force $F_g = \dfrac{GM_s m_e}{r^2}$ according to Question G becomes smaller in turn the centripetal force reduced but orbital velocity $v_0 = \sqrt{\dfrac{GM_s}{r}}$ or $v \propto \sqrt{G}$ reduced smaller i.e., centrifugal force becomes larger than the centripetal (gravitation $F_g \propto G$).

So the planet earth will move in larger orbit every time i.e., distance from sun increased, so temperature become smaller and orbit will not be close. So verifies option (*c, d*) and rejects (*b, a*).

Q8.14. Supposing Newton's law of gravitation for gravitation forces $\vec{F}_1$ and $\vec{F}_2$ between two masses m_1 and m_2 at positions $\vec{r}_1$ and $\vec{r}_2$ read $\vec{F}_1 = -\vec{F}_2 = -\dfrac{\vec{r}_{12}}{r_{12}^3} GM_0^2 \left[\dfrac{m_1 m_2}{M_0^2}\right]^n$ where M_0 is a constant of dimension of mass, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ and n is number. In such a case

(a) the acceleration due to gravity on earth will be different for different objects.

(b) none of the three laws of Kepler will be valid.

(c) only the third law will become invalid.

(d) for n negative, an object lighter than water will sink in water.

Ans. (*a, c, d*): $\because$ $\vec{F}_1 = -\vec{F}_2 = \dfrac{\vec{r}_{12}}{r_{12}^3} GM_0^2 \left[\dfrac{m_1 m_2}{M_0^2}\right]^n$ given $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$

Acceleration due to gravity $g = \dfrac{|F|}{\text{mass } (m)}$

$$\therefore \qquad g = \left| \frac{GM_0^2 \, (m_1 m_2)^n}{r_{12}^3 \, (M_0)^{2n}} \times \frac{\hat{r}_{12}}{m} \right|$$

$\hat{r}_{12}$ tells the direction of g from 1 to 2.

$$\therefore \qquad g = \frac{GM_0^2 \, (m_1 m_2)^n}{r_{12}^2 \, (M_0)^{2n}} \frac{1}{m}$$

Since 'g' depends on position vector $|r_{12}|$, mass of body m so 'g' on earth will be different for different bodies of different mass and their position also. Hence, verifies option (*a*).

As g is not constant. Hence, the third law of Kepler is not valid verifies option (*c*).

As the force is central in nature. So Kepler's I, II law valid rejects option (*b*).

For option (*d*), for n is negative.

$$g = \frac{GM_0^2 (m_1 m_2)^{-n}}{(r_{12})^2 (M_0)^{-2n}} \times \frac{1}{m}$$

$$g = \frac{GM_0^2 (m_1 m_2)^{-n}}{r_{12}^2 \, m} = \frac{GM_0^2}{(m_1 m_2)^n} \frac{M_0^{2n}}{r_{12}^2} \cdot \frac{1}{m}$$

$$g = \frac{GM_0^2}{r_{12}^2} \left[\frac{M_0^2}{m_1 m_2} \right]^n \cdot \frac{1}{m}$$

$\because$ $g > 0$. So $M_0 > m_1$ or m_2

Hence, in this case lighter object sink in water and verifies option (*d*).

Q8.15. Which of the following are true?

(*a*) A polar satellite goes around the earth's pole north-south direction.

(*b*) A geostationary satellite goes around the earth in east-west direction.

(*c*) A geostationary satellite goes around the earth in west-east direction.

(*d*) A polar satellite goes around the earth in east-west direction.

Ans. (*a, c*): A geostationary satellite appears stationary with respect to earth so it revolve along the direction of rotation west-east so the geostationary satellite goes around the earth west to east near the equator verifies the option (*c*) and rejects (*b*).

Polar satellite arounds about the poles of earth so it rounds north-south direction. Verifies the option (*a*) and rejects (*d*).

Q8.16. The centre of mass of an extended body on the surface of the earth and its centre of gravity

(*a*) are always at the same point for any size of the body

(*b*) are always at the same point only for spherical bodies.

(*c*) can never be at the same point.

(*d*) is close to each other for objects, say of size less than 100 m.

(*e*) both can change if the object is taken deep inside the earth.

Ans. (*d*): If the size of object is less than 100 m << R_e the centre of mass and centre of gravity very close to each other. If the size of object extended i.e., very large like lake, or mountain then the distance between centre of mass and centre of gravity increased. Hence, verifies the option (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q8.17. Molecules in air in the atmosphere are attracted by gravitational force of the earth. Explain why all of them do not fall into the earth just like an apple falling from the tree.

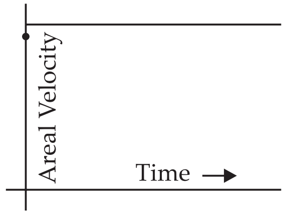
Ans. Air molecules are random in motion due to thermal (temperature) energy. Air molecules and apples both are attracted by earth by gravitational force but resultant velocity of air molecule is not exactly downward as apple as it has not any other motion except downward.

Q8.18. Give one example each of central force and non-central force.

Ans. Gravitational force, electrostatic force due to point mass and point charges are the example of central force.

Spin dependent nuclear forces, magnetic forces between two current carrying loops are the example of non central forces.

Q8.19. Draw areal velocity versus time graph for Mars.



Ans. Mars is a planet and obey's the Kepler's law. By Kepler's second law areal velocity by position vector of planet from sun to planet is always constant.

Q8.20. What is the direction of areal velocity of the earth around the sun?

Ans. Areal velocity of any planet around the sun is

$$\frac{dA}{dt} = \frac{L}{2m}$$

where L is angular momentum and m is the mass of the earth but $L = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

$$\frac{dA}{dt} = \frac{1}{2m}(\vec{r} \times m\vec{v}) = \frac{1}{2}\vec{r} \times \vec{v}$$

So the direction of areal velocity is perpendicular to plane of $\vec{r}$ and $\vec{v}$ as given by right hand grip rule of Maxwell.

Q8.21. How is the gravitational force between two point masses affected when they are dipped in water keeping the separation between them the same?

Ans. By Newton's Universal law of gravitational force of attraction (F) between two bodies of masses m_1, m_2 separated by distance r is

$$F = \frac{Gm_1m_2}{r^2}$$

G does not depend upon the medium. So force of attraction does not **changes** if the masses are kept in water or any medium.

Q8.22. Is it possible for a body to have inertia but no weight?

Ans. Each body has mass or inertia. But weight is measured by spring or weighing machine and is equal to mg, g in space can be zero or weight of freely falling object is zero. g at the centre of the earth is zero. So, the weight (mg) of a body can be zero, but inertia or mass (m) can never be zero.

Q8.23. We can shield charge from electric fields by putting it inside a hollow conductor. Can we shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

Ans. Gravitational force does not depend upon the nature of the medium, but electric force depend on intervening medium between them. So we cannot shield a body from gravitational force.

Q8.24. An astronaut inside a small spaceship orbiting around the earth cannot detect gravity. If the space station orbiting the earth has a large size, can he hope to detect gravity?

Ans. Astronaut inside a small spaceship experience a very small negligible constant acceleration and hence astronaut feel weightlessness. If the space station has too much large mass and size then he can experience acceleration due to gravity e.g. on moon.

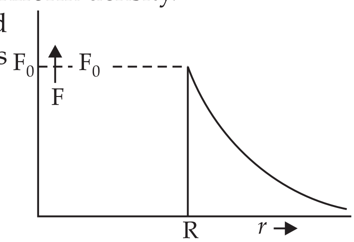
Q8.25. The gravitational force between a hollow spherical shell (of radius R and uniform density) and a point mass is F. Show that the nature of F versus r graph where r is the distance of the point from the centre of the hollow spherical shell of uniform density.

Ans. The gravitation force inside the and at the centre of hollow spherical shell is zero and on the surface is F_0 and inside

$$F_0 = \frac{Gm_1m_2}{R^2} \quad \text{at } r = R$$

and when $r > R$

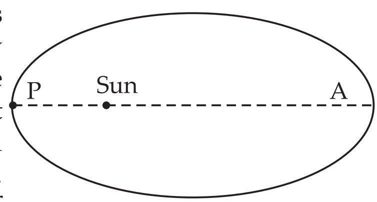
$$F \propto \frac{1}{r^2}$$



When point mass is inside the shell it becomes a part of shell.

Q8.26. Out of aphelion and perihelion, where is the speed of the earth more and why?

Ans. Earth revolve around the sun is elliptical orbit or by Kepler's first law and sun remains at its one focus. The position of earth at P and A at shortest and longest distance are called perihelion and Aphelion respectively. According to the second law of Kepler's the areal velocity of planet around the sun is constant.



$$\frac{dA}{dt} = \frac{L}{2m} = \frac{r \times p}{2m} = \frac{r \times mv}{2m} = \frac{1}{2} r \times v$$

Hence, if r increases at Aphelion the v decreases and *vice-versa* at P.

Q8.27. What is the angle between the equatorial plane and the orbital plane of: (*a*) Polar satellite? (*b*) Geostationary satellite?

Ans. (*a*) Polar satellite revolve along North south pole. So it plane makes 90° with equatorial plane.

(*b*) Geostationary satellite revolve along west to east direction along equatorial plane so angle between geostationary and equatorial plane is zero.

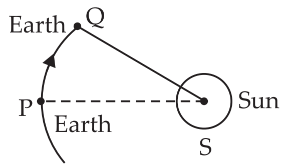
SHORT ANSWER TYPE QUESTIONS

Q8.28. Mean solar day is the time interval between two successive noon when sun passes through zenith point (meridian).

Sidereal day is the time interval between two successive transit of a distant star through the zenith point (meridian).

By drawing appropriate diagram showing earth's spin and orbital motion, show that mean solar day is four minutes longer than the sidereal day. In other words, distant stars would rise 4 minutes early every successive day.

[*Hint:* You may assume circular orbit for the earth.]



Ans. Consider that on a day at noon sun passes through zenith (meridian). After one revolution (360°) of earth about its own axis sun again passes through zenith.

During this time when earth revolve at it's own axis by 360° it changes its angle PSQ = 1°. So 361° rotation by earth is considered one solar day.

$\because$ In 361° corresponds to the = 24 hrs

$$1° \text{ will corresponds to } \frac{24}{361} \text{ hrs} = \frac{24}{361} \times 3600 \text{ sec}$$

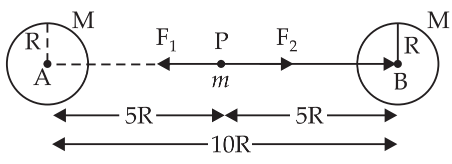
$$= 3 \text{ min } 59 \text{ sec} \cong 4 \text{ min.}$$

Hence, distant star rises 4 min. early every day.

Q8.29. Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the mid-point of the line joining their centres be in stable equilibrium or unstable equilibrium? Give reason for your answer.

Ans. $m_1 = m_2 = M \quad r = 10\,R$

Let mass m is placed at the mid point P of line joining the centres of A and B sphere



$$|F_2| = |F_1| = \frac{GMm}{(5R)^2}$$

$$|F_1| = |F_2| = \frac{GMm}{25R^2}$$

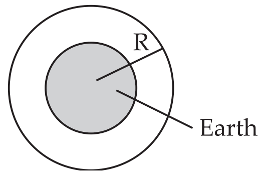
As the direction of force F_1 and F_2 are in opposite direction i.e., equal and opposite forces are acting on m at P. As net force $F_1 = -F_2$ or $F_1 + F_2 = 0$ is zero so the m is in equilibrium. If m is displaced x slightly from P to A then PA = $(5R - x)$ and PB = $(5R + x)$

i.e., $$F_1 = \frac{GMm}{(5R - x)^2} \quad \text{and} \quad F_2 = \frac{GMm}{(5R + x)^2}$$

$\therefore$ $F_2 < F_1$ i.e., resultant force acting on P is towards A. Hence, equilibrium is unstable equilibrium.

Q8.30. Show the nature of the following graph for a satellite orbiting the earth.

(*i*) K.E. *versus* orbital radius R.
(*ii*) P.E. *versus* orbital radius R.
(*iii*) T.E. *versus* orbital radius R.



Ans. Mass of earth = M_e

Radius of orbit of satellite = R

Mass of satellite = m

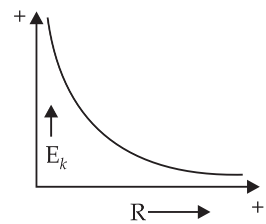
$$\text{Orbital Velocity } v_0 = \sqrt{\frac{GM}{R}}$$

(*a*) **E_k versus R:**

$$KE_k = \frac{1}{2} m v_0^2 = \frac{1}{2} m . \frac{GM}{R} = \frac{GMm}{2R}$$

$$E_k \propto \frac{1}{R}$$

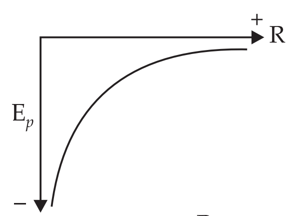
i.e., E_k decreases exponentially with R.



(*b*) **E_p versus R:** Potential energy

$$E_p = \frac{-GMm}{R}$$

$$E_p \propto -\frac{1}{R}$$



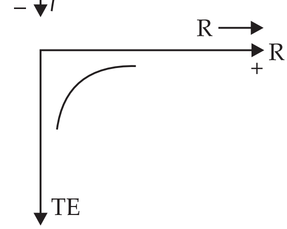
(*c*) **T.E. versus R:**

$$E_k = +\frac{1}{2}\frac{GMm}{R}$$

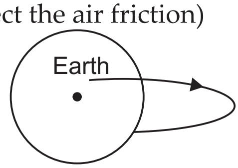
$$E_p = \frac{-GMm}{R}$$

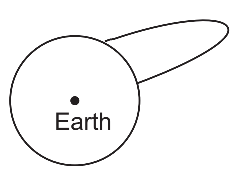
$$\text{T.E.} = E = \frac{1}{2}\frac{GMm}{R} - \frac{GMm}{R}$$

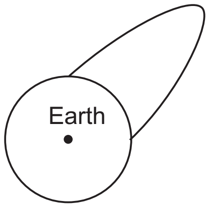
$$E = -\frac{1}{2}\frac{GMm}{R}$$

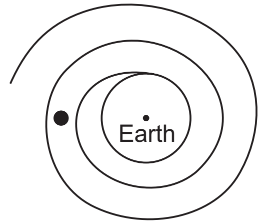


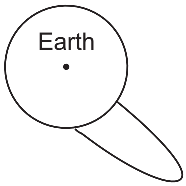
Q8.31. Shown are several curves (figure). Explain with reason, which ones amongst them can be possible trajectories traced by a projectile. (neglect the air friction)

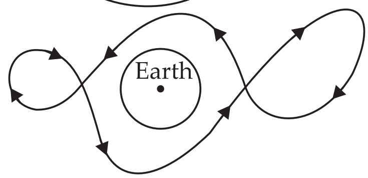
(*a*) 

(*b*) 

(*c*) 

(*d*) 

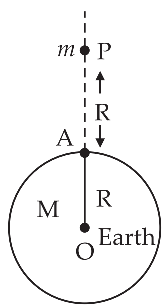
(*e*) 

(*f*) 

Ans. The trajectory of a projectile under gravitational force of earth is conic section or parabolic or elliptical or its part whose focus must be the centre of earth. Only (*c*) option in which centre of earth is the focus of trajectory.

Q8.32. An object of mass m is raised from the surface of the earth to a height equal to the radius of the earth, that is, taken from a distance R to 2R from the centre of the earth. What is the gain in its potential energy?

Ans. As the object of mass m is lifted upward from surface of earth to a height equal to the radius of earth. i.e., from $R \to 2R$



P.E. of body on the surface of earth = $\dfrac{-GMm}{R}$

P.E. of the object at a height equal to the radius of earth

$$= \frac{-GMm}{(2R)}$$

$$\text{Gain in P.E.} = E_{pf} - E_{pi}$$

$$\text{Gain in P.E.} = \frac{-GMm}{2R} - \left(\frac{-GMm}{R}\right)$$

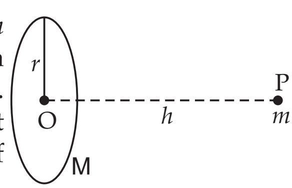
$$= \frac{GMm}{R}\left[-\frac{1}{2}+1\right]$$

$$= \frac{GMm}{2R}$$

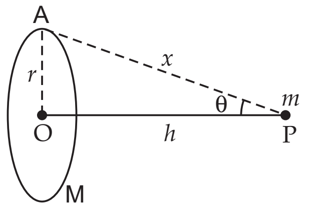
$$\boxed{GM = gR^2}$$

$$\text{Gain in P.E.} = \frac{gR^2 m}{2R} = \frac{1}{2}mgR$$

Q8.33. A mass m is placed at P a distance h along the normal through the centre O of a thin circular ring of mass M and radius r (figure). If the mass is removed further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h = r$?



Ans. A ring of radius r of mass M and a mass m is placed at a distance h axially at P. Then the gravitational force F at P



$$F_h = \frac{GMm\cos\theta}{AP^2} \quad \boxed{F_h = \frac{GMmh}{(r^2+h^2)^{3/2}}} \quad \left[\because \cos\theta = \frac{h}{(r^2+h^2)^{1/2}}\right]$$

$$\frac{F_r}{F_{2r}} = \frac{\dfrac{GMm.r}{(r^2+r^2)^{3/2}}}{\dfrac{GMm\,2r}{[r^2+(2r)^2]^{3/2}}} = \frac{1}{(2r^2)^{3/2}} \times \frac{(r^2+4r^2)^{3/2}}{2}$$

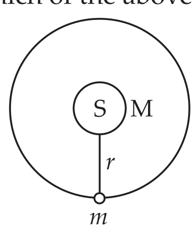
$$\frac{F_r}{F_{2r}} = \frac{1}{2\sqrt{2}r^3} \times \frac{(5r^2)^{3/2}}{2} = \frac{5\sqrt{5}r^3}{4\sqrt{2}r^3}$$

$$\frac{F_r}{F_{2r}} = \frac{5}{4}\sqrt{\frac{5}{2}} \quad \text{or} \quad \frac{F_{2r}}{F_r} = \frac{4}{5}\sqrt{\frac{2}{5}}$$

$\therefore$ Gravitational force on m at distance $2r$ from O is the $\frac{4}{5}\sqrt{\frac{2}{5}}$ time the gravitational force when m is placed at r distance from O.

LONG ANSWER TYPE QUESTIONS

Q8.34. A star like the sun has several bodies moving around it at different distances. Consider that all of them are moving in circular orbits. Let r be the distance of the body from the centre of the star and let its linear velocity be v, angular velocity ω, kinetic energy K, gravitational potential energy U, total energy E and angular momentum L. As the radius r of the orbit increases, determine which of the above quantities increase and which ones decrease.



Ans. Let us consider a body of mass m is rotating around the star S of mass m in circular path of radius r.

(*i*) Then orbital velocity

$$v_0 = \sqrt{\frac{GM}{r}} \quad \text{or} \quad v_0 \propto \frac{1}{\sqrt{r}}$$

Hence, on **increasing radius** of circular path **orbital velocity decreases**.

(*ii*) Angular velocity $\omega = \dfrac{2\pi}{T}$ and $T^2 \propto r^3$ by Kepler's third law

$$\therefore \qquad \omega = \frac{2\pi}{Kr^{3/2}} \quad \text{or} \quad \omega \propto \frac{1}{\sqrt{r^3}}$$

Hence, on increasing the radius of circular orbit the angular velocity **decreased**.

(*iii*) Kinetic energy $E_k = \frac{1}{2} m \frac{GM}{r}$

or $E_k \propto \frac{1}{r}$. Hence on increasing the radius of circular path the kinetic energy decreased.

(*iv*) Gravitation potential energy $E_p = \frac{-GMm}{r}$ or $E_p \propto -\left(\frac{1}{r}\right)$ so, on increasing radius of circular orbit the P.E. (E_p) increases.

(*v*) Total energy $E = E_k + E_p = \frac{GMm}{2r} + \left(\frac{-GMm}{r}\right)$

$$E = \frac{-GMm}{2r}$$

Hence, on increasing the radius of circular orbit the total energy E will also be increased.

(*vi*) Angular momentum = $L = mvr = m\sqrt{\frac{GM}{r}}\, r$

$L = m\sqrt{GMr}$ or $L \propto \sqrt{r}$

Hence, the increasing radius r of circular orbit increases the angular momentum.

Q8.35. Six point masses of mass m each are at the vertices of a regular hexagon of side l. Calculate the force on any of the masses.

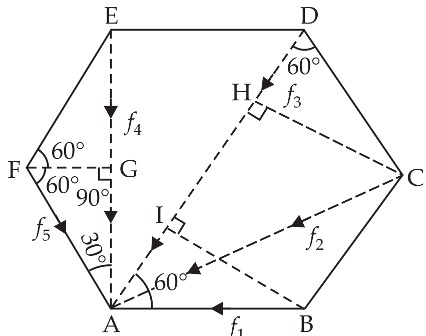
Ans. $AE = AG + EG = AG + AG = 2AG = 2l \cos 30°$

$$AE = 2l\frac{\sqrt{3}}{2} = \sqrt{3}l$$

$$\boxed{AE = AC = \sqrt{3}l}$$

$$AD = DH + HI + AI$$
$$= AI + l + AI = l + 2AI$$
$$= l + 2AB \cos 60°$$
$$= l + 2l \times \frac{1}{2} = 2l$$

$$\boxed{AD = 2l}$$



Force on A due to B = $f_1 = \frac{Gmm}{l^2} = \frac{Gm^2}{l^2}$ along B to A

Force on A due to C = $f_2 = \frac{Gmm}{(\sqrt{3}\,l)^2} = \frac{Gm^2}{3l^2}$ along C to A

Force on A due to D = $f_3 = \frac{Gmm}{(2l)^2} = \frac{Gm^2}{4l^2}$ along D to A

Force on A due to E = $f_4 = \frac{Gmm}{(\sqrt{3}\,l)^2} = \frac{Gm^2}{3l^2}$ along E to A

Force on A due to F = $f_5 = \dfrac{Gmm}{l^2} = \dfrac{Gm^2}{l^2}$ along F to A

$$\vec{F}_1^2 = \vec{F}_1 + \vec{F}_5 = (f_1)^2 + (f_5)^2 + 2f_1 f_5 \cos 120°$$

$$= \left(\frac{Gm^2}{l^2}\right)^2 + \left(\frac{Gm^2}{l^2}\right)^2 + 2\left(\frac{Gm^2}{l^2}\right)^2 \cos(90° + 30°)$$

$$= \left[\frac{Gm^2}{l^2}\right]^2 [1 + 1 - 2\sin 30°]$$

$$F_1 = \sqrt{\left(\frac{Gm^2}{l^2}\right)^2 \left[1 + 1 - 2 \times \frac{1}{2}\right]} = \sqrt{\left[\frac{Gm^2}{l^2}\right]^2}$$

$$F_1 = \frac{Gm^2}{l^2}$$

$$\vec{F}_2^2 = \left[\vec{F}_2 + \vec{F}_4\right] = f_2^2 + f_4^2 + 2f_2 f_4 \cos 60°$$

$$F_2^2 = \left(\frac{Gm^2}{3l^2}\right)^2 [1 + 1 + 1]$$

$$F_2 = \frac{Gm^2}{3l^2}\sqrt{3} = \frac{Gm^2}{\sqrt{3}\,l^2}$$

$$F_3 = \frac{Gm^2}{4l^2}$$

$$F = F_1 + F_2 + F_3 = \frac{Gm^2}{l^2} + \frac{Gm^2}{\sqrt{3}\,l^2} + \frac{Gm^2}{4l^2}$$

$$F = \frac{Gm^2}{l^2}\left[1 + \frac{1}{\sqrt{3}} + \frac{1}{4}\right] \text{ along DA.}$$

Q8.36. A satellite is to be placed in equatorial geostationary orbit around earth for communication.

(*a*) Calculate height of such a satellite.

(*b*) Find out the minimum number of satellites that are needed to cover entire earth so that at least one satellite is visible from any point on the equator. [$M = 6 \times 10^{24}$ kg, $R = 6400$ km, $T = 24$ h, $G = 6.67 \times 10^{-11}$ SI units]

Ans. (*a*) Mass of earth $M = 6 \times 10^{24}$ kg

Radius of earth $R = 6400 \text{ km} = 6.4 \times 10^6$ m

Time period $T = 24h = 24 \times 3600 \text{ s} = 24 \times 36 \times 10^2$ s

$G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}

Orbital Radius = $(R + h)$,

h is height of satellite from earth surface.

$$v_0 = \sqrt{\frac{GM}{R + h}} \Rightarrow v_0^2 = \frac{GM}{(R + h)}$$

$$\therefore \quad T = \frac{2\pi (R + h)}{v_0} \quad \text{or} \quad T^2 = \frac{4\pi^2 (R + h)^2 (R + h)}{GM}$$

$$T^2 = \frac{4\pi^2 (R+h)^3}{GM} \quad \text{or} \quad (R+h) = \left[\frac{GT^2M}{4\pi^2}\right]^{1/3}$$

$$h = \left[\frac{GT^2M}{4\pi^2}\right]^{1/3} - R$$

$$h = \left[\frac{6.67\times10^{-11}\times(24\times36)^2\times(10^2)^2\times6\times10^{24}}{4\times3.14\times3.14}\right]^{1/3} - 6.4\times10^6$$

$$h = \left[\frac{167\times24^2\times36^2\times6\times10^{-11+4+24+2}}{314\times314}\right]^{1/3} - 6.4\times10^6$$

$$h = \left[\frac{167\times24^2\times36^2\times6\times10^{-19}}{314\times314}\right]^{1/3} - 6.4\times10^6$$

$$h = x - 6.4\times10^6$$

$$\log x = \frac{1}{3}\left[\log N^r - \log D^r\right]$$

$\log N^r$	2.2227	$-\log D^r =$	2.4969
	1.3802		2.4969
	1.3802		4.9938
	1.5563		
	1.5563		
	0.7782		
	19.0000		
	27.8739		
$\log D^r$	– 4.9938		
	22.8801		

$$\log x = \frac{1}{3}\times22.8801$$

$$\log x = 7.6267$$

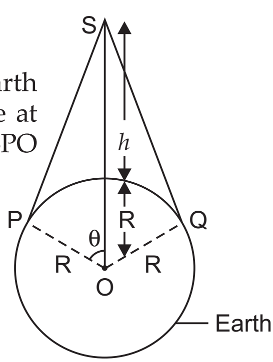
$$x = 4.2335\times10^7$$

$h = 42.335\times10^6 - 6.4\times10^6 = (42.335 - 6.4)\times10^6$ m

$= 35.94\times10^3\times10^3$ m $= 35.94\times10^3$ km

$h = 35{,}940$ km.

(*b*) Let a satellite S is at *h*m above the earth surface. Let angle subtended by satellite at centre of earth 2θ. Then in right angle ΔSPO



$$\cos\theta = \frac{R}{R+h} = \frac{R}{R\left[1+\frac{h}{R}\right]}$$

$$\cos\theta = \frac{1}{\left[1+\frac{h}{R}\right]}$$

$h = 3.59 \times 10^7$ m

(Height of geostationary satellite)

$R = 6.4 \times 10^6$ m

$$\cos\theta = \frac{1}{\left[1 + \frac{h}{R}\right]}$$

$$\cos\theta = \frac{1}{\left[1 + \frac{3.59 \times 10^7}{6.40 \times 10^6}\right]}$$

$$\cos\theta = \frac{1}{\left[1 + \frac{3590}{640} \times \frac{10^6}{10^6}\right]} = \frac{1}{1 + 5.6} = \frac{1}{6.6}$$

$$\cos\theta = 0.1515 \quad \Rightarrow \quad \theta = 81.28°$$

$2\theta = [81.28 \times 2]$ is covered by one satellite

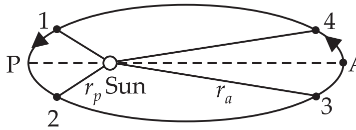
Total angle to be covered = 360°

$\therefore$ Number of satellite to cover 360° = $\dfrac{360°}{81.28 \times 2} = 2.21$

So number of satellite to cover whole parts of earth = 3.

Q8.37. Earth's orbit is an ellipse with eccentricity 0.0167. Thus, earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of the solar day is not constant through the year. Assume that earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain variation of length of the day during the year?

Ans. Let mass of earth is m, v_p, ω_p are the linear and angular velocities of earth around sun at perigee position P respectively v_a, ω_a are the linear velocity and angular velocity of earth at apogee position A respectively. According to Kepler's second law $\dfrac{dA}{dt} = \dfrac{L}{2m}$ = constt.



or
$$L_a = L_p$$
$$r_a \times p_a = r_p \times p_p$$
$$r_a \times mv_a = r_p \times mv_p$$
$$r_a \times m\omega_a r_a = r_p \times m\omega_p r_p$$
$$\omega_a r_a^2 = \omega_p r_p^2$$
$$\frac{\omega_p}{\omega_a} = \frac{r_a^2}{r_p^2} = \left(\frac{r_a}{r_p}\right)^2$$

$\because$
$$r_p = a(1 - e)$$
$$r_a = a(1 + e)$$

$\therefore$
$$\frac{\omega_p}{\omega_a} = \left[\frac{a(1+e)}{a(1-e)}\right]^2, \quad e = 0.0167 \qquad \text{(Given)}$$

$$\frac{\omega_p}{\omega_a} = \frac{(1+0.0167)^2}{(1-0.0167)^2} = \left(\frac{1.0167}{.9833}\right)^2 = (1.0339)^2 = 1.0691$$

If ω is the mean angular speed of earth then

$$\frac{\omega_p}{\omega} \times \frac{\omega}{\omega_a} = 1.0691$$

$$\frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} = \sqrt{1.0691} = 1.034$$

ω is mean angular velocity of earth around sun.

If ω is 1° it corresponds to one day. i.e., average angular speed $\omega_0 = 1.034°$ per day and $\omega_a = 0.9833°$, $\omega_p = 1.0167°$ per day 361° = 24 hrs mean solar day. We get 361.034° which corresponds to 24 hrs 8.14″ (8.1″ longer) and 360.967° corresponds to 23 hrs 59 min and 52″ (7.9″ smaller).

This does not explain the actual variation of the length of the day during the year.

Q8.38. A satellite is in an elliptic orbit around the earth with aphelion of 6R and perihelion of 2R where R = 6400 km is the radius of the earth. Find eccentricity of the orbit. Find the velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius 6R? [G = 6.67 × 10^{-11} SI units and M = 6 × 10^{24} kg]

Ans. $r_p = 2R \qquad r_a = 6R$

Hence,

$$r_p = a(1-e) = 2R \qquad \text{...}(i)$$

$$r_a = a(1+e) = 6R \qquad \text{...}(ii)$$

On dividing (*i*) by (*ii*)

$$\frac{1-e}{1+e} = \frac{2}{6}$$

$$3 - 3e = 1 + e$$

$$4e = 2 \quad \Rightarrow \quad e = \frac{1}{2}$$

There is not external force or torque on system.

So by the law of conservation of angular momentum.

$$L_1 = L_2$$

$$m_a v_a r_a = m_p v_p r_p \qquad m_a = m_p = m = \text{mass of satellite}$$

$$\therefore \quad \frac{v_a}{v_p} = \frac{r_p}{r_a} = \frac{2R}{6R} = \frac{1}{3}$$

So

$$\boxed{v_p = 3v_a} \qquad \text{...}(iii)$$

Apply conservation of energy at apogee and perigee

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

Multiplying $\frac{2}{m}$ to both side and put $r_p = 2R$ and $r_a = 6R$

$$v_p^2 - \frac{2GM}{2R} = v_a^2 - \frac{2GM}{6R} \qquad \text{(where M is mass of earth)}$$

$$v_a = \frac{v_p}{3} \qquad \text{[from } (iii)\text{]}$$

$$\therefore \qquad v_p^2 - v_a^2 = \frac{GM}{R} - \frac{1}{3}\frac{GM}{R}$$

$$v_p^2 - \left(\frac{v_p}{3}\right)^2 = \frac{GM}{R}\left[1 - \frac{1}{3}\right]$$

$$v_p^2\left[1 - \frac{1}{9}\right] = \frac{GM}{R}.\frac{2}{3}$$

$$v_p^2\frac{8}{9} = \frac{GM}{R}.\frac{2}{3}$$

$$v_p^2 = \frac{GM}{R}\frac{2}{3} \times \frac{9}{8} = \frac{3}{4}\frac{GM}{R}$$

$$v_p = \sqrt{\frac{3}{4}\frac{GM}{R}} = \sqrt{\frac{3 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4 \times 6.4 \times 10^6}}$$

$$= \sqrt{\frac{9 \times 667 \times 10^{24-6-11-1}}{128}}$$

$$v_p = \sqrt{\frac{6003 \times 10^{18-11-1}}{128}} = \sqrt{46.89 \times 10^6}$$

$$= 6.85 \times 10^3 \text{ m/s} = 6.85 \text{ km/s}$$

$$v_a = \frac{v_p}{3} = \frac{6.85}{3} = 2.28 \text{ km/s}$$

$$v_c = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6R}}$$

$$= \sqrt{\frac{6.67 \times 6 \times 10^{24-11}}{6 \times 6.4 \times 10^6}} = \sqrt{\frac{667}{640} \times 10^{13-6}}$$

$$= \sqrt{1.042 \times 10 \times 10^6} = \sqrt{10.42 \times 10^6}$$

$$v_c = 3.23 \text{ km/s}$$

Hence to transfer to a circular orbit at apogee we have to boost the velocity by Δv (3.23 – 2.28) = 0.95 km/s. This can be done by suitably firing rockets from the satellite.

❑❑❑

9 Mechanical Properties of Solids

MULTIPLE CHOICE QUESTIONS-I

Q9.1. Modulus of rigidity of ideal liquids is:

(*a*) infinity
(*b*) zero
(*c*) unity
(*d*) some finite small non-zero constant value.

Ans. (*b*): As liquid is ideal, so does not have frictional force hence tangential force are zero so there is no stress developed verifies option (b).

Q9.2. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will

(*a*) be double (*b*) be half
(*c*) be four times (*d*) remain same

Ans. (*d*): Breaking Stress = $\dfrac{\text{Breaking Force}}{\text{Area of Cross-section}}$

By reducing length half its area of cross-section remain same, and breaking stress does not depend on length.

So the breaking force remain same. Verifies option (d).

Q9.3. The temperature of a wire is doubled. The Young's modulus of elasticity

(*a*) will also double (*b*) will become four times
(*c*) will remain same (*d*) will decrease

Ans. (*d*): We know that $L_t = L_0\,(1 + \alpha\Delta t)$

$$\Delta L = L_t - L_0 = \alpha\, L_0 \Delta t$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL_0}{A\Delta L} = \frac{FL_0}{A(1+\alpha\Delta t)}$$

$$Y \,\alpha\, \frac{1}{\Delta t}$$

Hence, if temperature increases Young's modulus elasticity decreases.

Q9.4. A spring is stretched by applying a load to its free end. The strain produced in the spring is

(*a*) Volumetric (*b*) Shear
(*c*) Longitudinal and Shear (*d*) Longitudinal

Ans. (*c*): When a spring is stretched by a load its shape (shear) and length (longitudinal) changes. So strain produced is shearing and longitudinal strain.

Q9.5. A rigid bar of mass M is supported symmetrically by three wires each of length *l*. Those at each end are of copper and middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to

(*a*) $\dfrac{Y_{copper}}{Y_{iron}}$ (*b*) $\sqrt{\dfrac{Y_{iron}}{Y_{copper}}}$

(*c*) $\dfrac{Y_{iron}^2}{Y_{copper}^2}$ (*d*) $\dfrac{Y_{iron}}{Y_{copper}}$

Ans. (*b*): $$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{A\Delta L} = \frac{FL}{\pi\left(\frac{D}{2}\right)^2 \Delta L}$$

$$Y = \frac{4FL}{\pi D^2 \Delta L}$$

$\because$ L = *l* for the both wire given

ΔL Copper = ΔL iron (g)

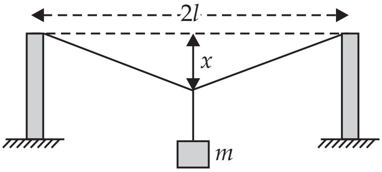
F = wt. of rod same both cases

$\therefore$ $$Y \,\alpha\, \frac{1}{D^2}$$

$$D^2 \,\alpha\, \frac{1}{Y} \quad \text{or} \quad D \,\alpha\, \sqrt{\frac{1}{Y}}$$

$$\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}}$$ verifies option (b).

Q9.6. A mild steel wire of length 2*l* and cross-sectional area A is stretched, well within elastic limit horizontally between two pillars (figure). A mass *m* is suspended from the mid-point of wire. Strain in the wire is



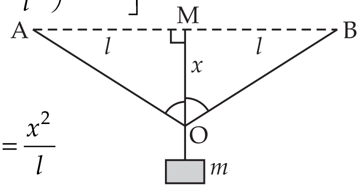
(*a*) $\dfrac{x^2}{2l^2}$ (*b*) $\dfrac{x}{l}$ (*c*) $\dfrac{x^2}{l}$ (*d*) $\dfrac{x^2}{2l}$

Ans. (*a*): $\Delta l = (AO + BO) - AB$

$$\Delta l = 2AO - 2l = 2[AO - l]$$

$$= 2[(l^2 + x^2)^{1/2} - l] = 2\left[l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - l\right]$$

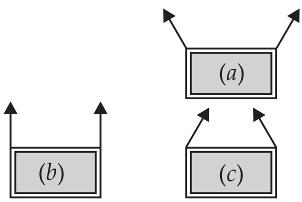
$$= 2l\left[\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 1\right]$$



$$\Delta l = 2l\left[1 + \frac{x^2}{2l^2} - 1\right] = 2l \cdot \frac{x^2}{2l^2} = \frac{x^2}{l}$$

$$\text{Strain} = \frac{\Delta l}{2l} = \frac{\frac{x^2}{l}}{2l} = \frac{x^2}{2l^2} \quad \text{verifies the option } (a).$$

Q9.7. A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports (figure). It can be done in one of the following three ways:



The tension in the strings will be:

(*a*) the same in all cases (*b*) least in (*a*)
(*c*) least in (*b*) (*d*) least in (*c*)

Ans. (*c*): Consider the free body diagram as under. As the frame is balanced

∴ Net forces acting on frame will be zero.

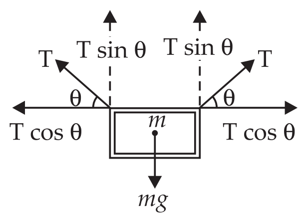
Vertical components

$$2\,\text{T} \sin \theta - mg = 0$$

or $$2\text{T} \sin \theta = mg$$

or $$\text{T} = \frac{mg}{2 \sin \theta}$$

or $$\text{T} \propto \frac{1}{\sin \theta}$$

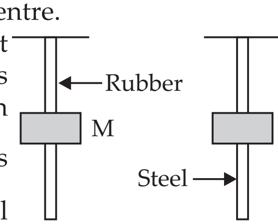


So tension is all 3 cases are different rejects option (*a*) for minimum tension sin θ must be sin θ = 1 or θ = 90° as in figure (*b*).

∴ verifies option (*c*).

Q9.8. Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods.

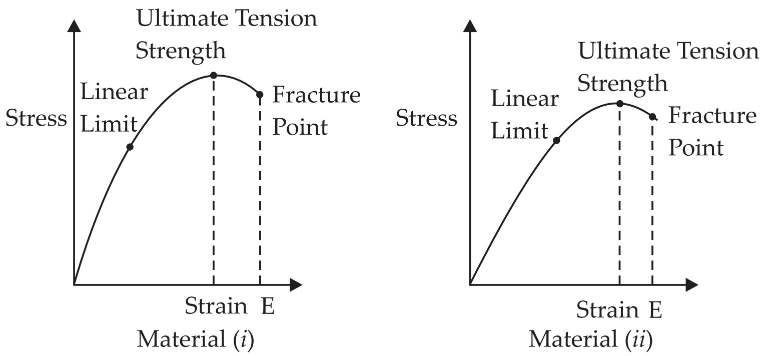
(*a*) Both the rods will elongate but there shall be no perceptible change in shape.
(*b*) The steel rod will elongate and change shape but the rubber rod will only elongate.
(*c*) The steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate and the shape of the bottom edge will change to an ellipse.
(*d*) The steel rod elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre.



Ans. (*d*): Mass M is attached at mid-point of rods of rubber and steel. As the Young's modulus of rigidity for steel is larger than rubber so $\frac{\Delta \text{L}}{\text{L}}$ for rubber $\left[\text{Y} = \frac{\text{F/A}}{\Delta\text{L/L}}\right]$ is larger than steel for same F/A. So in steel rod ΔL is very small not visible easily but in case of rubber it can be observed early due to change in its shape. Hence, verifies option (*d*).

MULTIPLE CHOICE QUESTIONS-II

Q9.9. The stress-strain graphs for two materials are shown in figure point (assume same scale).

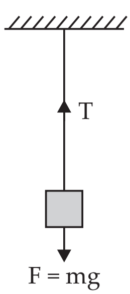


(*a*) Material (*ii*) is more elastic than material (*i*) and hence material (*ii*) is more brittle.
(*b*) Material (*i*) and (*ii*) have the same elasticity and the same brittleness.
(*c*) Material (*ii*) is elastic over a larger region of strain as compared to (*i*).
(*d*) Material (*ii*) is more brittle than material (*i*).

Ans. (*c, d*): On comparing ultimate tension strength of material, (*ii*) is greater than (*i*). Hence, material (*ii*) is elastic over larger region as compare to (*i*) so the material (*ii*) is elastic over a larger region of strain as compared to (*i*) (verifies option *c*).

As the fracture point of material (*ii*) is nearer than (*i*), hence the material (*ii*) is more brittle than material (*i*).

Q9.10. A wire is suspended from the ceiling and stretched under the action of a weight F suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight.

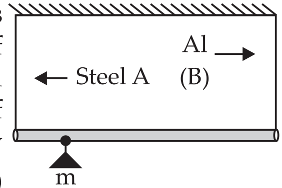


(*a*) Tensile stress at any cross-section A of the wire is F/A
(*b*) Tensile stress at any cross-section is zero
(*c*) Tensile stress at any cross-section A of the wire is 2F/A
(*d*) Tension at any cross section A of the wire is F

Ans. (*a, d*): Stress = $\frac{F}{A}$ verifies option (*a*).

Tension is balanced by force F. Hence, T = F verifies option (*d*).

Q9.11. A rod of length l and negligible mass is suspended at its two ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths (figure). The cross-sectional area of wires A and B are 1.0 mm^2 and 2.0 mm^2 respectively ($Y_{Al} = 70 \times 10^9$ N.m^{-2} and $Y_{steel} = 200 \times 10^9$ Nm^{-2})



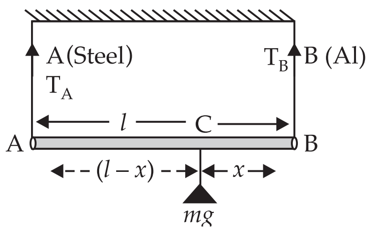
(*a*) Mass m should be suspended close to wire A to have equal stresses in both the wires.

(*b*) Mass m should be suspended close to B to have equal stresses in both the wires.

(*c*) Mass m should be suspended at the middle of the wires to have equal stresses in both the wires.

(*d*) Mass m should be suspended close to wire A to have equal strain in both wires.

Ans. (*b, d*): (*a*) Stress: Let mass m is hangs at distance x from wire B (Al) at C. As rod is balanced by weight mg (i.e. rod remains horizontal) taking moments about point A.



$$T_B(x) - T_A(l - x) = 0$$

$$T_B x = T_A(l - x)$$

$$\frac{T_B}{T_A} = \left(\frac{l - x}{x}\right) \qquad \text{(I)}$$

Stress in wire A $= \dfrac{T_A}{A_A}$

Stress in wire B $= \dfrac{T_B}{A_B} = \dfrac{T_B}{2A_A}$

$A_A = 1 \text{ mm}^2$

$A_B = 2.0 \text{ mm}^2$

$\therefore \quad A_B = 2A_A$

as the stress on steel (S_A) = Stress on Al (S_B)

$$\frac{T_A}{A_A} = \frac{T_B}{2A_A} \quad \text{or} \quad T_A = \frac{T_B}{2}$$

$$\Rightarrow \qquad T_B = 2T_A \quad \text{or} \quad \frac{T_B}{T_A} = 2 \qquad \text{(II)}$$

from I, II $\qquad \dfrac{l - x}{x} = \dfrac{2}{1}$

$$2x = l - x$$

$$3x = l \qquad x = \frac{l}{3} \text{ from B}$$

distance of m from A $= l - x = l - \dfrac{l}{3} = \dfrac{2l}{3}$

So m is near to B than A wire. Hence, verifies the option (*b*).

(B) Strain: Strain in both wire is equal because rod remain horizontal in balanced condition.

$$(\text{Strain})_A = (\text{Strain})_B$$

$$\therefore \qquad \frac{S_A}{Y_A} = \frac{S_B}{Y_B} \Rightarrow \frac{Y_A}{S_A} = \frac{Y_B}{S_B} \qquad \left[\because Y = \frac{\text{Stress}}{\text{Strain}}\right]$$

$$\frac{Y_{steel}}{T_A/A_A} = \frac{Y_{Al}}{T_B/A_B}$$

$$\frac{Y_{steel}}{Y_{Al}} = \frac{T_A}{T_B} \cdot \frac{A_B}{A_A} = \left(\frac{x}{l-x}\right)\left(\frac{2A_A}{A_A}\right)$$

$$\frac{200 \times 10^9}{70 \times 10^9} = \frac{2x}{(l-x)} \qquad \Rightarrow \quad 14x = 20l - 20x$$

$$34x = 20l \qquad x = \frac{20l}{34} = \frac{10l}{17} \quad \text{from B}$$

$$(l-x) \text{ from A} = \left(l - \frac{10l}{17}\right) = \frac{7l}{17} \quad \text{from A}$$

for equal strain mass *m* should be closer to A.

Hence, verifies the option (*d*).

Q9.12. For an ideal liquid:

(*a*) The bulk modulus is infinite
(*b*) The bulk modulus is zero
(*c*) The shear modulus is infinite
(*d*) The shear modulus is zero

Ans. (*a, d*): An ideal liquid is not compressible

$$\text{Bulk modulus (K)} = \frac{-p(V)}{\Delta V} \qquad (\because \quad \Delta V = 0)$$

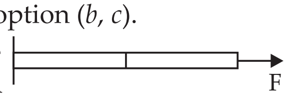
$\because$ $\Delta V = 0$ for ideal liquid
$\therefore$ $K = \infty$ for ideal liquid

As there is no net tangential force on liquid (S.T. is all around on a particle) so shearing strain $\Delta\theta = 0$ and $F = 0$.

$$\eta = \frac{F/A}{\Delta\theta} = \frac{0}{0} = \text{indeterminant value}$$

Hence, verifies option (*a*) and (*d*) and rejects option (*b, c*).

Q9.13. A copper and a steel wire of the same diameter are connected end to end. A deforming force F is applied to this composite wire which causes a total elongation of 1 cm. The two wires will have:



(*a*) The same stress
(*b*) Different stress
(*c*) The same strain
(*d*) Different strain

Ans. (*a, d*): $\because$ $\text{Stress} = \dfrac{F}{A}$

$\because$ area of cross section for both wire same and stretched by same force. So their stress are equal verifies option (*a*).

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

as stress for both wire same so

$$(\text{Strain})_{\text{steel}} \propto \frac{1}{Y_{\text{steel}}} \quad \text{and} \quad (\text{Strain})_{\text{Al}} \propto \frac{1}{Y_{\text{Al}}}$$

$$\frac{(\text{Strain})_{\text{steel}}}{(\text{Strain})_{\text{Al}}} = \frac{Y_{\text{Al}}}{Y_{\text{Steel}}}$$

$$Y_{AL} < Ys \quad \text{So} \quad \frac{\text{Al}}{\text{steel}}$$

or $\quad (\text{Strain})_{\text{steel}} < (\text{Strain})_{\text{Al}}$

verifies option (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q9.14. The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress?

Ans. $Y = \dfrac{\text{Stress}}{\text{Strain}}$ As per question strain (longitudinal) are equal.

$\therefore \quad Y \propto \text{Stress}$

$$\therefore \quad \frac{Y_{\text{Steel}}}{Y_{\text{Rubber}}} = \frac{(\text{Stress})_{\text{Steel}}}{(\text{Stress})_{\text{Rubber}}} \quad \text{As the } Y_{\text{Steel}} > Y_{\text{Rubber}}$$

$$\therefore \quad \frac{Y_{\text{Steel}}}{Y_{\text{Rubber}}} > 1$$

$\therefore \quad (\text{Stress})_{\text{Steel}}$ is large than $(\text{Stress})_{\text{Rubber}}$.

Q9.15. Is stress a vector quantity?

Ans. $\text{Stress} = \dfrac{\text{Magnitude of restoring force by solid}}{\text{Area of cross-section}}$

as deforming and restoring force are equal and opposite so no net direction.

Hence, the stress is not a vector quantity just like pressure.

Q9.16. Identical springs of steel and copper are equally stretched. On which, more work will have to be done?

Ans. The two identical springs are equally stretched by same force. So

Spring I (steel)	Spring II (copper)	
$F_1 = F$	$F_2 = F$	(equally stretched)
$Y_1 = Y_S$	$Y_2 = Y_{Cu}$	
$L_1 = L$	$L_2 = L$	
$A_1 = A$	$A_2 = A$	identical wires

$$\Delta L = \Delta L_1 \qquad\qquad \Delta L' = \Delta L_2$$

$$\because \quad Y = \frac{FL}{A\Delta L} \qquad \therefore \quad \Delta L = \frac{FL}{AY}$$

$\therefore$ Wires $\Delta L \propto \dfrac{1}{Y}$

$$\therefore \quad \frac{\Delta L_1}{\Delta L_2} = \frac{Y_C}{Y_S} \qquad [\because \ WD = F \cdot \Delta L \ \text{ or } \ F_1 = F_2 = F \ \text{ so } WD \propto \Delta L]$$

$$\frac{WD_S}{WD_C} = \frac{Y_C}{Y_S} \qquad (\because \quad Y_S > Y_C)$$

so $\dfrac{Y_C}{Y_S} < 1$

or $\dfrac{WD_S}{WD_C} < 1 \qquad WD_S < WD_C$

So the W.D. is more to stretch copper spring.

Q9.17. What is the Young's modulus for a perfect rigid body?

Ans. $Y = \dfrac{FL}{A\Delta L}$ a rigid body cannot be deshaped by applying any deforming force

$\therefore \quad \Delta L = 0$

$\Rightarrow \quad Y = \dfrac{FL}{AX0}$ = infinity i.e. for a perfect rigid body Young's modulus is infinity.

Q9.18. What is the Bulk modulus for a perfect rigid body?

Ans. Bulk Modulus $= \dfrac{-p(V)}{\Delta V}$ as the perfect rigid body does not change it's shape even an infinite (deforming a stretching) force. Hence, $\Delta V = 0$

$$\Rightarrow \qquad B = \frac{pV}{\Delta V} = \frac{pV}{0} = \infty$$

So the bulk modulus is infinity.

SHORT ANSWER TYPE QUESTIONS

Q9.19. A wire of length L and radius *r* is clamped rigidly at one end. When the other end of the wire is pulled by a force *f*, its length increases by *l*. Another wire of same material of length 2L and radius 2*r*, is pulled by a force 2*f*. Find the increase in length of this wire.

Ans.

Wire I	Wire II
$L_1 = L$	$L_2 = 2L$
$r_1 = r$	$r_2 = 2r$
$A_1 = \pi r^2$	$A_2 = \pi(2r)^2 = 4\pi r^2$
$F_1 = f$	$F_2 = 2f$

$$\Delta L_1 = l \qquad\qquad \Delta L_2 = ?$$

$$Y_1 = Y \qquad\qquad Y_2 = Y \text{ (Same material)}$$

$$\because \qquad Y = \frac{FL}{A\Delta L} \text{ or } \Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L_2}{\Delta L_1} = \frac{\frac{F_2L_2}{A_2Y_2}}{\frac{F_1L_1}{A_1Y_1}} = \frac{F_2L_2}{F_1L_1} \times \frac{A_1Y_1}{A_2Y_2} = \frac{2f\ 2L}{fL} \times \frac{\pi r^2 \times Y}{4\pi r^2 \times Y}$$

$$\frac{\Delta L_2}{l} = \frac{4}{4} = 1 \qquad \therefore \quad \Delta L_2 = l$$

So the change in the length in IInd wire is also same i.e. l.

Q9.20. A steel rod ($Y = 2 \times 10^{11}$ N.m^{-2}; and $\alpha = 10^{-5}$ C^{-1}) of length 1 m and area of cross-section 1 cm^2 is heated from 0°C to 200°C, the without being allowed to extend or bend. What is the tension produced in the rod?

Ans. $\therefore \qquad L_t = L_0(1 + \alpha\,\Delta t)$

$$L_t - L_0 = L_0\,\alpha \cdot \Delta t$$

$$\Delta L = 1 \times 10^{-5} \times 200 = 2 \times 10^{-3}$$

$$Y = \frac{FL_0}{A\Delta L} \qquad\qquad L_0 = 1 \text{ m}$$

$$\therefore \qquad F = \frac{YA\Delta L}{L_0} \qquad\qquad A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$$

$$\Delta L = 2 \times 10^{-3} \text{ m}$$

$$= \frac{2 \times 10^{11} \times 10^{-4} \times 2 \times 10^{-3}}{1} \qquad\qquad Y = 2 \times 10^{11} \text{ N m}^{-2}$$

$$= 4 \times 10^{11-7} = 4 \times 10^4 \text{ N.}$$

Q9.21. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1% (The bulk modulus of rubber is 9.8×10^8 Nm^{-2}; and the density of sea water is 10^3 Kg m^{-3}).

Ans. Bulk modulus = B = 9.8×10^8 N.m^{-2}

Density of water = $\rho = 10^3$ kg m^{-3}

$$\text{Percentage change volume} = \frac{\Delta V}{V} \times 100 = 0.1$$

$$\text{or} \qquad \frac{\Delta V}{V} = \frac{1}{1000}$$

Let the rubber ball is taken upto depth h in sea

$$p = h\rho g = h \times 10^3 \times 9.8 \text{ N/m}^2$$

$$\text{or} \qquad B = \left|\frac{-p}{\frac{\Delta V}{V}}\right| \quad \text{or} \quad p = B \times \frac{\Delta V}{V}$$

$$\text{or} \qquad h \times 9.8 \times 10^3 = 9.8 \times 10^8 \times \frac{1}{1000}$$

$$\therefore \qquad h = \frac{9.8 \times 10^8}{1000 \times 9.8 \times 10^3} = 10^{+8-6} = 10^2$$

$$h = 100 \text{ m}$$

So the ball is taken 100 m below in sea.

Q9.22. A truck is pulling a car out of a ditch by means of a steel cable that is 9.1 m long and has a radius of 5 mm. When the car just begins to move, the tension in the cable is 800 N. How much has the cable stretched? (Young's modulus for steel is 2×10^{11} N.m^{-2})

Ans. Length of cable $L_0 = 9.1$ m

$$r = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}, \ A = \pi r^2$$

Tension in cable = F = 800 N

$$Y = 2 \times 10^{11} \text{ Nm}^{-2}$$

$$\Delta L = \frac{FL}{AY} = \frac{800 \times 9.10}{3.14 \times 10^{-3} \times 10^{-3} \times 5 \times 5 \times 2 \times 10^{11}}$$

$$\Delta L = \frac{800 \times 910 \times 10^{-11+6}}{314 \times 5 \times 5 \times 2}$$

$$= \frac{728}{157} \times 10^{-5} \text{m} = 4.64 \times 10^{-5} \text{m}$$

$$\Delta L = 4.64 \times 10^{-5} \text{ m}.$$

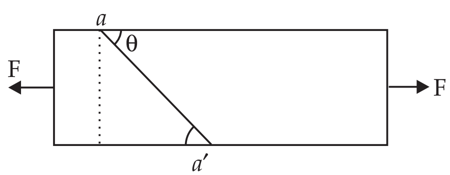
Q9.23. Two identical solid balls, one of ivory and the other of wet-clay, are dropped from the same height on the floor. Which one will rise to a greater height after striking the floor and why?

Ans. Since the ivory ball is more elastic then wet clay ball, and both are dropped from same height so their velocity before striking the floor will be same. Hence ivory ball tries to regain its original shape quickly and change in shape is negligible for ivory ball as compared to wet clay ball.

Hence, more energy is transferred to ivory ball as compared to wet clay ball. So ivory ball rise more than clay ball.

LONG ANSWER TYPE QUESTIONS

Q9.24. Consider a long steel bar under a tensile stress due to forces $\vec{F}$ acting at the edges along the length of the bar (figure). Consider a plane making an angle θ with the length. What are the tensile and shearing stresses on this plane?



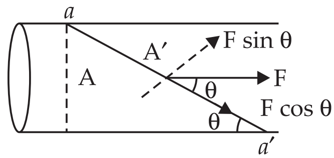
(*a*) For what angle is the tensile stress a maximum?

(*b*) For what angle is the shearing stress a maximum?

Ans. (*a*) Tensile Stress = $\dfrac{\text{Force normal to surface } (F_P)}{\text{Area of surface}}$

A = Area of cross section perpendicular to the length of bar.

A' = Area of cross section plane cut along aa'



$$\therefore \qquad \sin\theta = \frac{A}{A'}$$

$$A' = \frac{A}{\sin\theta}$$

Component of F along perpendicular to the plane A' or $aa' = F\sin\theta = F_P$

$$\therefore \qquad \text{Tensile stress} = \frac{F\sin\theta}{A}\cdot\sin\theta = \frac{F}{A}\sin^2\theta$$

For maximum tensile strength $\sin^2\theta = 1$

or $\qquad \sin\theta = 1 \quad \text{or} \quad \sin\theta = \sin 90°$

$$\theta = \frac{\pi}{2}$$

(b) $\quad$ Shearing stress $= \dfrac{\text{Force along the plane } F_P}{\text{Area of plane}}$

$$= \frac{F\cos\theta}{A'} = \frac{F\cos\theta\sin\theta}{A} = \frac{F}{2A}\cdot 2\sin\theta\cos\theta$$

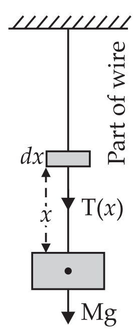
$$\text{Shearing stress} = \frac{F}{2A}\sin 2\theta$$

For maximum shearing stress

$$\sin 2\theta = 1 \quad \text{or} \quad \sin 2\theta = \sin 90°$$

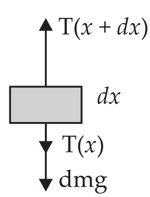
$$\therefore \qquad 2\theta = 90° \quad \theta = \frac{90°}{2} = 45° = \frac{\pi}{4}.$$

Q9.25. (*a*) A steel wire of mass μ per unit length with a circular cross-section has a radius of 0.1 cm. The wire is of length 10 m when measured lying horizontal, and hangs from a hook on the wall. A mass of 25 kg is hung from the free end of the wire. Assuming the wire to be uniform and lateral strains << longitudinal strains, find the extension in the length of the wire. The density of steel is 7860 kg m^{-3} (Young modulus $Y = 2 \times 10^{11}$ $N.m^{-2}$)



(*b*) If the yield strength of steel is 2.5×10^8 $N.m^{-2}$, what is the maximum weight that can be hung at the lower end of the wire?

Ans. Consider an small element dx, of mass dm from the length L of wire at distance x, from the end of hung weight. μ is mass per unit length



$\therefore \qquad dm = \mu\cdot dx \quad r = 0.1\text{ cm} = 0.1 \times 10^{-2}\text{ m} = 10^{-3}\text{ m}$

$A = \pi r^2 = \pi(10)^{-6} \quad M = 25\text{ kg};\ L = 10\text{ m}$

(*a*) downward force on dx = wt below it (dx)

$$T(x) = (x\cdot\mu)g + Mg \qquad \text{(I)}$$

$$Y = \frac{\frac{T(x)}{A}}{\frac{dr}{dx}}$$

where dr is increase in length of wire by $T(x)$

$$Y = \frac{T(x) \cdot dx}{A \cdot dr} \quad \text{or} \quad \frac{dr}{dx} = \frac{T(x)}{AY}$$

$$dr = \frac{1}{AY}(x\mu g + Mg)\,dx \qquad \text{From } (i)$$

integrating both sides

$$\int_0^r dr = \frac{1}{AY}\int_0^L (x\mu g + Mg)\,dx$$

$$r(\text{change in length is L}) = \frac{1}{AY}\left[\mu g\frac{x^2}{2} + Mgx\right]_0^L$$

$$\text{extension in wire of L} = \frac{gL}{2\pi r^2 Y}[\mu L + 2M] \qquad \because \quad \mu L = m$$

m the whole mass of wire

$$\therefore \quad \text{extension} = \frac{gL}{2A \cdot Y}[m + 2M]$$

$$\therefore \text{extension} = \frac{10 \times 10\,[0.25 + 2 \times 25]}{2 \times 3.14 \times 10^{-6} \times 2 \times 10^{11}}$$

$$\text{extension} = \frac{100[.25 + 50]}{2 \times 6.28 \times 10^5}$$

$$= \frac{100 \times 5025 \times 10^{-5}}{2 \times 628}$$

$$= \frac{502500}{2 \times 628} \times 10^{-5}$$

$$= 400 \times 10^{-5}\, m$$

$$\text{extension} = 4.00 \times 10^{-3}\, m$$

$$m = \text{vol} \times \text{density}$$
$$= (A \cdot L) \times \rho$$
$$= \pi r^2 \times 10 \times 7860$$
$$= 3.14 \times (10^{-3})^2 \times 7860 \times 10$$
$$= 3.14 \times (7860) \times 10^{-6} \times 10$$
$$= 3.14 \times 786 \times 10^{-4}\text{ kg}$$
$$= 2468.04 \times 10^{-4}\text{ kg}$$
$$m \cong 0.25\text{ kg}$$

(*b*) Tension in the wire will be maximum at $x = L$

$$T(x) = \mu g x + Mg$$

$$T(L) = \mu g L + Mg$$

$$T = \mu L g + Mg \qquad (\because \quad \mu L = m)$$

$$\therefore \quad T = (m + M)g$$

$$\text{Yield force} = [\text{Yield strength (Y)}] \times A$$

$$= 2.5 \times 10^8 \times 3.14 \times 10^{-6}$$

$$\text{Yield force} = 250 \times 3.14$$

$$\text{Yield force} = \text{Tension (maximum)}$$

$$250 \times 3.14 = (m + \text{M})g$$

$$\frac{250 \times 3.14}{g} = [0.25 + \text{M}]$$

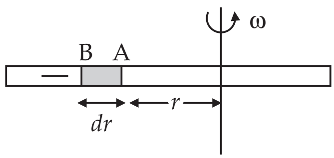
$$\frac{785}{10} - 0.25 = \text{M}$$

$$\text{M} = 78.5 - 0.25$$

$$\text{M} = 78.25 \text{ Kg.}$$

Q9.26. A steel rod of length $2l$, cross-sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the centre. If Y is the Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform)

Ans. Consider in given figure an element (dr) of rod at a distance r from the centre.



Let T(r) and T($r + dr$) are the tensions external force to rod extend at A and B ends of element (small) respectively. Centrifugal force on element dr due to tension difference

$$= \text{T}(r + dr) - \text{T}(r)$$

Centrifugal Force $= -d\text{T}$ (outward)

Centripetal Force due to rotation on element $dr = dm\, rw^2$

$\therefore$ $$-d\text{T} = dm\, w^2 r$$ (Let μ = mass per unit length)

then $$-d\text{T} = w^2 r(dr \cdot \mu)$$

$$-d\text{T} = \mu w^2\, r \cdot dr$$

Integrating both sides

$$-\int_0^T d\text{T} = \mu w^2 \int_r^l r dr$$

Tension in rod at distance r from centre so limits will varies from r to l

$$\therefore \quad -\text{T}(r) = \mu w^2 \left[\frac{r^2}{2}\right]_r^l = \frac{\mu w^2}{2}(l^2 - r^2) \qquad \text{(I)}$$

Let the increase in length of dr element at distance r from centre is δr then

$$\text{Y} = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{T}(r)/\text{A}}{\delta r/dr} = \frac{\text{T}(r)}{\text{A}} \cdot \frac{dr}{\delta r}$$

$$\frac{\delta r}{dr} = \frac{\text{T}(r)}{\text{AY}} = \frac{-\mu w^2}{2\text{AY}}(l^2 - r^2)$$

$\because$ Negative sign shows only the direction extension is opposite to restoring force

$$\therefore \quad \delta r = \frac{\mu w^2}{2\text{AY}}(l^2 - r^2) dr$$

$$\int_0^\delta \delta r = \int_0^l \frac{\mu w^2}{2\text{AY}}(l^2 - r^2) dr$$ (for rod one from centre)

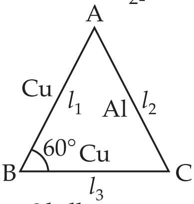
$$\delta = \frac{\mu w^2}{2AY}\left(l^3 - \frac{l^3}{3}\right) = \frac{\mu w^2}{2AY}\frac{2}{3}l^3$$

$$\delta = \frac{\mu w^2}{3AY}l^3$$

$\therefore$ Total extension in rod both sides $= 2\delta = \dfrac{2\mu w^2 l^3}{3AY}$

Q9.27. An equilateral triangle ABC is formed by two Cu rods AB and BC and one Al rod. It is heated in such a way that temperature of each rod increases ΔT. Find the change in angle ABC. [Coefficient of linear expansion for Cu is α_1, coefficient of linear expansion for Al is α_2].

Ans. By trigonometry



$$\cos\theta = \frac{l_1^2 + l_3^2 - l_2^2}{2l_1 l_3}$$

$$2l_1\, l_3 \cos\theta = l_1^2 + l_3^2 - l_2^2$$

Differentiating both sides

$$2[d(l_1\, l_3)\cdot\cos\theta + l_1 l_3\, d(\cos\theta)] = 2l_1\, dl_1 + 2l_3 dl_3 - 2l_2 dl_2$$

$$2[(l_1\, dl_3 + l_3 dl_1)\cos\theta - l_1 l_3 \sin\theta\, d\theta] = 2(l_1\, dl_1 + l_3 dl_3 - l_2 dl_2)$$

$$(l_1 dl_3 + l_3 dl_1)\cos\theta - l_1 l_3 \sin\theta\, d\theta = l_1 dl_1 + l_3 dl_3 - l_2 dl_2 \qquad \text{(I)}$$

$$L_t = L_0(1 + \alpha\Delta t)$$

$$L_t - L_0 = L_0\alpha\Delta t$$

$$\Delta L = L\alpha\cdot\Delta t$$

$$dl_1 = l_1\,\alpha_1\,\Delta t,\ dl_3 = l_2\alpha_1\Delta t$$

and $$dl_2 = l_2\alpha_2\Delta t$$

$$l_1 = l_2 = l_3 = l$$

$\therefore$ $$dl_1 = l\alpha_1\Delta t,\ dl_3 = l\alpha_1\,\Delta t$$

and $dl_2 = l\alpha_2\,\Delta t$

Substitute their value in (I)

$$\cos\theta\,(l^2\cdot\alpha_1\,\Delta t + l^2\,\alpha_1\,\Delta t) - l^2\sin\theta\, d\theta = l^2\,\alpha_1\,\Delta t + l^2\alpha_1\Delta t - l^2\alpha_2\Delta t$$

$$2l^2\alpha_1\Delta t\cos\theta - l^2[\sin\theta\cdot d\theta] = l^2[\alpha_1 + \alpha_1 - \alpha_2]\Delta t$$

$$l^2[2\alpha_1\Delta t\cos 60° - \sin 60°\, d\theta] = l^2[2\alpha_1 - \alpha_2]\Delta t$$

$$2\alpha_1\Delta t\times\frac{1}{2} - 2\alpha_1\Delta t + \alpha_2\Delta t = \frac{\sqrt{3}}{2}d\theta$$

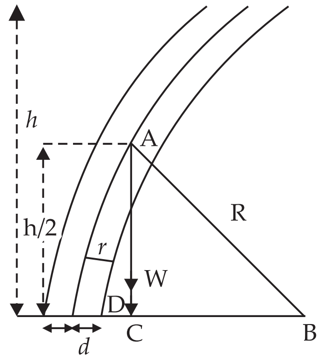
$$\frac{\sqrt{3}}{2}d\theta = [\alpha_1 - 2\alpha_1 + \alpha_2]\Delta t$$

$$d\theta = \frac{2(\alpha_2 - \alpha_1)\Delta t}{\sqrt{3}}$$

$[\because \Delta t = \Delta T \text{ (given)}]$

$$d\theta = \frac{2(\alpha_2 - \alpha_1)\Delta T}{\sqrt{3}}$$

Q9.28. In nature, the failure of structural members usually result from large torque because of twisting or bending rather than due to tensile or compressive strains. This process of structural breakdown is called buckling and in cases of tall cylindrical structures like trees, the torque is caused by its own weight bending the structure. Thus the vertical through the centre of gravity does not fall within the base. The elastic torque caused because of this bending about the central axis of the tree is given by [$Y\pi r^4/4R$]. Y is Young's modulus, r is the radius of the trunk and R is the radius of curvature of the bent surface along the height of the tree containing the centre of gravity (the neutral surface). Estimate the critical height of a tree for a given radius of the trunk.



Ans. By Pythagoras theorem in right angled ΔABC where C point is just outside the base of Trunk i.e. point C is at D

$$R^2 = (R-d)^2 + \left(\frac{h}{2}\right)^2$$

$$R^2 = R^2 + d^2 - 2Rd + \frac{h^2}{4}$$

$\because$ $d <<< R$ $\quad$ ($\therefore$ d^2 can be neglected)

$$2Rd = \frac{h^2}{4} \quad \text{or} \quad \boxed{d = \frac{h^2}{8R}} \qquad \text{(I)}$$

Let weight of Trunk per unit volume = W_0

The weight of trunk = Volume $\times W_0 = (\pi r^2 h)\, W_0$

Torque by bending the trunk = Force $\times \perp$ dist.

$$\tau = \pi r^2 h W_0 \times d$$

$$\tau = \frac{\pi r^4 Y}{4R} \quad \text{(given)}$$

$$\therefore \quad \pi r^2 h W_0 \times \frac{h^2}{8R} = \frac{\pi r^4 Y}{4R}$$

$$h^3 = \frac{\pi r^4 Y \times 8R}{4R\pi r^2 W_0} = \frac{2r^2 Y}{W_0}$$

$$h = \left[\frac{2Y}{W_0}\right]^{1/3} r^{2/3}$$

Hence, h is the critical height given in this expression.

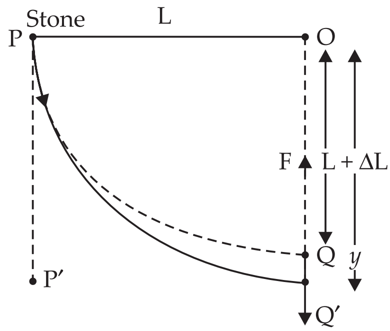
Q9.29. A stone of mass m is tied to an elastic string of negligible mass and spring constant K. The unstretched length of the string is L and has negligible mass. The other end of the string is fixed to a nail at a point P.

Initially the stone is at the same level as the point P. The stone is dropped vertically from point P:

(a) Find the distance y from the top when the mass comes to rest for an instant, for the first time.

(b) What is the maximum velocity attained by the stone is this drop?

(c) What shall be the nature of the motion after the stone has reached its lowest point?



Ans. A stone is tied at P with string of length L. String is fixed with nail at 'O'. Stone is lifted upto height L, so that string stretched as shown in given fig.

When stone fall under gravity. It tries to follow path PP′ but due to elastic string it will go a part of circular path P to Q. Like this is a centrifugal force stretched the string outward and increases its length (ΔL). So the change in P · E of stone at Q′ and P converts into mechanical energy in string of spring constant K.

So P · E of stone = mechanical E of string

$$mgy = \frac{1}{2}K(y - L)^2$$

$$mgy = \frac{1}{2}K(y^2 + L^2 - 2yL)$$

$$2mgy = K[y^2 + L^2 - 2yL]$$

$$2\,mgy = Ky^2 - 2KyL + KL^2$$

or $$Ky^2 - 2KyL - 2mgy + KL^2 = 0$$

$$Ky^2 - 2(KL + mg)y + KL^2 = 0$$

Solve this equation by quadratic formula

$$D = b^2 - 4ac \qquad (a = K \quad b = -2(KL + mg) \quad c = KL^2)$$

$$D = [-2(KL + mg)]^2 - 4(K)(KL^2)$$

$$D = +4[(KL)^2 + (mg)^2 + 2(KL)(mg)] - 4K^2L^2$$

$$D = 4[K^2L^2 + m^2g^2 + 2KLmg] - 4K^2L^2$$

$$= 4K^2L^2 + 4m^2g^2 + 8KLmg - 4K^2L^2$$

$$\sqrt{D} = \sqrt{4mg[mg + 2KL]} = 2\sqrt{mg(mg + 2KL)}$$

$$\therefore \qquad y = \frac{-b \pm \sqrt{D}}{2a} = \frac{+2(KL + mg) \pm 2\sqrt{mg(2KL + mg)}}{2K}$$

$$y = \frac{2[(KL + mg) \pm \sqrt{mg(2KL + mg)}]}{2K}$$

$$y = \frac{(KL + mg) \pm \sqrt{mg(2KL + mg)}}{K}$$

(*b*) At maximum velocity at its lowest point acceleration is zero.

$\therefore$ F = 0

So the spring or string force Kx is balanced by gravitational force mg. So, these two forces will be equal and opposite

$\therefore mg = Kx$ ... I where x is extension in string

Let v be the maximum velocity of stone at bottom of journey.

By law of conservation of energy

KE of stone + PE gain by string = P · E lost by stone from P to Q′

$$\frac{1}{2}mv^2 + \frac{1}{2}Kx^2 = mg(L + x)$$

$$mv^2 + Kx^2 = 2mg\,(L + x)$$

$$mv^2 = 2mgL + 2mgx - Kx^2$$

$$mg = Kx \text{ (from I)} \quad \Rightarrow \quad x = \frac{mg}{K}$$

$$\therefore \qquad mv^2 = 2mgL + 2mg \cdot \frac{mg}{K} - K\frac{m^2g^2}{K^2}$$

$$= 2mgL + \frac{2m^2g^2}{K} - \frac{m^2g^2}{K}$$

$$mv^2 = m\left[2gL + \frac{mg^2}{K}\right]$$

$$\therefore \qquad v = \left[2gL + \frac{mg^2}{K}\right]^{1/2}$$

(*c*) At lowest point from figure in part (*a*)

$$F = mg\downarrow - K(y - L)\uparrow \text{ (by string)}$$

$$\therefore \qquad m\frac{d^2z}{dt^2} = mg - K(y - L)$$

$$\frac{d^2z}{dt^2} - g + \frac{K}{m}(y - L) = 0 \qquad \frac{d^2y}{dt^2} + \frac{K}{m}\left[(y - L) - \frac{mg}{K}\right]$$

Make a transformation of variables:

$$z = \left[(y - L) - \frac{mg}{K}\right] \qquad ...(i)$$

then $$\frac{d^2z}{dt^2} + \frac{K}{m}z = 0$$

It is differential equation of second order which represents S.H.M.

$$\therefore \qquad \frac{d^2z}{dt^2} + \omega^2 z = 0$$

where w is angular frequency so $\omega = \sqrt{\frac{K}{m}}$

Solution of above differential equation is of type

$$z = A\cos(\omega t + \theta)$$

where $\omega = \sqrt{\frac{K}{m}}$ and θ is phase difference.

$$z = \left(L + \frac{m}{K}g\right) + A'\cos(\omega t + \theta)$$

So the stone performs SHM with angular frequency w about the point at $y = 0$

$$|z_0| = \left|-\left(L + \frac{mg}{K}\right)\right| \qquad \text{[from } (i)\text{]}$$

$$\therefore \qquad z_0 = \left(L + \frac{mg}{K}\right)$$

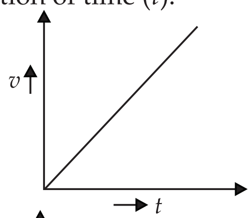
❑❑❑

10 Mechanical Properties of Fluids

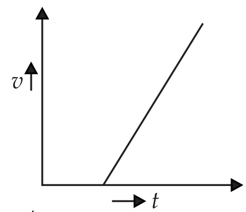
MULTIPLE CHOICE QUESTIONS-I

Q10.1. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero velocity. From the plot shown in figure, indicate the one that represents the velocity v of the pebble as a function of time (t).

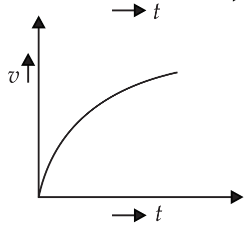
(*a*)



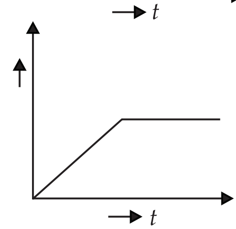
(*b*)



(*c*)



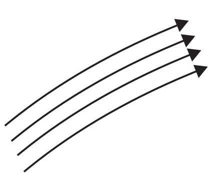
(*d*)



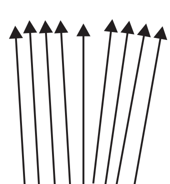
Ans. (*c*): When the pebble is dropped from the top of cylinder filled with viscous oil, pebble falls under gravity with constant acceleration, but as it is dropped it enters in oil and dragging force F = 6 $\pi\eta rv$ due to viscosity of oil so acceleration decreases from g to zero *i.e.* velocity increases, but acceleration decreases, when acceleration decreased to zero, velocity becomes constant (terminal vel.). These conditions are verified in option (*c*).

Q10.2. Which of the following diagrams does not represent a streamline flow?

(*a*)

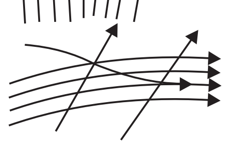


(*b*)



(*c*)

(*d*)



Ans. (*d*): In streamlined flow the velocity of fluid at any given point remains constant (at a particular line) across any cross-sectional area. Hence in streamline flow layers do not cross each other. Hence option (*d*) is not streamline.

Q10.3. Along a streamline

(*a*) the velocity of a fluid particle remains constant.
(*b*) the velocity of all fluid particles crossing a given position is constant.
(*c*) the velocity of all fluid particles at a given instant is constant.
(*d*) the speed of a fluid particle remains constant.

Ans. (*b*): In stream line flow, the speed at a point in a cross-section is always constant because Av = constant. Hence, verifies the option only (*b*).

Q10.4. An ideal fluid flows through a pipe of cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in the two pipes is

(*a*) 9 : 4 (*b*) 3 : 2 (*c*) $\sqrt{3}:\sqrt{2}$ (*d*) $\sqrt{2}:\sqrt{3}$

Ans. (*a*): According to equation or law of continuity
(Law of Conservation of Mass)

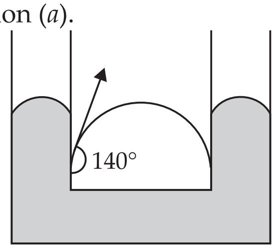
$$a_1 v_1 = a_2 v_2$$

$$\frac{v_1}{v_2} = \frac{a_2}{a_1} = \frac{\pi\left(\frac{d_2}{2}\right)^2}{\pi\left(\frac{d_1}{2}\right)^2} = \frac{d_2^2}{4}\cdot\frac{4}{d_1^2} = \frac{d_2^2}{d_1^2}$$

$$\frac{v_1}{v_2} = \frac{(3.75)^2}{(2.50)^2} = \left[\frac{3}{2}\right]^2 \quad \Rightarrow \quad \frac{v_1}{v_2} = \frac{9}{4}$$

or $v_1 : v_2 :: 9 : 4$ verifies the option (*a*).

Q10.5. The angle of contact at the interface of water-glass is 0°, Ethylalcohol-glass is 0°, Mercury-glass is 140°, and Methyliodide glass is 30°. A glass-capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is:



(*a*) water (*b*) ethyl alcohol
(*c*) mercury (*d*) methyliodide

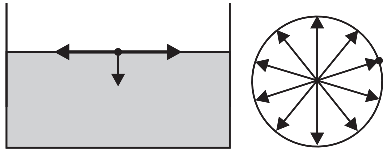
Ans. (*c*): Meniscus of liquid is convex so angle of contact will be obtuse angle which in case of mercury-glass. Hence, verifies the option only (*c*).

MULTIPLE CHOICE QUESTIONS—II

Q10.6. For a surface molecule

(*a*) the net force on it is zero
(*b*) there is a net downward force
(*c*) the potential energy is less than that of a molecule inside
(*d*) the potential energy is more than that of a molecule inside.

Ans. (*b, d*): Fig. (*b*) A molecule as viewed from the top to surface of the fluid. Forces are equal and opposite to a molecule by all nearby molecule so net force horizontally is zero.



(*a*) (*b*)

In Fig. (*a*) as a molecule is viewed horizontally to the surface of fluid a vertically downward force acts on molecule due to which it does not escape from surface. Hence the right option is (*b*).

There is a net force of attraction between a molecule on surface acts downward i.e. attractive. So the P.E of molecule on the surface is negative to lower one but in magnitude PE (*mg h*) is more than that of lower or inside molecule verifies option (*d*).

Q10.7. Pressure is a scalar quantity because

(*a*) it is the ratio of force to area and both force and area are vectors.
(*b*) it is the ratio of the magnitude of the force to area.
(*c*) it is the ratio of the component of the force normal to the area.
(*d*) it does not depend on the size of the area chosen.

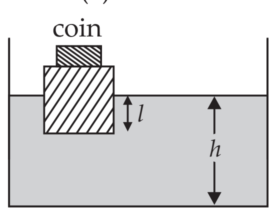
Ans. (*c, d*): Pressure is defined as the ratio of magnitude of component of force normal to area to the per unit area, verifies option (*c*).

As pressure is magnitude of force per unit area so does not depend on size of area verifies the option (*d*). As in pressure, magnitude of force is taken perpendicular to area of surface. Hence it is scalar.

Q10.8. A wooden block with a coin placed on its top, floats in water as shown in figure. The distances l and h are shown in the figure. After some time the coin falls into the water. Then

(*a*) l decreases (*b*) h decreases (*c*) l increases (*d*) h increases

Ans. (*a, b*): According to law of floatation weight of floating body is equal to weight of displaced fluid, when coin falls into the water, net weight of floating body decreased so the floats body displace less amount of water so block rise up l will decreased and height h of water will decrease as block rises up so verifies the option (*a, b*)



Q10.9. With increase in temperature, the viscosity of

(*a*) gases decreases (*b*) liquids increases
(*c*) gases increases (*d*) liquids decreases

Ans. (*c, d*): For coefficient of viscosity of liquid $\eta \propto \frac{1}{\sqrt{T}}$. For viscosity of gases $\eta \propto \sqrt{T}$ as temperature increase KE of gas molecule increases so collisions increases the viscosity.

In case of liquid on increasing temperature the KE of fluid increases. So layers speed increases but relative speed of layer remains almost same. So option (*c*) and (*d*) verifies.

Q10.10. Streamline flow is more likely for liquids with

(*a*) high density (*b*) high viscosity

(*c*) low density (*d*) low viscosity

Ans. (*b, c*): Stream line flow is more likely for liquids of low density (or lower gravitational force).

If viscosity of a liquid is larger caused more velocity gradient, hence each line of flow can be easily differentiated.

On increasing η and decreasing ρ, Reynold number $R_e = \dfrac{\rho v D}{\eta}$ decreases, which makes the flow towards streamline.

Hence verifies the option (*c*) and (*b*).

VERY SHORT ANSWER TYPE QUESTIONS

Q10.11. Is viscosity is a vector?

Ans. It is the property of liquid which is equal to the magnitude of dragging force per unit area between the two layers of liquid whose velocity gradient is unity. So it has no direction (only magnitude of force) so it is not vector.

Q10.12. Is surface tension a vector?

Ans. Surface tension $\left[\sigma = \dfrac{\text{W.D.}}{\text{Change in Surface Area}}\right]$. So, σ is not a vector as it is equal to the work done per unit change in surface area.

Q10.13. Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged if the density of ice is $\rho_i = 0.917$ gm/cm^3?

Ans. As iceberg is floating on surface of sea

$\therefore$ Weight of ice berg = Weight of displace liquid

$$V \cdot \rho_{ic} g = V \rho_w g$$

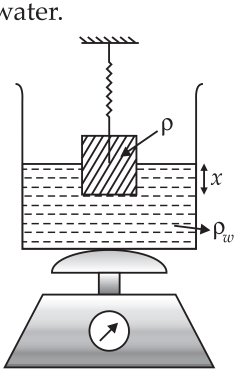
V = Vol. of ice berg

V′ = Vol. of ice berg inside water or vol. of displaced water by iceberg

$$\frac{\text{Volume of iceberg submerged}}{\text{Volume of iceberg}} = \frac{V'}{V} = \frac{\rho_{ice}}{\rho_w} = \frac{.917}{1}$$

Hence .917 part of iceberg body submerged into water.

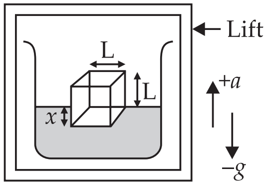
Q10.14. A vessel filled with water is kept on a weighing pan and the scale adjusted to zero. A block of mass M and density ρ is suspended by a massless spring of spring constant *K*. This block is submerged inside into the water in the vessel. What is the reading of the scale?



Ans. Consider the diagram. Beaker filled with water is placed on weighing pan and then scale is adjusted at zero.

As the block is submerged into water the Bouyant force (upward) acts on block by water.

This bouyant force acts as reaction force so reaction force by Newton's third law block will apply reaction force downward due to which reading on scale increase equal to the buoyant force = $V\rho_w g$



V = Volume of water displaced by block

ρ_w = density of water

mass of block = $M = V\cdot\rho$ or $V = \dfrac{M}{\rho}$

$\therefore$ Reading of weighing scale = $\dfrac{M}{\rho}\cdot\rho_w g = \dfrac{\rho_w}{\rho}Mg$

Q10.15. A cubical block of density ρ is floating on the surface of water. Out of its height L, fraction *x* is submerged in water. The vessel is in an elevator accelerating upward with acceleration *a*. What is the fraction immersed?

Ans. (*a*) When cubical block submerged into water then by principle of floatation.

$$V\rho g = V'\rho_w g$$

V′ = Volume of water displaced by block

V′ = Volume of block inside water

= area of base of block × height (inside water)

$V' = L^2 x$

V = Volume of block L^3, ρ_B = Density of block

$$\therefore \quad L^3\rho_B = L^2 x\,\rho_w \quad \text{or} \quad \frac{\rho_B}{\rho_w} = \frac{x}{L} \qquad ...(i)$$

$$x = \frac{\rho_B}{\rho_w}L.$$

(*b*) When the immersed block is in lift moving upward then net acceleration on system = $(g + a)$

Weight of block = $m(g + a) = V \times \rho_B(g + a)$

$= L^3\rho_B(g + a)$

Now let part of block submerge into water in moving lift (upward) is x_1

Weight of block = Buoyant Force

$$L^3\rho_B(g + a) = x_1 L^2\rho_w(g + a)$$

$$\therefore \quad \frac{\rho_B}{\rho_w} = \frac{x_1}{L}$$

$$x_1 = L\cdot\frac{\rho_B}{\rho_w} \qquad ...(ii)$$

from (*i*), (*ii*) we observe that submerged part of cube inside water in both case is $\left(\dfrac{\rho_B}{\rho_w}L\right)$ which constant or it is independent of acceleration of lift ($+a$, $-a$ or zero) i.e. motion of lift upward or downward or rest.

SHORT ANSWER TYPE QUESTIONS

Q10.16. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$ m. The surface tension of sap is $T = 7.28 \times 10^{-2}$ Nm^{-1} and the angle of contact is 0°. Does the surface tension alone account for the supply of water to the top of all trees?

Ans. Capillarity $r = 2.5 \times 10^{-5}$ m

$$S = T = 7.28 \times 10^{-2} \text{ Nm}^{-1} \quad g = 9.8 \text{ m/s}^2$$

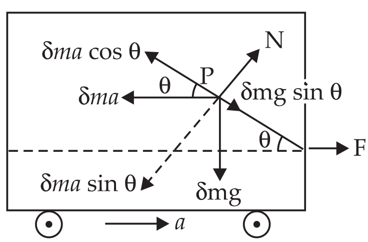
$$\theta = 0° \quad \rho = 10^3 \text{ kg/m}^3$$

$$h = \frac{2S\cos\theta}{r\rho g} = \frac{2 \times 7.28 \times 10^{-2}\cos 0°}{2.5 \times 10^{-5} \times 10^3 \times 9.8} = \frac{2 \times 728 \times 10^{-2+5}}{25 \times 98 \times 10^3}$$

$$h = \frac{104}{175} \times \frac{10^{+3}}{10^3} = \frac{104}{175} = 0.594 \text{ m} \cong .6 \text{ m}$$

Most of trees are of more than 0.6 m height. So capillary action alone cannot account for the rise of water in all other trees.

Q10.17. The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating, the free surface will be tilted by an angle θ. If the acceleration is $a$ms^{-2}, what will be the slope of the free surface?



Ans. Consider the tanker is pulled by force F which produces acceleration in the truck in forward direction.

Consider only an small element of mass δm at P. When tanker is pulled by forward acceleration 'a' then this element of mass also experience it in forward direction but due to inertia of rest it tries to remain in rest due to same acceleration in backward direction as it is free not rigidly connected to tanker. Hence force acting on δm are $F_1 = \delta ma$ in horizontally backward direction due to tanker's acceleration a, $F_2 = \delta mg$ vertically downward due to gravity.

The resolution of component of F_1 and F_2 along an perpendicular to the inclined surface of oil are resolved N-normal reaction is balanced by component $\delta ma \sin\theta$ of F_1 and when surface is inclined at maximum angle then

$$\delta ma\cos\theta = \delta mg\sin\theta$$

So $$\frac{\sin\theta}{\cos\theta} = \frac{a}{g} \text{ is required slope}$$

∴ $$\tan\theta = \frac{a}{g} \text{ is required slope.}$$

Q10.18. Two mercury droplets of radii 0.1 cm and 0.2 cm. Collapse into one single drop. What amount of energy is released? The surface tension of mercury $T = 435.5 \times 10^{-3}\ \text{Nm}^{-1}$.

Ans. Energy due to Surface Tension $E = \sigma\Delta A$

By law of conservation of mass vol of drop $V_1 + V_2 = V$

$$r_1 = 0.1 \text{ cm} = 0.1 \times 10^{-2} \text{ m} = 10^{-3} \text{ m}$$
$$r_2 = 0.2 \text{ cm} = 2 \times 10^{-3} \text{ m}$$
$$\Delta A = 4\pi r_1^2 + 4\pi r_2^2 - 4\pi R^2 = 4\pi[r_1^2 + r_2^2 - R^2]$$

R is the radius of new drop formed by the combination of two smaller drops.

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$$
$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi[r_1^3 + r_2^3] \quad \Rightarrow \quad R^3 = r_1^3 + r_2^3$$
$$R^3 = [(1 \times 10^{-3})^3 + (2 \times 10^{-3})^3] = \left[10^{-9} + 8 \times 10^{-9}\right] = 9 \times 10^{-9}$$
$$R \approx 2.1 \times 10^{-3} \text{ m}$$
$$E = \Delta A\sigma = 4 \times 3.14\,[(10^{-3})^2 + (2.0 \times 10^{-3})^2 - (2.1 \times 10^{-3})^2] \times 435.5 \times 10^{-3}$$
$$E = 4 \times 3.14 \times 435.5 \times 10^{-3} \times (10^{-3})^2\ [1 + 4 - (2.1)^2]$$
$$= 4 \times 3.14 \times 435.5 \times 10^{-9}\ [5 - 4.41]$$
$$E = 1742.0 \times 3.14 \times 10^{-9}\ [0.59] = 5469.88 \times 0.59 \times 10^{-9}$$
$$E = 3227.23 \times 10^{-9} = 32.2723 \times 10^{-7} \text{ J}$$
$$E = 32.27 \times 10^{-7} \text{ J}$$

energy is released due to formation of bigger drop from smaller drops as finally area will be smaller.

Q10.19. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of droplets. Let a drop of radius R, break into N small droplets each of radius r. Estimate the drop in temperature.

Ans. $\Delta E = \sigma$ (Final area – initial area) of surface

$\Delta E = ms\Delta t,$

By the law of conservation of mass, final volume = initial volume

One drop of radius R splits in N drops of radius r

$$\therefore \quad \frac{4}{3}\pi R^3 = N \cdot \frac{4}{3}\pi r^3 \quad \text{or} \quad R^3 = Nr^3$$
$$r = \frac{R}{(N)^{1/3}}$$

$\Delta E = \sigma\Delta A = \sigma$[area of N drops of radius r – area of big drop]

$$ms\Delta t = \sigma[N \cdot 4\pi r^2 - 4\pi R^2]$$
$$V \cdot \rho\, s\Delta t = 4\pi\sigma[Nr^2 - R^2]$$
$$N \cdot \left(\frac{4}{3}\pi r^3\right)\rho s\Delta t = 4\pi\sigma[Nr^2 - R^2]$$

m = mass of all smaller drops
ρ = density of liquid
s = specific heat of liquid
Δt = change in temperature

$$\Delta t = \frac{4\pi\sigma \times 3}{N \cdot 4\pi r^3 \rho s}[Nr^2 - R^2] \qquad (\because R^3 = Nr^3)$$

$$\Delta t = \frac{3\sigma}{N\rho s}\left[\frac{Nr^2}{r^3} - \frac{R^2}{r^3}\right] \qquad \left(\therefore r^3 = \frac{R^3}{N}\right)$$

$$\Delta t = \frac{3\sigma}{N\rho s}\left[\frac{N}{r} - \frac{R^2 N}{R^3}\right] = \frac{3\sigma N}{\rho N s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\Delta t = \frac{3\sigma}{\rho s}\left[\frac{1}{r} - \frac{1}{R}\right] \qquad \text{as } R > r$$

$\therefore$ Δt will be positive i.e. $\therefore$ $\frac{1}{R} < \frac{1}{r}$

hence formation of smaller drops require the temperature of drops to increase. This energy is taken from surroundings whose temperature decrease.

Q10.20. The surface tension and vapour pressure of water at 20°C is 7.28×10^{-2} Nm^{-1} and 2.33×10^3 Pa respectively. What is the radius of smallest spherical water droplet which can form without evaporating at 20°C?

Ans. The drop will evaporate if the water pressure on liquid, is greater than vapour pressure above the surface of liquid. Let a water droplet of radius R can be formed without evaporating then

Vapour pressure = Excess pressure in a drop

$$p = \frac{2\sigma}{R} \quad \text{(only one surface in drop)}$$

$$R = \frac{2 \times 7.28 \times 10^{-2}}{\text{Vapour pressure}} = \frac{2 \times 7.28 \times 10^{-2}}{2.33 \times 10^3} = \frac{1456 \times 10}{233 \times 10^5}$$

$$R = 6.25 \times 10^{-5} \text{ m.}$$

LONG ANSWER TYPE QUESTIONS

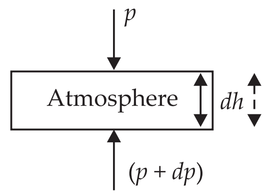
Q10.21. (*a*) Pressure decreases as one ascends the atmosphere. If the density of air is ρ. What is the change in pressure dp over a differential height dh?

(*b*) Considering the pressure p to be proportional to the density, find the pressure p at a height h if the pressure on the surface of the earth is p_0.

(*c*) If $p_0 = 1.013 \times 10^5$ $N \cdot m^{-2}$, $\rho_0 = 1.29$ kg/m^3 and $g = 9.8$ ms^{-2}, at what height will the pressure drop to $\left(\frac{1}{10}\right)$ the value at the surface of the earth?

(*d*) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

Ans. Consider a part (packet) of atmosphere of thickness dh. As the pressure at a point in fluid is equal in all direction. So the pressure on upper layer is p downward and on lower layer is $(p + dp)$ upward. Force due to pressure is balanced by Buoyant force by air



$$(p + dp)\,\mathrm{A} - p \cdot \mathrm{A} = -\mathrm{V}\rho g$$
$$p \cdot \mathrm{A} + dp\,\mathrm{A} - p \cdot \mathrm{A} = -\mathrm{A} \cdot dh\,\rho\,g$$
$$dp\mathrm{A} = -\rho g dh \mathrm{A}$$
$$dp = -\rho g dh \qquad ...(i)$$

Negative sign shows that pressure decreases as height increases.

(*b*) Let ρ_0 is the density of air on surface of earth

$\because$ Pressure p at a point is directly proportional to density.

$$\therefore \quad p \propto \rho \quad \text{or} \quad \frac{p}{p_0} = \frac{\rho}{\rho_0} \quad \text{or} \quad \rho = \left(\frac{p}{p_0}\right)\rho_0 \qquad ...(ii)$$

$$dp = -\rho g dh \text{ from } (i)$$

(ρ is density of air in atmosphere)

$$dp = -\left(\frac{p}{p_0}\right)\rho_0 g dh \quad \Rightarrow \quad \frac{dp}{p} = \frac{-\rho_0 g}{p_0} dh$$

integrating both sides $\displaystyle\int_{p_0}^{p} \log p\, dp = -\int_0^h \frac{\rho_0 g}{p_0} dh$

$$\log\left(\frac{p}{p_0}\right) = \frac{-\rho_0}{p_0} gh \qquad ...(iii)$$

$$\Rightarrow \qquad \frac{p}{p_0} = e^{-\frac{\rho_0 gh}{p_0}} \quad \Rightarrow \quad p = p_0 e^{\frac{-\rho_0 gh}{p_0}}$$

(*c*) $\because$ from (*iii*) $\displaystyle \log\frac{p}{p_0} = -\frac{\rho_0 gh}{p_0}$ put $p = \dfrac{p_0}{10}$ (given)

$$\log\frac{p_0/10}{p_0} = \frac{-\rho_0 gh}{p_0} \quad \Rightarrow \quad \log\frac{1}{10} = \frac{-\rho_0 gh}{p_0}$$

$$\log 10^{-1} = \frac{-\rho_0 gh}{p_0} \quad \Rightarrow \quad -\log 10 = \frac{-\rho_0 gh}{p_0}$$

or
$$h = \frac{p_0}{\rho_0 g}\log 10$$

$$h = \frac{p_0 \log_{10} 10 \times 2.303}{\rho_0 g}$$

$$= \frac{1.013 \times 10^5 \times 2.303}{1.29 \times 9.8} = 0.184 \times 10^5\,\mathrm{m}$$

$$h = 18.4 \times 10^3\,\mathrm{m} = 18.4\,\mathrm{km} \quad \text{Ans.}$$

(*d*) Temperature (T) remains constant only near the surface of the earth not at greater height.

Q10.22. Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decreases with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporisation for water L_V = 540 k cal. Kg^{-1}, the mechanical equivalent of heat J = 4.2 J cal^{-1}, density of water $\rho_w = 10^3$ kg/*l*. Avogadro's No. $N_A = 6.0 \times 10^{26}$ K $mole^{-1}$ and the molecular weight of water M_A = 18 kg for 1 K mole.

(*a*) estimate the energy required for one molecule of water to evaporate.

(*b*) show that the intermolecular distance for water is $d = \left[\frac{M_A}{N_A} \cdot \frac{1}{\rho_w}\right]^{1/3}$ and find its value.

(*c*) 1 g of water in vapour state at 1 atm occupies 1601 cm^3. Estimate the intermolecular distance at boiling point, in the vapour state.

(*d*) during vaporisation molecule overcomes a force F, assumed constant, to go from an inter molecular distance *d* to *d'*. Estimate the value of F, where $d = 3.1 \times 10^{-10}$m.

(*e*) calculate F/*d* which is a measure of the surface tension.

Ans.
$$L_V = 540\ K \text{ cal Kg}^{-1} = 540 \times 10^3 \text{ cal Kg}^{-1}$$
$$= 540 \times 4.2 \times 10^3 \text{ J Kg}^{-1}$$

Energy required to evaporate 1 kg water = L_v K cal.
Energy required to evaporate M_A Kg of water = $L_A M_A$ K cal.
In M_A Kg Number of molecule = N_A

$\therefore$ Energy required to evaporate 1 molecule = $\frac{L_V M_A}{N_A}$ K.cal

$$U = \frac{M_A L_V}{N_A} \text{ K cal.} = \frac{M_A L_V \times 10^3 \times 4.2 \text{ J}}{N_A}$$

$$U = \frac{18 \times 540 \times 10^3 \times 4.2}{6 \times 10^{26}} = 12.6 \times 540 \times 10^{3-26}$$
$$= 6804 \times 10^{-23} \text{ J}$$
$$U = 6.8 \times 10^{-20} \text{ J.}$$

(*b*) Let the water molecules to be point size and separated by distance '*d*' from each other.

Volume of N_A molecules = $\frac{\text{Mass of } N_A \text{ molecule}}{\text{density}} = \frac{M_A}{\rho_w}$

Volume occupied by 1 molecule = $\frac{M_A}{\rho_w N_A}$

Volume occupied by 1 molecule = $d^3 = \frac{M_A}{N_A \rho_w}$

$$d = \left[\frac{M_A}{N_A \rho_w}\right]^{1/3}$$

1 g (*i.e.* 10^{-3} Kg) of vapour occupies volume = 1601 cm^3 = 1601×10^{-6} m^3

(*c*) 1 Kg of vapour occupies volume = 1601 cm^3 = 1601×10^{-3} m^3
18 Kg of vapours occupies volume = $18 \times 1601 \times 10^{-3}$ m^3

18 Kg of water = 6×10^{26} molecules

$\therefore$ 6×10^{26} molecules occupies volume = $18 \times 1601 \times 10^{-3}$ m^3

1 molecule occupies volume = $\dfrac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}}$

$$\therefore \quad d'^3 = \left[\frac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}}\right] = [3 \times 1601 \times 10^{-29}]$$

$$d'^3 = (30 \times 1601) \times 10^{-30}$$

$$d' = (30 \times 1601)^{1/3} \times 10^{-10} \text{ m} = 36.3 \times 10^{-10} \text{ m}$$

(*d*) W.D. to change distance between molecules from d to d' or required work done = F($d' - d$)

Required WD = F($d' - d$)

Energy required to evaporate 1 molecule

$$\therefore \quad F(d' - d) = 6.8 \times 10^{-20}$$

$$F(36.3 \times 10^{-10} - 3.1 \times 10^{-10}) = 6.8 \times 10^{-20}$$

$$F \times 33.2 \times 10^{-10} = 6.8 \times 10^{-20}$$

$$F = \frac{6.8 \times 10^{-20}}{33.2 \times 10^{-10}} = 0.205 \times 10^{-10} \text{ N}$$

$$F = 2.05 \times 10^{-11} \text{ N}$$

(*e*) Surface Tension = $\dfrac{F}{d} = \dfrac{2.05 \times 10^{-11}}{3.1 \times 10^{-10}} = 6.6 \times 10^{-2}$ N/m

$$\sigma = 6.6 \times 10^{-2} \text{ N/m.}$$

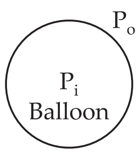
Q10.23. A hot air balloon is a sphere of radius 8 m. The air inside is at a temperature of 60°C. How large a mass can the balloon lift when the outside temperature is 20°C? (Assume air is an ideal gas, R = 8.314 J mole^{-1} K^{-1}, 1 atm = 1.013×10^5 N.m^{-2} or Pa the membrane tension is 5 Nm^{-1}.)

Ans. Pressure inside (P_i) balloon is larger than outer pressure (P_0) of atmosphere.

$$\therefore \quad P_i - P_0 = \frac{2\sigma}{R}$$

σ = surface tension in membrane of balloon

R = radius of balloon.



gas or air inside is perfect (considered)

$$\therefore \quad P_i V = n_i RT_i$$

V = Volume of balloon

n_i = no. of moles of gas in balloon

R = gas constant

T_i = temperature of balloon

$$n_i = \frac{P_i V}{RT_i} = \frac{\text{mass of air balloon } (M_i)}{\text{molecular mass } (M_A)}$$

$$n_i = \frac{M_i}{M_A} = \frac{P_i V}{RT_i} \quad ...(i)$$

Similarly $n_0 = \dfrac{P_0 V}{RT_0}$

By principle of floatation $W + M_i g = M_o g$

W = weight lifted by balloon $W = M_0 g - M_i g$

$W = (M_0 - M_i)g$

where n_0 = no. of molecules of air displaced by balloon

V = Volume of air displaced by balloon equal to volume of balloon

If M_0 mass of air displaced by balloon

M_A = molecular mass inside or outside balloon

$$\therefore \quad n_0 = \frac{M_0}{M_A} \quad \text{or} \quad n_0 = \frac{M_0}{M_A} = \frac{P_0 V}{RT_0} \quad \Rightarrow M_0 = \frac{P_0 V M_A}{RT_0}$$

from (i) $M_i = \dfrac{P_i V \cdot M_A}{RT_i}$

$$\therefore \quad W = \left[\frac{P_0 V M_A}{RT_0} - \frac{P_i V M_A}{RT_i}\right] g$$

$$W = \frac{V M_A}{R}\left[\frac{P_0}{T_0} - \frac{P_i}{T_i}\right] g$$

$M_A = 21\%\ O_2 + 79\%$ of N_2

$M_A = .21 \times 32 + .79 \times 28 = 4[.21 \times 8 + .79 \times 7]$

$M_A = 4[1.68 + 5.53] = 4[7.21] = 28.84\ g$

$M_A = .02884$ kg $\quad P_i = P_0 + \dfrac{2\sigma}{R}$

$$W = \frac{\frac{4}{3}\pi \times 8 \times 8 \times 8 \times .02884}{8.314}\left[\frac{1.013 \times 10^5}{(273+20)} - \frac{P_i}{273+60}\right] g$$

$P_i = P_0 + P$ due to S.T of membrane $= P_0 + \dfrac{2\sigma}{R}$

$$P_i = \left[1.013 \times 10^5 + \frac{2 \times 5}{8}\right] = [101300 + 1.25]$$

$P_i = 101301.25 = 1.0130125 \times 10^5 \cong 1.013 \times 10^5$

$$\therefore \quad W = \frac{4 \times 3.14 \times 8 \times 8 \times 8 \times 0.02884}{3 \times 8.314}\left[\frac{1.013 \times 10^5}{293} - \frac{1.013 \times 10^5}{333}\right] g$$

$$W = \frac{4 \times 3.14 \times 8 \times 8 \times 8 \times 0.02884 \times 1.013 \times 10^5}{3 \times 8.314}\left[\frac{1}{293} - \frac{1}{333}\right] g$$

$$W = \frac{4 \times 3.14 \times 8 \times 8 \times 8 \times 0.02884 \times 1.013 \times 10^5 \times 9.8}{3 \times 8.314}\left[\frac{333 - 293}{293 \times 333}\right]$$

$$W = \frac{3.14 \times 64 \times 32 \times 0.02884 \times 1.013 \times 10^5 \times 9.8 \times 40}{3 \times 8.314 \times 293 \times 333} = 3044.2\ \text{N}.$$

❑❑❑

11 Thermal Properties of Matter

MULTIPLE CHOICE QUESTIONS-I

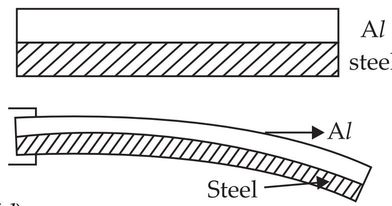
Q11.1. A bimetallic strip is made of aluminium and steel ($\alpha_{Al} > \alpha_{Steel}$) on heating, the strip will

(*a*) remain straight (*b*) get twisted

(*c*) will bend with aluminium on concave side

(*d*) will bend with steel on concave side.

Ans. (*d*): Both strips of Al and steel are fixed together initially in bimetallic strip. When both are heated then expansion in steel will be smaller than aluminium. So Al strip will be convex side and steel on concave side verifies the option (*d*).



Q11.2. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly

(*a*) its speed of rotation increases

(*b*) its speed of rotation decreases

(*c*) its speed of rotation remains same

(*d*) its speed increases because its moment of inertia increases.

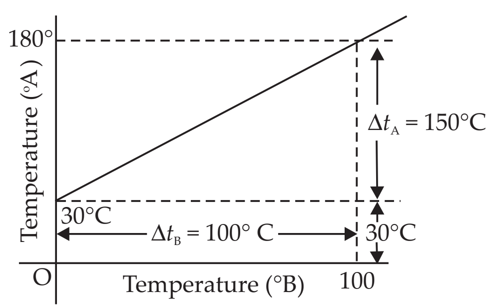
Ans. (*b*): On heating a uniform metallic rod its length will increase so moment of inertia of rod increased from I_1 to I_2 (i.e. $I_1 < I_2$). Due to law of conservation of angular momentum

$$I_1 \omega_1 = I_2 \omega_2$$

$\because \quad I_1 < I_2 \quad \Rightarrow \quad \omega_1 > \omega_2$, so angular speed decreased.

Verifies option (*b*).

Q11.3. The graph between two temperature scales A and B is shown in figure. Between upper fixed point and lower fixed point there are 150 equal division on scale A and 100 on scale B. The relationship for conversion between the two scales is given by



(*a*) $\frac{t_A - 180}{100} = \frac{t_B}{150}$ (*b*) $\frac{t_A - 30}{150} = \frac{t_B}{100}$

(*c*) $\frac{t_B - 180}{150} = \frac{t_A}{100}$ (*d*) $\frac{t_B - 40}{100} = \frac{t_A}{180}$

Ans. (*b*): It is clear from graph in t_A scale lower fixed point (LFP) is 30° and upper fixed point (UFP) is 180°. Similarly in scale °B, UFP = 100° and LFP = 0°.

Hence Formula $\frac{t_A - (\text{LFP})_A}{(\text{UFP})_A - (\text{LFP})_A} = \frac{t_B - (\text{LFP})_B}{(\text{UFP})_B - (\text{LFP})_B}$

$$\frac{t_A - 30}{180 - 30} = \frac{t_B - 0}{100 - 0}$$

or $$\frac{t_A - 30}{150} = \frac{t_B}{100}$$

Hence it tallies with option (*b*).

Q11.4. An aluminium sphere is dipped into water. Which of the following will be true?

(*a*) Buoyancy will be less in water at 0°C than that in water at 4°C
(*b*) Buoyancy will be more in water at 0°C than that in water at 4°C
(*c*) Buoyancy in water at 0°C will be same as that in water at 4°C
(*d*) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere.

Ans. (*a*): We know that Buoyant force on (B.F) a body of volume V_B and density of ρ_B, when immersed in liquid of density ρ_l is = $V'\rho_l g$

V' = Volume of displace liquid by dipped body = V

The density of water at 4°C is maximum all over the range of temperature.

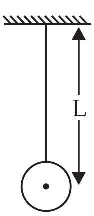
So the density of water at 4°C is maximum i.e. at 4°C buoyant force will be maximum.

or B.F α ρ_l (of liquid)

$$\frac{F_4}{F_0} = \frac{\rho_4}{\rho_0} \quad \text{As, } \rho_4 > \rho_0$$

Hence $\frac{F_4}{F_0} > 1$ or $F_4 > F_0$ Hence verifies option (*a*).

Q11.5. As the temperature is increased, the time period of a pendulum,



(*a*) increases as its effective length increases even though its centre of mass still remains at the centre of the bob.
(*b*) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob.

(*c*) increases as its effective length increases due to shifting of centre of mass below the centre of the bob.

(*d*) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob.

Ans. (*a*): As the temperature increased the length L increase due to expansion (Linear) and $T = 2\pi\sqrt{\frac{L}{g}}$ or $T \propto \sqrt{L}$

so on increasing temperature, T also increases verifies option (*a*).

Q11.6. Heat is associated with

(*a*) kinetic energy of random motion of molecules.

(*b*) kinetic energy of the orderly motion of molecules.

(*c*) total kinetic energy of random and orderly motion of molecules.

(*d*) kinetic energy of random motion in some cases and kinetic energy of orderly motion in order.

Ans. (*a*): We know that as the temperature increases vibration of molecules about their mean position increases. Hence the kinetic energy associated with random motion of molecule increases.

Q11.7. The radius of a metal sphere at room temperature T is R, and the coefficient of linear expansion of the metal is α. The sphere is heated a little by a temperature Δt so that its new temperature is $T + \Delta t$. The increase in the volume of the sphere is approximately.

(*a*) $2\pi R\alpha\Delta t$ (*b*) $\pi R^2\alpha\Delta t$ (*c*) $4\pi R^3\alpha\,\Delta t/3$ (*d*) $4\pi R^3\,\alpha\Delta t$

Ans. (*d*): $V_0 = \frac{4}{3}\pi R^3$

Coefficient of linear expansion is α.

Coefficient of cubical expansion = $3\,\alpha = \gamma$

$$\gamma = \frac{\Delta V}{V\Delta t} \quad \Rightarrow \quad \Delta V = \gamma V \cdot \Delta t$$

$$\Delta V = 3\alpha \cdot \frac{4}{3}\pi R^3 \Delta t = 4\pi R^3 \alpha \cdot \Delta t$$

Verifies option (*d*).

Q11.8. A sphere, a cube and a thin circular plate, all of the same material and same mass are initially heated to same high temperature. which object cool slowest and which one cool fastest?

(*a*) Plate will cool fastest and cube the slowest.

(*b*) Sphere will cool fastest and cube the slowest.

(*c*) Plate will cool fastest and sphere the slowest.

(*d*) Cube will cool fastest and plate the slowest.

Ans. (*c*): loss of heat on cooling increases as

(*i*) temperature difference between body and surrounding

(*ii*) surface area exposed to surrounding

(*iii*) material of object

We know that surface area of sphere is minimum and of circular plate is maximum.

So sphere cool slowest and circular plate fastest.

MULTIPLE CHOICE QUESTIONS-II

Q11.9. Mark the correct options.

(*a*) A system X is in thermal equilibrium with Y but not with Z. System Y and Z may be in thermal equilibrium with each other.

(*b*) A system X is in thermal equilibrium with Y but not with Z. Systems Y and Z are not in thermal equilibrium with each other.

(*c*) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z must be in thermal equilibrium with each other.

(*d*) A system X is neither in thermal equilibrium with Y nor with Z. The system Y and Z may be in thermal equilibrium with each other.

Ans. (*b, d*): (*a*) $T_x = T_y$ and $T_x \neq T_z$ $\therefore$ $T_x = T_y \neq T_z$

So Y and Z not in thermal equilibrium (so given statement is incorrect)

(*b*) $T_x = T_y$ and $T_x \neq T_z$ $\Rightarrow$ $T_y \neq T_z$

system Y and Z are not in thermal equilibrium, so statement (*b*) is correct

(*c*) $T_x \neq T_y$ and $T_x \neq T_z$ $\therefore$ $T_y \neq T_z$

So Y and Z are not in thermal equilibrium, so (*c*) is not correct.

(*d*) $T_x \neq T_y$ and $T_x \neq T_z$ T_z may be = to T_y. Hence (*d*) is correct.

Q11.10. 'Gulab Jamuns' (assumed to be spherical) are to be heated in an oven. They are also available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice big (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following:

(*a*) Both size gulab jamuns will get heated in the same time.

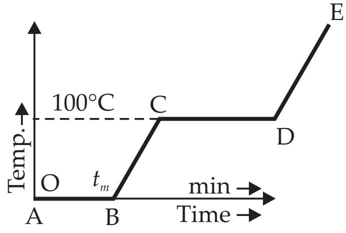
(*b*) Smaller gulab jamuns are heated before bigger one.

(*c*) Smaller pizzas are heated before bigger ones.

(*d*) Bigger pizzas are heated before smaller ones.

Ans. (*b, c*): Smaller one has smaller thickness through which heat to be transfer upto depth get heated before bigger one having larger depths. Hence option (*b, c*) are verified.

Q11.11. Refer to the plot of temperature *versus* time (figure). Showing the changes in the state of ice on heating (not to scale). Which of the following is correct?



(*a*) The region AB represents ice and water in the thermal equilibrium.

(*b*) At B water starts boiling.

(*c*) At C all the water gets converted into steam.

(*d*) C to D represents water and steam in equilibrium at boiling point.

Ans. (*a, d*): When a substance heat continuously but its temperature does not change, then its state (solid, liquid or gas) changes.

So part AB (0°C) and CD (100°C) represents the conversion of solid into liquid and liquid to gas respectively. So part AB contain ice and water both upto B, and part CD represents water and steam.

Q11.12. A glass full of hot milk poured on the table. It begins to cool gradually. Which of the following is correct ?

(*a*) The rate of cooling is constant till milk attains the temperature of the surrounding.

(*b*) The temperature of milk falls off exponentially with time.

(*c*) While cooling, there is a flow of heat from milk to the surrounding as well as from surrounding to the milk but the net flow of heat is from milk to the surrounding and thats why it cools.

(*d*) All three phenomenon, conduction, convection and radiation are responsible for the loss of heat from milk to the surroundings.

Ans. (*b, c, d*): When hot milk spread on table it transfer heat to surrounding by conduction, convection and radiation [verifies option *d*].

By Newtons law of cooling the heat of milk falls exponentially [verifies option (*b*)].

Lost of heat is directly proportional to the temperature difference with surrounding and body. So as milk cool, temperature difference decreases so rate of cooling decreases with time [reject option (*a*)].

While cooling very small amount of heat also flows from surrounding to milk as compared to heat lost by milk to surrounding [verifies option (*c*)].

VERY SHORT ANSWER TYPE QUESTIONS

Q11.13. Is the bulb of thermometer made of diathermic walls or adiabetic walls?

Ans. Adiabetic walls does not allow to pass heat into mercury of bulb and diathermic allows to conducts heat through it. So in the bulb of thermometer diathermic walls are used.

Q11.14. A student records the initial length (L), change in temperature Δt and change in length ΔL of the rod as follows:

S.No.	L(m)	Δt (°C)	ΔL (m)
1	2	10	4×10^{-4}
2	1	10	4×10^{-4}
3	2	20	2×10^{-4}
4	3	10	6×10^{-4}

If the first observation is correct, what can you say about observations 2, 3 and 4.

Ans. As here in experiment there is a rod and linear expansion (α) of a material or rod remains same i.e. equal to each °C as L and Δt is same. From Ist observation

$$\alpha = \frac{\Delta L}{L\Delta t} = \frac{4 \times 10^{-4}}{2 \times 10} = 2 \times 10^{-5}\ {}^\circ C^{-1}$$

For observation S.No. 2

$$\Delta L = \alpha \cdot L\,\Delta t = 2\times10^{-5}\times1\times10 = 2\times10^{-4}\text{ m} \neq 4\times10^{-4}\text{ m (wrong)}$$

For observation S. No. 3

$$\Delta L = \alpha L \Delta t$$
$$\Delta L = 2\times10^{-5}\times2\times20 = 8\times10^{-4}\text{ m} \neq 2\times10^{-4}\text{ m (wrong)}$$

For observation S. No. 4

$$\Delta L = \alpha \cdot L\Delta t$$
$$\Delta L = 2\times10^{-5}\times3\times10 = 6\times10^{-4} = 6\times10^{-4}\text{ m (correct).}$$

Q11.15. Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appears cooler than wooden bar if they are both colder than room temperature.

Ans. (*i*) It is due to facts that conductivity of metal bar is very high as of wood. So the rate of transferring the heat in metal is very large than in wood.

(*ii*) The specific heat of metal is very low as compared to wood, so metal requires very smaller quantities of heat than wood to change each degree of temperature. So due to larger conductivity and smaller specific heat, metals become more colder when placed in colder when region as compared to wood and become more hot when placed in hot region.

Q11.16. Calculate the temperature which has same numeral value on Celsius and Fahrenheit scale.

Ans. Let the required temperature is x °C = x° F

$$\frac{C}{100} = \frac{F-32}{180} \Rightarrow \frac{x}{5} = \frac{x-32}{9}$$
$$5x - 160 = 9x \Rightarrow -9x + 5x = 160$$
$$-4x = 160 \quad x = \frac{160}{-4} = -40°$$
$$\therefore \quad -40°F = -40°C$$

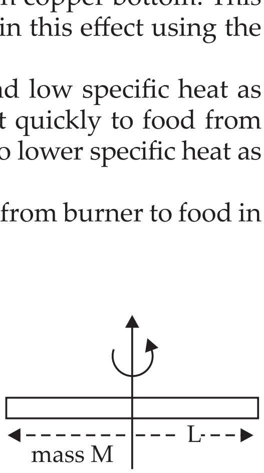
Q11.17. These days people use steel utensil with copper bottom. This is supposed for uniform heating of food, explain this effect using the fact that copper is better conductor.

Ans. As the copper has higher conductivity and low specific heat as compared of steel. So copper base transfer heat quickly to food from bottom and also need low quantity of heat due to lower specific heat as compared to steel.

So copper base supply quick and larger heat from burner to food in utensil.

SHORT ANSWER TYPE QUESTIONS

Q11.18. Find out the increase in moment of inertia I of a uniform rod. (coefficient of linear expansion α) about its perpendicular bisector when its temperature is slightly increased by ΔT.



Ans. I of rod about its axis along perpendicular bisector $= \frac{1}{12}ML^2$

$$\Delta L = \alpha L \Delta T$$

$$\therefore \qquad I' = \frac{\quad}{12}M(L + \quad L) \quad = \frac{1}{12}M[L^2 + \Delta L^2 + 2L\Delta L]$$

(neglecting ΔL^2 very small term)

$$I' = \frac{M}{12}(L^2 + 2L\Delta L) = \frac{ML^2}{12} + \frac{ML\Delta L}{6} \times \frac{2L}{2L}$$

$$I' = \frac{ML^2}{12} + \frac{ML^2}{12} \cdot \frac{2\Delta L}{L} = I + I \cdot \frac{2\alpha L \Delta T}{L}$$

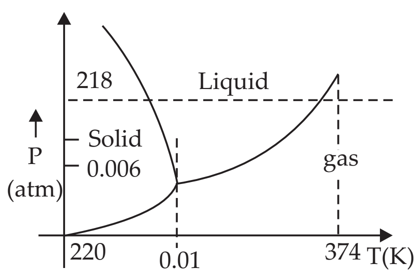
$$I' = I + I \cdot (2\alpha \Delta T)$$

So new moment of inertia increased by ($2I\ \alpha\ \Delta T$).

Q11.19. During summers in India, one of the common practice to keep cool is to make ice balls of crushed ice, dip it in flavoured sugar syrup and sip it. For this a stick is inserted into crushed ice and is squeezed in the palm to make into the ball. Equivalently in winter in those areas where it snows, people make snow balls and throw around. Explain the formation of ball out of crushed ice or snow in the light of P–T diagram of water.

Ans. From the P–T graph or diagram of water and double headed arrow. Increasing pressure at 0°C and 1 atm takes ice into liquid state and decreasing pressure in liquid state at 0°C and 1 atm takes water to ice state.

When crushed ice is squeezed, some of its parts melts into water at 0°C and filling up the gaps between the ice flakes. During squeezing the ice flakes, the m.p. increased and water at between the flakes also freezes into ice and binds all ice flakes making the ball more stable.



Q11.20. 100 g of water is supercooled to –10°C. At this point, due to some disturbance mechanized or otherwise, some of it suddenly freezes to ice. What will be the temperature of resultant mixture and how much mass would freeze?

[S_w = 1 cal/g/°C and L^w_{fusion} = 80 cal/g]

Ans. Water mass = 100 g

At –10°C ice and water mixture exists.

Heat required (given out) by – 10°C ice to 0°C ice = $ms\Delta t$

$$= 100 \times 1 \times [0 - (-10)]$$

$$Q = 1000 \text{ cal}$$

Let m gm of ice melted $Q = mL$

$$m = \frac{Q}{L} = \frac{1000}{80} = 12.5\,g$$

So, there is $m = 12.5\,g$ water and $100 - 12.5\,g$ ice in mixture. Hence temperature of mixture remains 0°C.

Q11.21. One day in the morning, Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some time say 5–10 minutes before he could take bath. Now he had two options: (*i*) fill the remaining bucket completely by cold water and then attend to the work, (*ii*) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer? Explain.

Ans. By Newton's law of cooling we know that rate of cooling or loss of heat energy is directly proportion to the difference in the temperature of body and surroundings.

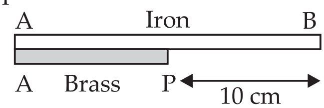
Hence First option would have to keep water warmer, because difference between the temperature of surrounding and water is small. So less amount of loss of heat energy in Ist option.

In IInd option the difference between water and surrounding is large, so larger amount of heat energy lost.

LONG ANSWER TYPE QUESTIONS

Q11.22. We would like to prepare a scale whose length does not change with temperature. It is proposed to prepare a unit scale of this type whose length remains, say 10 cm. We can use a bimetallic strip made of brass and iron each of different length whose length (both components) would change in such a way that difference between their lengths remain constant. If $\alpha_{iron} = 1.2 \times 10^{-5}$/K and $\alpha_{brass} = 1.8 \times 10^{-5}$/K, what should we take as length of each strip?

Ans. Required scale can be made with the help of iron and brass rod so that their one end are at A coincide and the other ends remains at distance PB = 10 cm at any temperature. Let initial length of iron and brass rods at any temperature T are L_{1I} and L_{1B} respectively.



$$L_{1I} - L_{1B} = 10 \text{ cm} \quad \ldots(i)$$

$$\therefore \quad \alpha = \frac{\Delta L}{L_0 \Delta T} \quad \text{or} \quad \alpha = \frac{L_2 - L_1}{L_1 \Delta T}$$

$$L_2 - L_1 = L_1\, \alpha \Delta T$$

$$L_2 = L_1(1 + \alpha \Delta T)$$

If combination of rods are heated by temperature ΔT, then length becomes L_{2I} and L_{2B}

$$L_{2I} - L_{2B} = 10 \text{ cm}$$
(again according to question)

$$L_{1I}(1 + \alpha_I \Delta T) - L_{1B}(1 + \alpha_B \Delta T) = 10$$

$$L_{1I} + \alpha_I L_{1I}\Delta T - L_{1B} - L_{1B}\alpha_B \Delta T = 10$$

$$L_{1I} - L_{1B} + (\alpha_I L_{1I} - \alpha_B L_{1B})\Delta T = 10$$

$$10 + (\alpha_I L_{1I} - \alpha_B L_{1B})\Delta T = 10 \qquad [\text{from } (i)]$$

or $$\alpha_I L_{1I} - \alpha_B L_{1B} = 0$$

$$\alpha_I L_{1I} = \alpha_B L_{1B}$$

$$\frac{L_{1I}}{L_{1B}} = \frac{\alpha_B}{\alpha_I} = \frac{1.8 \times 10^{-5}}{1.2 \times 10^{-5}} = \frac{18}{12} = \frac{3}{2}$$

$\therefore$ $$\frac{L_{1I}}{L_{1B}} = \frac{3}{2}$$

Then let $$L_{1I} = 3x \text{ and } L_{1B} = 2x$$

$$L_{1I} - L_{1B} = 10$$

$$3x - 2x = 10$$

$$x = 10$$

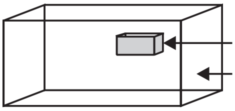
$\therefore$ $$\text{Length of iron rod} = 3 \times 10 = 30 \text{ cm}$$

$$\text{Length of brass rod} = 2 \times 10 = 20 \text{ cm}$$

The difference between IInd end remain = 10 cm.

Q11.23. We would like to make a vessel whose volume does not change with temperature (take the hint from the problem above). We can use brass and iron ($\gamma_{v\,\text{brass}} = 6 \times 10^{-5}\ \text{K}^{-1}$ and $\gamma_{v\,\text{iron}} = 3.55 \times 10^{-5}\ \text{K}^{-1}$, to create a volume of 100 c.c. How do you think you can achieve this?

Ans. We have to make double container whose difference in volume is 100 c.c. Let initial volumes of container of iron and brass $V_{1i} - V_{1b} = 100$ c.c.



Now the containers are heated by ΔT K then new volumes becomes V_{2i} and V_{2b} but then difference remains 100 c.c.

$$V_{2i} - V_{2b} = 100 \text{ c.c}$$

$$\gamma = \frac{\Delta V}{V\Delta T}$$

$\therefore$ $$V_2 - V_1 = \gamma V_1 \Delta T$$

$$V_2 = V_1 + \gamma V_1 \Delta T = V_1(1 + \gamma \Delta T)$$

$\therefore$ $$V_{2i} = V_{1i}(1 + \gamma_i \Delta T) \text{ and } V_{2b} = V_{1b}(1 + \gamma_b \Delta T)$$

$\therefore$ $$V_{1i} + V_{1i}\gamma_i \Delta T - (V_{ib} + V_{ib}\gamma_b \Delta T) = 100 \text{ c.c}$$

$$V_{1i} - V_{1b} + V_{1i}\gamma_i \Delta T - V_{1b}\gamma_b \Delta T = 100$$

$$100 + (V_{1i}\gamma_i - V_{1b}\gamma_b)\Delta T = 100$$

$$\gamma_i V_{1i} - V_{1b}\gamma_b = 0$$

$$\frac{V_{1i}}{V_{1b}} = \frac{\gamma_b}{\gamma_i} = \frac{6 \times 10^{-5}}{3.55 \times 10^{-5}}$$

$$\frac{V_{1i}}{V_{1b}} = \frac{6}{3.55} = \frac{600}{355} = \frac{120}{71}$$

Let $V_{1i} = 120x$ and $V_{1b} = 71x$

and $V_{1i} - V_{1b} = 100$

$$120x - 71x = 100 \Rightarrow 49x = 100 \Rightarrow x = \frac{100}{49}$$

$$x = 2.04$$

$$V_{1i} = 120 \times 2.04 = 245 \text{ c.c}$$

$$V_{1b} = 71 \times 2.04 = 145 \text{ c.c}$$

Q11.24. Calculate the stress developed inside a tooth cavity filled with copper, when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and $\alpha = 1.7 \times 10^{-5}$/K. bulk modulus for copper = 140×10^9 N/m^2.

Ans. Change in temperature $\Delta T = 57 - 37 = 20°C$

Linear expansion α of (tooth) body = 1.7×10^{-5}/K

Cubical expansion $\gamma = 3\alpha = 3 \times 1.7 \times 10^{-5} = 5.1 \times 10^{-5}$ K^{-1}

Let the volume of the cavity be V and its volume increased by ΔV due to increase in temperature ΔT.

$$\Delta V = \gamma V \cdot \Delta T$$

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$\text{Thermal stress produced} = B \times \text{Volumetric strain}$$

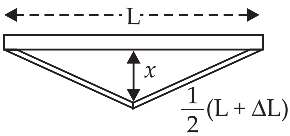
$$= B \cdot \frac{\Delta V}{V} = B \cdot \gamma \Delta T$$

$$\text{Thermal stress} = 140 \times 10^9 \times 5.1 \times 10^{-5} \times 20 = 14280 \times 10^4$$

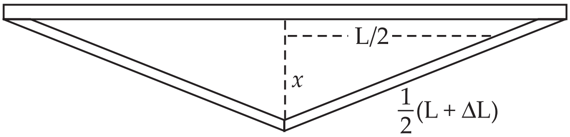
$$= 1.428 \times 10^8 \text{ Nm}^{-2}$$

This stress is about 10^3 times of atmospheric pressure (*i.e.*, 1.01×10^5 Nm^{-2}).

Q11.25. A rail track made up of steel having length 10 m is clamped on a railway line at its two ends (figure). On a summer day due to rise in temperature by 20°C, it is deformed as shown in figure. Find x (displacement of the centre) if $\alpha_{steel} = 1.2 \times 10^{-5}$/°C.



Ans. $\alpha = 1.2 \times 10^{-5}$/°C $L_0 = 10$ m $\Delta T = 20°C$



By Pythagoras Theorem

$$x^2 = \left[(L + \Delta L)\frac{1}{2}\right]^2 - \left(\frac{L}{2}\right)^2$$

$$= \frac{1}{4}\left[L^2 + \Delta L^2 + 2L\Delta L\right] - \frac{L^2}{4}$$

$$x = \frac{L^2}{4} + \frac{\Delta L^2}{4} + \frac{2L\Delta L}{4} - \frac{L^2}{4} \qquad (\Delta L^2 << L \text{ neglecting } \Delta L^2)$$

$$x^2 = \frac{2L\Delta L}{4}$$

$$x = \frac{1}{2}\sqrt{2L\Delta L}$$

$$\Delta L = L_0\ \alpha \cdot \Delta T \ = 10 \times 1.2 \times 10^{-5} \times 20 = \ 240.0 \times 10^{-5}$$

$$x = \frac{1}{2}\sqrt{2 \times 10 \times 240 \times 10^{-5}}$$

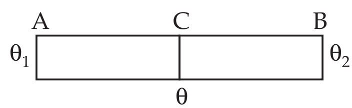
$$= \frac{1}{2}\sqrt{10 \times 10 \times 2 \times 2 \times 2 \times 2 \times 10^{-1} \times 10^{-4} \times 3}$$

$$= \frac{10 \times 4}{2}\sqrt{.3} \times 10^{-2} \ = 20 \times .54 \times 10^{-2} = 10.8 \times 10^{-2}\ \text{m}$$

$$x = 10.8\ \text{cm}$$

Q11.26. A thin rod having length L_0 at 0°C and coefficient of linear expansion α has its two ends maintained at temperatures θ_1 and θ_2 respectively. Find its new length.

Ans. As the temperature of rod varies from θ_1 to θ_2 from one end to another. So mean temperature of rod $= \theta = \dfrac{\theta_1 + \theta_2}{2}$



at C. So rate of flow of heat from A to C and C to B are equal

$\because \quad \theta_1 > \theta > \theta_2$

$$\therefore \qquad \frac{d\theta}{dt} = \frac{KA(\theta_1 - \theta)}{L_0/2} = \frac{KA(\theta - \theta_2)}{L_0/2}$$

K is coefficient of thermal conductivity

$$\therefore \qquad \theta_1 - \theta = \theta - \theta_2$$

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

$$L = L_0\ (1 + \alpha\theta) = L_0\left[1 + \alpha\left(\frac{\theta_1 + \theta_2}{2}\right)\right].$$

Q11.27. According to Stefan's law of radiation, a black body radiates energy σT^4 from its unit surface area every second where T is the surface temperature of the black body and $\sigma = 5.67 \times 10^{-8}$ W/m^2/K^4 is known as Stefan's constant. A nuclear weapon may be thought of as a ball of radius 0.5 m. When detonated, it reaches temperature of 10^6 K and can be treated as a black body.

(*a*) Estimate the power its radiates.

(*b*) If surrounding has water at 30°C, how much water can 10% of energy produced evaporate in 1s?

[$S_w = 4186.0 J/KgK$, $L_V = 22.6 \times 10^5$ J/Kg]

(*c*) If all this energy U is in the form of radiation, corresponding momentum is $p' = \frac{U}{c}$. How much momentum per unit time does it impart on unit area at a distance of 1 Km?

Ans. (*a*) $E = \sigma T^4$ per second per sq. m

Total E = radiated from all surface area A per sec will be power radiated by nuclear weapon

$$P = \sigma A T^4$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4, \quad R = 0.5 \text{ m}, \quad T = 10^6 \text{ K}$$

$$P = 5.67 \times 10^{-8} \times (4 \times \pi R^2)(10^6)^4$$

$$= 5.67 \times 4 \times 3.14 \times .5 \times .5 \times 10^{-8} \times 10^{24}$$

$$= 5.67 \times 4 \times 3.14 \times .5 \times .5 \times 10^{-8} \times 10^{24}$$

$$= 5.67 \times 3.14 \times 10^{24-8} \times 1.00$$

$$P \cong 18 \times 10^{16} \text{ Watt} = 1.8 \times 10^{17} \text{ J/s} \qquad \text{(I)}$$

(*b*) $\therefore \quad P = 18 \times 10^{16}$ Watt

10% of this power is required to evaporate water

$$E = \frac{10}{100} \times 18 \times 10^{16} \text{ Watt} = 1.8 \times 10^{16} \text{ J/s}$$

Energy required by *m* kg water at 30° C to evaporate 100°C.

= E required to heat up water from 30°C to 100°C
+ E required to evaporate water into vapour

$$= mS_w(T_2 - T_1) + mL = m(S_w(T_2 - T_1) + L)$$

$$1.8 \times 10^{16} = m[4180\,(100 - 30) + 22.6 \times 10^5]$$

$$= m[4186 \times 70 + 22.6 \times 10^5]$$

$$m(2.93020 \times 10^5 + 22.6 \times 10^5) = 1.8 \times 10^{16}$$

$$m(2.93020 + 22.6) \times 10^5 = 1.8 \times 10^{16}$$

$$m\,25.5 \times 10^5 = 1.8 \times 10^{16}$$

$$m = \frac{1.8 \times 10^{16}}{25.5 \times 10^5} \cong 7 \times 10^9 \text{ kg.}$$

(*c*) Momentum per unit time $p' = \frac{U}{c} = \frac{1.8 \times 10^{17}}{3 \times 10^8} = .6 \times 10^9$

from (I) $\qquad p' = 6 \times 10^8$ Kg ms^{-2}

p per unit time per unit area at a distance 1 km $= \frac{6 \times 10^8}{4\pi R^2}$

$$\therefore \qquad p = \frac{6 \times 10^8}{4 \times 3.14 \times (10^3)^2}$$

$$\frac{6 \times 10^8}{4 \times 3.14 \times 10^6} = \frac{6 \times 100}{12.56} = 47.77 \text{ Kgms}^{-2}/\text{m}^2$$

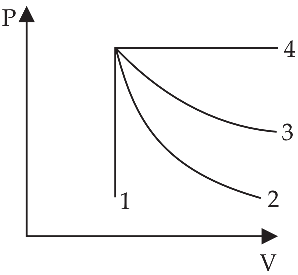
p marked per sec at 1 km away on $1m^2 = 47.8$ N/m^2.

❑❑❑

12 Thermodynamics

MULTIPLE CHOICE QUESTIONS-I

Q12.1. An ideal gas undergoes four different processes from the same initial state (figure). Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1, 2, 3, and 4, which one is adiabatic?



(*a*) 4 (*b*) 3
(*c*) 2 (*d*) 1

Ans. (*c*): 4 is isobaric process, 1 is isochoric. Out of 3 and 2, 3 has the smaller slope (magnitude) hence is isothermal. Remaining process 2 is adiabatic.

Q12.2. If an average person jogs, he produces 14.5×10^3 cal/min. This is removed by the evaporation of sweat. The amount of sweat evaporated per minute (assuming 1 kg requires 580×10^3 calories for evaporation).

(*a*) 0.025 kg (*b*) 2.25 kg (*c*) 0.05 kg (*d*) 0.20 kg

Ans. (*a*): 580×10^3 calories are needed to convert 1 kg H_2O into steam

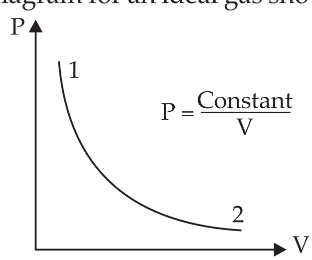
$$1 \text{ cal will produce sweat} = \frac{1 \text{ kg}}{580 \times 10^3}$$

$$14.5 \times 10^3 \text{ cal will produce sweat} = \frac{14.5 \times 10^3}{580 \times 10^3} \text{ kg}$$

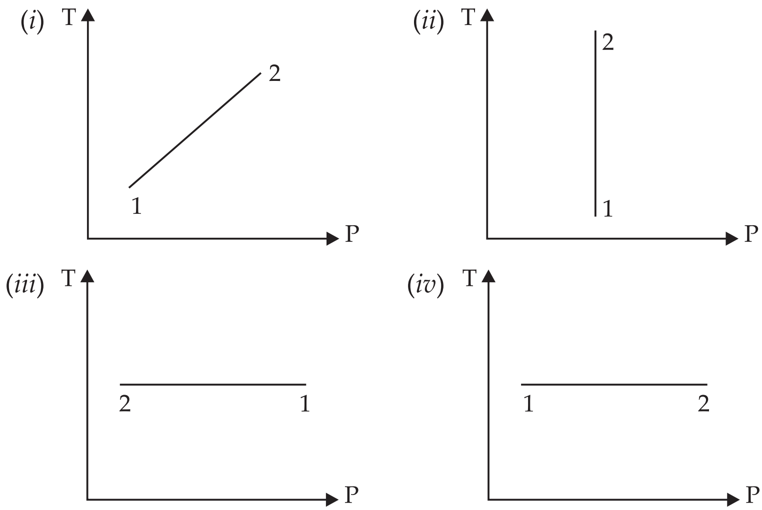
$$= \frac{14.5}{580} \text{ kg per minute}$$

$$= \frac{145}{5800} \text{ kg per minute } = 0.025 \text{ kg per min}$$

Q12.3. Consider P-V diagram for an ideal gas shown in the given figure.



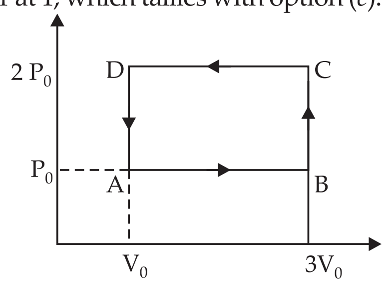
Out of the following diagrams (figure ahead), which represents the T-P diagram?



(*a*) (*iv*) (*b*) (*ii*) (*c*) (*iii*) (*d*) (*i*)

Ans. (*c*): According to P-V diagram at constant temperature, P increases as V decreases. So, it is Boyle's law in options (*iii*) and (*iv*). If P increases at constant temperature, volume V decreases. As in (iii) T-P diagram, P is smaller at 2 and larger at 1, which tallies with option (*c*).

Q12.4. An ideal gas undergoes cyclic process ABCDA as shown in the given PV diagram. The amount of work done by the gas is:



(*a*) 6 P_0V_0 (*b*) $-2P_0V_0$

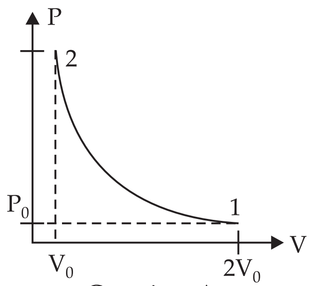
(*c*) $+2P_0V_0$ (*d*) $+4P_0V_0$

Ans. (*d*): The direction of arrows is anticlockwise so work done is negative equal to the area of loop = $-(3V_0 - V_0)\ (2P_0 - P_0) = -2P_0V_0$ verifies the option (*b*). New work implies external work is done on the system.

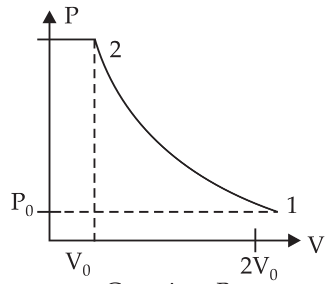
Q12.5. Consider two containers A and B containing identical gases at the same pressure, volume and temperature. The gas in container A is compressed to half of its original volume isothermally while the gas in container B is compressed to half of its original value adiabatically. The ratio of final pressure of gas in B to that of gas in A is:

(*a*) $2^{\gamma-1}$ (*b*) $\left(\frac{1}{2}\right)^{\gamma-1}$ (*c*) $\left(\frac{1}{1-\gamma}\right)^2$ (*d*) $-\left(\frac{1}{\gamma-1}\right)^2$

Ans. (*a*): Consider P-V diagram for container A and B. In both processes compression of gas involve. For isothermal process (gas A) during $(1 \rightarrow 2)$



Container A
(isothermal)



Container B
(adiabatic)

$$P_1V_1 = P_2V_2$$
$$P_0(2V_0) = P_2(V_0)$$
$$P_2 = 2P_0$$

For adiabatic process (gas B) during $(1 \to 2)$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$P_0(2V_0)^{\gamma} = P_2(V_0)^{\gamma}$$
$$P_2 = \left(\frac{2V_0}{V_0}\right)^{\gamma} P_0 = 2^{\gamma}P_0$$
$$\frac{(P_2)_B}{(P_1)_A} = \frac{2^{\gamma}P_0}{2P_0} = 2^{\gamma-1}$$

Hence verifies the option (*a*).

Q12.6. Three copper blocks of masses M_1, M_2 and M_3 kg respectively are brought into thermal contact till they reach equilibrium. Before contact, they were at T_1, T_2, T_3 $(T_1 > T_2 > T_3)$. Assuming there is no heat loss to the surroundings, the equilibrium temperature T is (*s* is specific heat of copper).

(*a*) $T = \dfrac{T_1 + T_2 + T_3}{3}$ (*b*) $T = \dfrac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$

(*c*) $T = \dfrac{M_1T_1 + M_2T_2 + M_3T_3}{3(M_1 + M_2 + M_3)}$ (*d*) $T = \dfrac{M_1T_1s + M_2T_2s + M_3T_3s}{M_1 + M_2 + M_3}$

Ans. (*b*): Let the equilibrium temperature of the system = T

Let $T_1\ T_2 < T < T_3$

As there is no loss to the surroundings.

$$\text{heat lost by } M_3 = \text{Heat gain by } M_1 + \text{Heat gain by } M_2$$
$$M_3s(T_3 - T) = M_1s(T - T_1) + M_2s(T - T_2)$$
$$M_3sT_3 - M_3sT = M_1sT - M_1sT_1 + M_2sT - M_2sT_2$$
$$T(M_3 + M_1 + M_2) = [M_3T_3 + M_1T_1 + M_2T_2]$$

$$T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$$

Hence verifies option (*b*).

MULTIPLE CHOICE QUESTIONS-II

Q12.7. Which of the processes described below are irreversible?

(*a*) The increase in temperature of an iron rod by hammering it.

(*b*) A gas in a small container at a temperature T_1 is brought in contact with a big reservoir at a higher temperature T_2 which increases the temperature of the gas.

(*c*) A quasi-static isothermal expansion of an ideal gas in cylinder fitted with a frictionless piston.

(*d*) An ideal gas is enclosed in a piston cylinder arrangement with adiabatic walls. A weight W is added to the piston, resulting in compression of gas.

Ans. (*a*, *b*, *d*)

(*a*) During hammering the rod work is done on rod in hammering, this work converts into heat, raises the temperature of rod, this heat energy cannot be converted into work so process is not reversible.

(*b*) The heat of bigger container transfers to smaller container, till the temperature of both become equal which is average of both. Now, heat from smaller container cannot flow to larger as heat flows from higher temperature to lower.

(*d*) When weight is added to piston then pressure is increased volume decreased it cannot be reversed back itself.

Q12.8. An ideal gas undergoes isothermal process from some initial state *i* to final state *f*. Choose the correct alternatives.

(*a*) $dU = 0$ (*b*) $dQ = 0$

(*c*) $dQ = dU$ (*d*) $dQ = dW$

Ans. (*a*, *d*): As process is isothermal $\Delta T = 0$ or T constant for an ideal gas dU = change in internal energy $dU = nC_v dT$

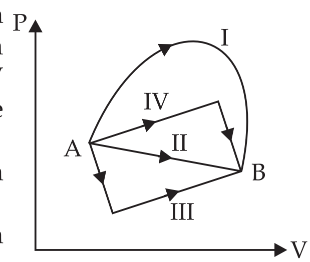
as $dT = 0 \quad \therefore \quad dU = 0$

n = number of moles

$$dQ = dU + dW \quad \Rightarrow \quad dQ = dW$$

verifies option (*a*) and (*d*).

Q12.9. Figure shows the P-V diagram of an ideal gas undergoing a change of state from A to B. Four different parts I, II, III and IV as shown in the figure may lead to the same change of state



(*a*) Change in internal energy is same in IV and III cases, but not in I and II.

(*b*) Change in internal energy in same in all the four cases (*c*) W.D is maximum in case I

(*d*) Work done is minimum in case II.

Ans. (*b, c*): **Main concept used:** *d*U does not depend on P-V path, it depends on initial and final position. WD is P–V is equal to area enclosed with V-axis.

The initial and final position are same for different parts I, II, III, IV. So ΔU is same. Hence option (*a*) rejected verifies option (*b*). As the area enclosed by path I is maximum with V-axis, so W.D. during path I is maximum and minimum is in III.

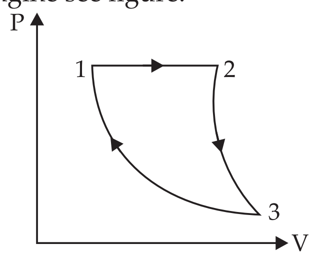
Hence option (*c*) verifies and option *d* rejected.

Q12.10. Consider a cycle followed by an engine see figure.

1 to 2 is isothermal

2 to 3 is adiabatic

3 to 1 is adiabatic



Such a process does not exist because

(*a*) Heat is completely converted to mechanical energy in such a process, which is not possible.

(*b*) Mechanical energy is completely converted to heat in this process, which is not possible.

(*c*) Curves representing two adiabatic processes don't intersect.

(*d*) Curves representing an adiabatic process and an isothermal process don't intersect.

Ans. (*a, c*): (*a*) The given process is cyclic which starts from 1 and ends up at 1 again rd

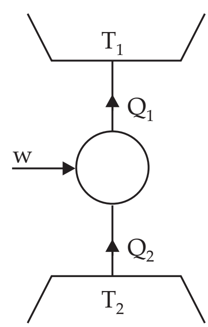
so $\quad dU = 0 \quad$ i.e. $\quad dQ = dU + dW$

$\Rightarrow \quad dQ = dW$

Hence heat energy supply to system converts totally into mechanical work which is not possible by second law of thermodynamics verifies option (*a*).

(*c*) P-V curve 2 to 3 and 3 to 1 both are adiabatic it cannot be possible without supply energy hence process 3 to 1 cannot be adiabatic. Verifies option (*c*) and rejects option (*d*).

Q12.11. Consider a heat engine as shown in figure, Q_1 and Q_2 are heat added to heat bath T_1 and heat taken from T_2 in one cycle of engine. W is mechanical work done on the engine.



If W > 0, then possibilities are:

(*a*) $Q_1 > Q_2 > 0$ (*b*) $Q_2 > Q_1 > 0$

(*c*) $Q_2 < Q_1 < 0$ (*d*) $Q_1 < 0$ and $Q_2 > 0$

Ans. (*a, c*): From fig $Q_1 = W + Q_2$

$\therefore \quad W > 0 \quad$ So $\because Q_1 - Q_2 > 0 \quad$ or $\quad Q_1 > 0$

$\therefore \quad Q_1 > Q_2 > 0$ if both Q_1, Q_2 positive verifies option (*a*).

or $\quad Q_2 < Q_1 < 0$ if both Q_1 Q_2 negative verifies option (*c*).

VERY SHORT ANSWER QUESTIONS

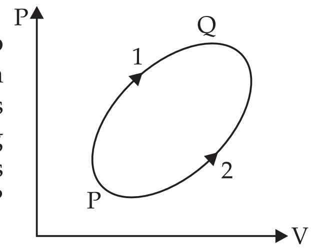
Q12.12. Can a system be heated and its temperature remains constant?

Ans. It is given that $\Delta T = 0 \quad \Rightarrow \quad \Delta U = 0$

$\therefore \qquad \Delta Q = \Delta U + \Delta W$

$\Rightarrow \qquad \Delta Q = \Delta W$ So heat supplied to the system is utilized in expansion system is isothermal.

Q12.13. A system goes from P to Q by two different paths in P-V diagram as shown in figure. Heat given to the system in path 1 is 1000 J. The work done by the system along path 1 is more than path 2 by 100 J. What is the heat exchanged by the system in path 2?



(given)

Ans. For path (1) $Q_1 = +1000$ J

$$\text{W.D.} = W_1 - W_2 = 100$$
$$W_1 = \text{WD through path 1}$$
$$W_2 = \text{WD through path 2}$$
$$\therefore \quad W_2 = W_1 - 100$$

As change in internal energy by path 1 and 2 are same

$$\Delta U = Q_1 - W_1 = Q_2 - W_2$$
$$1000 - W_1 = Q_2 - (W_1 - 100)$$
$$1000 = Q_2 + 100$$
$$Q_2 = 900 \text{ J.}$$

Q12.14. If a refrigerator's door is kept open, will the room become cool or hot? Explain.

Ans. If a refrigerator door is kept open the room will become hotter, because amount of heat absorbed from inside the refrigerator and work done on refrigerator by electricity both will be rejected by refrigerator in room.

Q12.15. Is it possible to increase the temperature of a gas without adding heat to it? Explain.

Ans. During adiabatic compression the temperature of gas increase while no heat is given to system

In adiabatic compression $dQ = 0$

$$dQ = dU + dW$$
$$\therefore \quad dU = -dW$$

So in compression work is done on system (WD (–)ve). So dU +ve and increase the temperature of system.

So as internal energy of gas (ideal) increases its temperature increases.

Q12.16. Air pressure in a car tyre increases during driving. Explain.

Ans. During driving, reaction force due to force on tyres, temperature of gas increases so gas inside tyres expands as volume inside the tyre remains constant (Charle's law) so temperature of car tyre increases during driving (as $P \propto T$).

SHORT ANSWER TYPE QUESTIONS

Q12.17. Consider a Carnot's cycle operating between $T_1 = 500$ K and $T_2 = 300$ K producing 1 kJ of mechanical work per cycle. Find the heat transferred to the engine by the reservoirs.

Ans. Efficiency of Carnot's engine $\eta = 1 - \frac{T_2}{T_1}$

Temperature of source or reservoir $= T_1 = 500$ K

Temperature of sink $= T_2 = 300$ K

$$\therefore \quad \eta = 1 - \frac{T_2}{T_1}$$

$$\frac{\text{Output work}}{\text{Input work (E)}} = 1 - \frac{300}{500}$$

$$\frac{1000\text{ J}}{x} = 1 - 0.6$$

$$\frac{1000}{x} = 0.4$$

$$x = \frac{1000}{0.4} = 2500\text{ J}.$$

Q12.18. A person of mass 60 kg wants to lose 5 kg by going up and down a 10 m high stairs. Assume he burns twice as much fat while going up than coming down. If 1 kg of fat is brunt on expending 7000 k cal, how many times must he go up and down to reduce his weight by 5 kg?

Ans. Energy produced by 1 kg fat = 7000 k cal

Energy produced by 5 kg fat $= 5 \times 7000$ k cal = 35000 k cal.

$= 35 \times 10^6$ cal

Energy consumed to go up one time $= mgh$

Energy consumed to come down one time $= \frac{1}{2} mgh$ (given)

$\therefore$ Energy consumed to go up and down one time $= mgh + \frac{1}{2} mgh$

$$E = \frac{3}{2} mgh = \frac{3}{2} \times 60 \times 10 \times 10 = 9000\text{ J}$$

Energy consume 1 time $= \frac{9000}{4.2}$ cal.

Let he go up and down n time to consumed energy 35×10^6 cal

$$n \times \frac{9000}{4.2} = 35 \times 10^6$$

$$n = \frac{35 \times 1000 \times 1000 \times 42}{9000 \times 10} = \frac{3500}{3} \times 14$$

$$= \frac{49000}{3} = 16.3 \times 10^3\text{ times}.$$

Q12.19. Consider a cycle tyre being filled with air by a pump. Let V be the volume of tyre (fixed) and at each stroke of the pump $\Delta V[<<V]$ of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from P_1 to P_2?

Ans. Air is transferred into tyre adiabatically let initial volume of air in tyre V and after pumping one stroke it become (V + dV) and pressure increased from P to (P + dP) then

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$P(V + dV)^\gamma = (P + dP)V^\gamma$$

$$PV^\gamma\left[1+\frac{dV}{V}\right]^\gamma = P\left[1+\frac{dP}{P}\right]V^\gamma$$

as volume of tyre V remains constant

$$PV^\gamma\left[1+\gamma\frac{dV}{V}\right] = PV^\gamma\left[1+\frac{dP}{P}\right]$$

[on expanding by binomial theorem neglecting the higher terms of ΔV as $\Delta V << V$]

$$1+\gamma\frac{dV}{V} = 1+\frac{dP}{P}$$

$$dV = \frac{VdP}{\gamma P}$$

Integrating both side in limits W_1 to W_2 and $P_1 \rightarrow P_2$

$$\int PdV = \int_{P_1}^{P_2}\frac{VdP}{\gamma}$$

$$\int_{W_1}^{W_2} dW = \frac{V}{\gamma}(P_2 - P_1) \qquad \text{(V = constant)}$$

$$W = \frac{(P_2 - P_1)V}{\gamma}$$

Q12.20. In a refrigerator one removes heat from lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor of 1 kW power, and heat is transferred – 3°C to 27°C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.

Ans. Carnot's engine is perfect heat engine operating between two temperature T_1 and T_2 (source and sink). Refrigerator is also Carnot's engine working in reverse order its efficiency is η

$$\eta = 1-\frac{T_2}{T_1} = 1-\frac{273-3}{273+27} = 1-\frac{270}{300} = 1-.9 = .1 = \frac{1}{10}$$

Efficiency of refrigerator's 50% of perfect engine

$\therefore$ Efficiency of refrigerator = 50% of 1 = .5

Net efficiency $= \eta' = 0.5 \times 0.1 = 0.05$

$\therefore$ Coefficient of performance $\beta = \dfrac{Q_2}{W} = \dfrac{1-\eta'}{\eta'}$

$$\beta = \frac{1-0.05}{0.05} = \frac{0.95}{0.05} = 19$$

Q_2 = 19% W.D. by motor on refrigerator

$= 19 \times 1\ \text{kW} = 19\ \text{kJ/s}$

Q12.21. If the coefficient of performance of a refrigerator is 5 and operates at the room temperature (27°C), find the temperature inside the refrigerator.

Ans. We know,

$$\beta = \frac{T_2}{T_1 - T_2} \qquad \begin{pmatrix} \beta = 5 \\ T_1 = 27 + 273 = 300\ \text{K} \end{pmatrix}$$

$$5 = \frac{T_2}{300 - T_2} \quad \Rightarrow \quad T_2 = 1500 - 5T_2$$

$$T_2 + 5T_2 = 1500 \quad \Rightarrow \quad 6\,T_2 = 1500$$

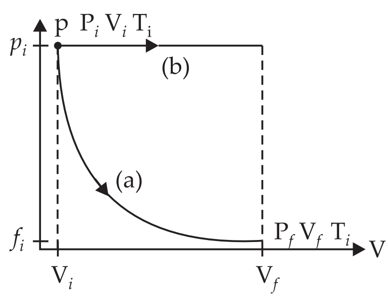
$$T_2 = \frac{1500}{6} = 250\ \text{K} = 250 - 273 = -23°\text{C}.$$

Q12.22. The initial state of a certain gas is $(P_iV_iT_i)$. It undergoes expansion till its volume become V_f. Consider the following two cases.

(*a*) The expansion takes place at constant temperature.

(*b*) The expansion takes place at constant pressure.

Plot P–V diagram for each case. In which of the two cases, is the work done by the gas more?



Ans. (*a*) The expansion from V_i to V_f temperature T_i remains constant so isothermal expansion i.e. $P_iV_i = P_fV_f$ constant T.

(*b*) The expansion is at constant pressure p_i, so isobaric process so graph P-V will be parallel to V axis till its volume becomes V_f. As the area enclosed by graph (*a*) is less than (*b*) with volume axis so W.D. by process (*b*) is more than of (*a*).

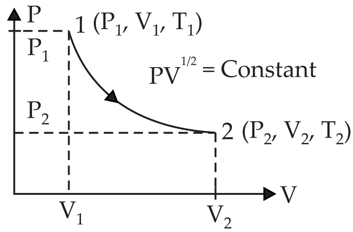
LONG ANSWER TYPE QUESTIONS

Q12.23. Consider a P-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in figure.

(*a*) Find work done when the gas is taken from state (1) to state (2).

(*b*) What is the ratio of temperature T_1/T_2, if $V_2 = 2V_1$?

(*c*) Given the internal energy for one mole of gas at temperature T is (3/2) RT, find the heat supplied to the gas when it is taken from state (1) to (2) with $V_2 = 2V_1$.



Ans. $\because PV^{1/2}$ = constant = K(given) or $P_1V_1^{1/2} = P_2V_2^{1/2} = K$ and $P = K/V^{1/2}$

(*a*) Work done for process from 1 to 2

$$WD = \int_{V_1}^{V_2} P \cdot dV = \int_{V_1}^{V_2} \frac{K}{V^{1/2}} dV = K \int_{V_1}^{V_2} V^{-(1/2)} dV$$

$$WD = K\left[\frac{V^{1/2}}{\frac{1}{2}}\right]_{V_1}^{V_2} = 2K\left[\sqrt{V_2} - \sqrt{V_1}\right]$$

WD from V_1 to V_2, i.e., $dW = 2P_1V_1^{1/2}\left[\sqrt{V_2} - \sqrt{V_1}\right]$

$$= 2P_2V_2^{1/2}\left[\sqrt{V_2} - \sqrt{V_1}\right]$$

(*b*) from gas equation of ideal gas $PV = nRT$

$$\Rightarrow \quad T = \frac{PV}{nR} = \frac{P\sqrt{V}\sqrt{V}}{nR} = \frac{K\sqrt{V}}{nR}$$

$$T_1 = \frac{K\sqrt{V_1}}{nR} \quad \text{and} \quad T_2 = \frac{K\sqrt{V_2}}{nR}$$

$$\frac{T_1}{T_2} = \frac{\frac{K\sqrt{V_1}}{nR}}{\frac{K\sqrt{V_2}}{nR}} = \frac{\sqrt{V_1}}{\sqrt{V_2}} = \sqrt{\frac{V_1}{2V_1}} \quad (\because \quad V_2 = 2V_1 \text{ given})$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{\sqrt{2}} \qquad \text{...}(ii)$$

required ratio is $1 : \sqrt{2}$.

(c) Given that internal energy U of gas is

$$U = \left(\frac{3}{2}\right)RT$$

$$\Delta U = \frac{3}{2}R\,dT = \frac{3}{2}R(T_2 - T_1)$$

$$\because \quad T_2 = \sqrt{2}\,T_1, \text{ from part } (b)$$

$$\Delta U = \frac{3}{2}R\left[\sqrt{2}T_1 - T_1\right] = \frac{3}{2}RT_1(\sqrt{2} - 1)$$

from part (a) $dW = 2P_1V^{1/2}\left(\sqrt{V_2} - \sqrt{V_1}\right)$

$\because \quad V_2 = 2V_1$ (given)

so $\quad \sqrt{V_2} = \sqrt{2}\sqrt{V_1}$ then

$$dW = 2P_1V_1^{1/2}\left(\sqrt{2}\sqrt{V_1} - \sqrt{V_1}\right)$$

$$= 2P_1V_1^{1/2}\sqrt{V_1}[\sqrt{2} - 1]$$

$$dW = 2P_1V_1(\sqrt{2} - 1)$$

$$dW = 2nRT_1(\sqrt{2} - 1) \qquad (\because \quad P_1V_1 = nRT_1)$$

$\because \quad n = 1 \quad \therefore \quad dW = 2RT_1\,(\sqrt{2} - 1)$

$$\therefore \quad dQ = dW + dU = 2RT_1(\sqrt{2} - 1) + \frac{3}{2}RT_1\,(\sqrt{2} - 1)$$

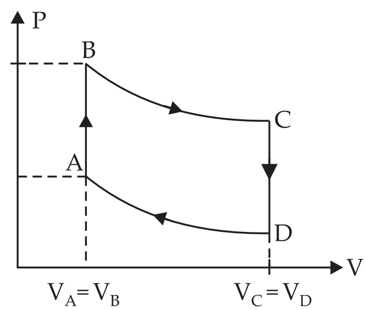
$$= (\sqrt{2} - 1)\,RT_1\left[2 + \frac{3}{2}\right]$$

$$dQ = -(\sqrt{2} \quad 1)\,RT\,.$$

Q12.24. A cycle followed by an engine [made of one mole of perfect gas in a cylinder with piston] is shown in figure.

A to B : constant volume
B to C : adiabatic
C to D : volume constant
D to A : adiabatic

$$V_C = V_D = 2V_A = 2V_B$$



(a) In which part of the cycle heat is supplied to the engine from outside?

(b) In which part of the cycle heat is being given to the surrounding by the engine?

(c) What is the work done by the engine in one cycle? Write your answer in term of P_A, P_B, V_A?

(d) What is the efficiency of the engine?

$$\left[\gamma = \frac{5}{3} \text{ for the gas}\right], \left[C_V = \frac{3}{2}R \text{ for one mole}\right]$$

Ans. (a): Heat is supplied to engine in part AB in which

$dV = 0$ so $\because$ $dW = P \cdot dV$ $\therefore$ $dW = P \cdot 0$ $\therefore$ $dW = 0$

By Ist law of thermodynamics $dQ = dU + dW$

$$dQ = dU$$

i.e. heat energy supplied to system does not work, but increase the internal energy dU of gas or system

$$\therefore \quad P = \frac{nRT}{V} \quad \because \quad V = \text{constant} \quad \therefore \quad P \propto T$$

as pressure is increased at constant volume it temperature i.e. internal energy increased (*b*) Heat is given out by system in part CD. (inverse of AB). In part CD, $dV = 0$, pressure decreases so temperature also decreases ($P \propto T$) i.e. energy given out by the system to surrounding.

(c) WD by system = $\int_A^B PdV + \int_B^C PdV + \int_C^D PdV + \int_D^A PdV$

in A to B and C to D $dV = 0$

$$\int_A^B PdV = 0 \quad \text{and} \quad \int_C^D PdV = 0$$

for adiabatic change $PV^\gamma = K$ ...(*i*)

$$P = \frac{K}{V^\gamma}$$

$$\int_B^C PdV = \int_{V_B}^{V_C} \frac{K}{V^\gamma} dV = K \int_{V_B}^{V_C} V^{-\gamma} dV = K\left[\frac{V^{-\gamma+1}}{1-\gamma}\right]_{V_B}^{V_C}$$

$$\int_B^C PdV = \frac{K}{1-\gamma}\left[V_C^{1-\gamma} - V_B^{1-\gamma}\right] \quad \text{...}(ii)$$

Similarly D to A is also adiabatic so

$$\int_D^A PdV = K\frac{(V_A^{1-\gamma} - V_D^{1-\gamma})}{1-\gamma} = \frac{K}{1-\gamma}\left[V_A^{1-\gamma} - V_D^{1-\gamma}\right]$$

$$= \frac{K V_A^{1-\gamma} - KV_D^{1-\gamma}}{1-\gamma}$$

For adiabatic

$$\int_D^A PdV = \frac{P_A V_A^\gamma V_A^{1-\gamma} - P_D V_D^\gamma V_D^{1-\gamma}}{1-\gamma} \qquad [\because \; PV^\gamma = K \text{ from } (i)]$$

$$\int_D^A PdV = \frac{P_A V_A - P_D V_D}{1-\gamma}$$

Similarly

$$\int_B^C PdV = \frac{P_C V_C - P_B V_B}{1-\gamma}$$

$$\therefore \quad \text{Total} \quad WD = 0 + \frac{P_C V_C - P_B V_B}{1-\gamma} + 0 + \frac{P_A V_A - P_D V_D}{1-\gamma}$$

$$= \frac{1}{1-\gamma}[P_C V_C - P_B V_B + P_A V_A - P_D V_D]$$

For adiabatic change $P_B V_B^\gamma = P_C V_C^\gamma$

$$P_C = \frac{P_B V_B^\gamma}{V_C^\gamma} = P_B\left(\frac{V_B}{V_C}\right)^\gamma = P_B\left(\frac{V_B}{2V_B}\right)^\gamma$$

$$P_C = P_B \frac{1}{2^\gamma} = P_B 2^{-\gamma} \quad \text{similarly } P_D = P_A 2^{-\gamma}$$

$$\therefore \quad \text{Net WD} = \frac{1}{1-\gamma}[P_B V_C 2^{-\gamma} - P_B V_B + P_A V_A - P_A V_D 2^{-\gamma}]$$

$$= \frac{1}{1-\gamma}[P_B 2V_B 2^{-\gamma} - P_B V_B + P_A V_A - P_A 2V_A 2^{-\gamma}]$$

$$= \frac{1}{1-\gamma}[-P_B V_B[-2^{1-\gamma} + 1] + P_A V_A [1 - 2^{1-\gamma}]]$$

$$= \frac{1}{1-\gamma}[-2^{1-\gamma} + 1][-P_B V_B + P_A V_A]$$

$$= \frac{1}{1-\gamma}(-2^{1-\gamma} + 1)[-P_B V_A + P_A V_A]$$

$$\because 2V_B = 2V_A \quad \Rightarrow \quad V_A = V_B$$

$$= \frac{1}{1-\frac{5}{3}}[-2^{1-5/3} + 1][-P_B + P_A]V_A$$

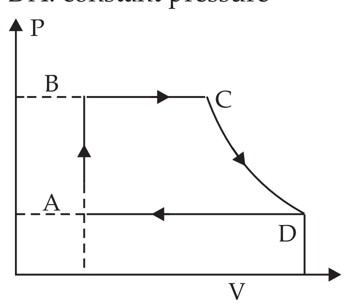
$$= +\frac{3}{2}(-2^{-2/3} + 1)[P_B - P_A]V_A$$

$$= +\frac{3}{2}\left[1 - \left(\frac{1}{2}\right)^{2/3}\right][P_B - P_A]V_A .$$

Q12.25. A cycle followed by an engine (made of one mole of an ideal gas in a cylinder with a piston) is shown in figure. Find heat exchanged by the engine, with the surroundings for each section of the cycle. $\left(C_V = \frac{3}{2}R\right)$.

(*a*) AB: constant volume (*b*) BC: constant pressure

(*c*) CD: adiabatic (*d*) DA: constant pressure



Ans. (*a*): For $A \to B$, $dV = 0$

so $\quad dW = \int P \cdot dV = \int P \times 0 = 0$

$$dW = 0$$

By 1st law of thermodynamics

$$dQ = dU + dW = dU + 0$$

$$\therefore \quad dQ = dU$$

$$(\because dQ = nC_V dT)$$

$$n = 1; \quad C_V = \frac{3}{2}R$$

so $\quad dQ = 1\frac{3}{2}R(T_B - T_A) \quad ...(i)$

$$dU = dQ = \frac{3}{2}(RT_B - RT_A) = \frac{3}{2}(P_B V_B - P_A V_A)$$

$\therefore$ Heat exchange [to system]

$$dQ_1 = dU = \frac{3}{2}(P_B V_B - P_A V_A)$$

(*b*) For B to C, $\Delta P = 0 \quad n = 1$

$$dQ = dU + dW = C_V(dT) + P_B\, dV$$

$$dQ_2 = \frac{3}{2}R(T_C - T_B) + P_B(V_C - V_B)$$

$$= \frac{3}{2}(T_C R - RT_B) + P_B V_C - P_B V_B$$

$$= \frac{3}{2}[P_C V_C] - \frac{3}{2}[P_B V_B] - P_B V_B + P_B V_C$$

$$V_A = V_B \quad \text{and} \quad P_B = P_C$$

$$\therefore \qquad dQ_2 = \frac{3}{2}P_B V_C - \frac{3}{2}P_B V_A - P_B V_A + P_B V_C$$

$$= \frac{5}{2}P_B V_C - \frac{5}{2}P_B V_A$$

$$dQ_2 = \frac{5}{2}P_B[V_C - V_A].$$

(*c*) For diagram C → D, adiabatic change

$\therefore \qquad dQ_3 = 0$ (No exchange of heat)

(*d*) For diagram D → A, $\Delta P = 0$ Compression of gas from volume V_D to V_A at constant pressure hence heat exchange similar to part (*b*) i.e. Heat exchange $dQ_3 = \frac{5}{2}P_A(V_A - V_D)$

Q12.26. Consider that an ideal gas (*n* moles) is expanding in a process given by $P = f(V)$, which passes through a point (V_0, P_0). Show that the gas is absorbing heat at (P_0, V_0) if the slope of the curve $P = f(V)$ is larger than the slope of the adiabatic passing through (P_0, V_0).

Ans. Slope of graph at $(V_0, P_0) = \left(\frac{dP}{dV}\right)_{(V_0 P_0)}$

$P = f(V)$ for adiabatic process PV^{γ} = constant (K)

$$\text{or} \qquad P = \frac{K}{V^{\gamma}} \quad \text{or} \quad \frac{dP}{dV} = K(-\gamma)\, V^{-\gamma-1}$$

$$\frac{dP}{dV} = -\gamma P V^{\gamma} V^{-\gamma} V^{-1} = -\frac{\gamma P}{V}$$

$$\left(\frac{dP}{dV}\right)_{(P_0, V_0)} = \frac{-\gamma P_0}{V_0}$$ Heat absorbed by in the process $P = f(V)$

$$dQ = dU + dW$$

$$dQ = nC_V dT + P dV \qquad \ldots(i)$$

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{V}{nR} f(V)$$

$$\frac{dT}{dV} = \frac{1}{nR}[f(V) + Vf'(V)]$$

$$\frac{dQ}{dV} = nC_V \frac{dT}{dV} + P \cdot \frac{dV}{dV} = \frac{nC_V}{nR}[f(V) + Vf'(V)] + P$$

$$\left(\frac{dQ}{dV}\right)_{V=V_0} = \frac{C_V}{R}[f(V_0) + V_0 \ f'(V_0)] + f(V_0) \quad [\because \ P = f(V) \text{ given}]$$

$$= f(V_0)\left[\frac{C_V}{R} + 1\right] + V_0 f'(V_0)\frac{C_V}{R}$$

$$C_p - C_V = R \ \Rightarrow \ \frac{C_p}{C_V} - 1 = \frac{R}{C_V}$$

$$\therefore \ \gamma - 1 = \frac{R}{C_V} \ \Rightarrow \ C_V = \frac{R}{\gamma - 1} \ \Rightarrow \ \frac{C_V}{R} = \frac{1}{\gamma - 1}$$

$$\left(\frac{dQ}{dV}\right)_{V=V_0} = f(V_0)\left[\frac{1}{\gamma - 1} + 1\right] + V_0 f'(V_0)\frac{1}{\gamma - 1}$$

$$= f(V_0)\left[\frac{1 + \gamma - 1}{\gamma - 1}\right] + \frac{V_0 f'(V_0)}{\gamma - 1}$$

$$= \frac{\gamma}{(\gamma - 1)} f(V_0) + V_0 \frac{f'(V_0)}{(\gamma - 1)}$$

$$= \frac{1}{(\gamma - 1)}[\gamma f(V_0) + V_0 f'(V_0)] \quad (\because \ f(V_0) = P_0)$$

$$\left(\frac{dQ}{dV}\right)_{V=V_0} = \frac{1}{(r - 1)}[\gamma P_0 + V_0 f'(V_0)]$$

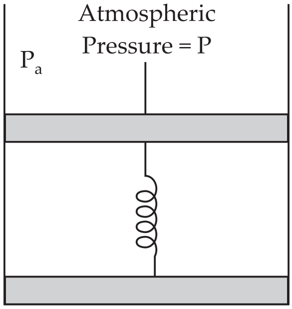
$$\therefore \ \left(\frac{dQ}{dV}\right)_{V=V_0} > 1 \ \therefore \ \text{and} \ \ \gamma > 1 \ \text{ so } \frac{1}{\gamma - 1} \text{ is +ve}$$

$$\therefore \ \gamma P_0 + V_0 f'(V_0) > 0$$

$$V_0 f'(V_0) > -\gamma P_0$$

$$f'(V_0) > \frac{-\gamma P_0}{V_0}.$$

Q12.27. Consider one mole of perfect gas in a cylinder of unit cross-section with a piston attached (figure). A spring (spring constant k) is attached (unstretched length L) to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas causing an increase of volume from V_0 to V_1.



(*a*) What is the initial pressure of the system?

(*b*) What is the final pressure of the system?

(*c*) Using the first law of thermo-dynamics, write down a relation between Q, P_a, V_1, V_0 and k.

Ans. (*a*): It is considered that piston is mass less and piston is balanced by atmospheric pressure (P_a). So the initial pressure of system inside the cylinder = P_a.

(*b*) On supply heat Q. Volume of gas increases from V_0 to V_1 and spring stretched also.

So increase in volume = $V_1 - V_0$

If displacement of piston is x then volume increase in cylinder

$$= \text{Area of base} \times \text{height} = A \times x$$

$$A \times x = V_1 - V_0 \qquad (A = \text{area of cross section of cylinder})$$

$$\therefore \qquad x = \frac{V_1 - V_0}{A}$$

$$\text{Force exerted by spring } F_S = Kx = \frac{K(V_1 - V_0)}{A}$$

as the piston is of unit area of cross-section $\quad \therefore A = 1$

Force due to spring = $K(V_1 - V_0)$ on unit area can be say press due to spring = $K(V_1 - V_0)$

Final total pressure on gas $P_f = P_a + K(V_1 - V_0)$

(*c*) By Ist law of thermodynamics $dQ = dU + dW$

$$dU = C_V(T - T_0)$$

T = final temperature of gas

T_0 = initial temperature of gas

$n = 1$

$$T_f = T = \frac{P_f V_f}{R} = \frac{[P_a + k(V_1 - V_0)]V_1}{R}$$

W.D. by gas = $p.\,dV$ + increase in PE of spring

$$dW = P_a(V_1 - V_0) + \frac{1}{2}kx^2$$

Now
$$dQ = dU + dW$$
$$= C_V\,(T - T_0) + P_a(V_1 - V_0) + \frac{1}{2}kx^2$$
$$dQ = C_V(T - T_0) + P_a(V_1 - V_0) + \frac{1}{2}k\,(V_1 - V_0)^2$$

It is required relation.

❑❑❑

13 Kinetic Theory

MULTIPLE CHOICE QUESTIONS-I

Q13.1. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 ms^{-1} in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground.

(*a*) remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas.

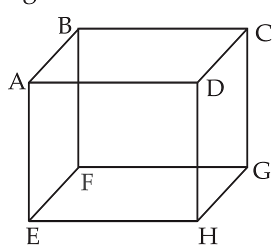
(*b*) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.

(*c*) Will increase by a factor equal to $[v_{rms}^2 + (500)^2]/v_{rms}^2$ where v_{rms} was the original mean square velocity of the gas.

(*d*) Will be different on the top wall and bottom wall of the vessel.

Ans. (*b*): As the relative velocity of molecule with respect to the walls of container does not change in rocket, due to the mass of a molecule is negligible with respect to the mass of whole system, and system of gas moves as a whole and $g = 0$ on molecule everywhere. Hence the pressure inside the vessel of gas as observed by us on the ground remain same.

Q13.2. 1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300 K (figure). One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,



(*a*) the pressure on EFGH would be zero.

(*b*) the pressure on all the faces will be equal.

(*c*) the pressure of EFGH would be double the pressure on ABCD.

(*d*) the pressure on EFGH would be half that on ABCD.

Ans. (*d*): Pressure on the wall due to force exerted by molecule on walls due to its rate of transfer of momentum to wall. The molecule bounces back due to elastic collision and momentum transferred to wall by each molecule is $2mv$ but wall EFGH absorbs those molecule which strike to it so rate of change in momentum to it become only mv so the pressure of EFGH would be half of ABCD.

Q13.3. Boyle's law is applicable for an

(*a*) adiabatic process (*b*) isothermal process

(*c*) isobaric process (*d*) isochoric process

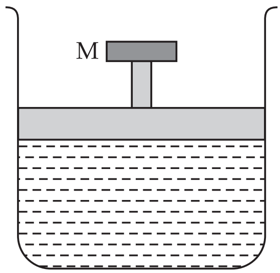
Ans. (*b*): Boyle's law is applicable at constant temperature, and temperature remains constant in isothermal process

$$PV = nRT \qquad (n, R, T \text{ are constant})$$

$$\therefore \quad PV = \text{constant}$$

$$P \propto \frac{1}{V}$$

Q13.4. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction (figure). If the temperature is increased



(*a*) both P and V of the gas will change

(*b*) only P will increase according to Charles Law

(*c*) V will change but not P.

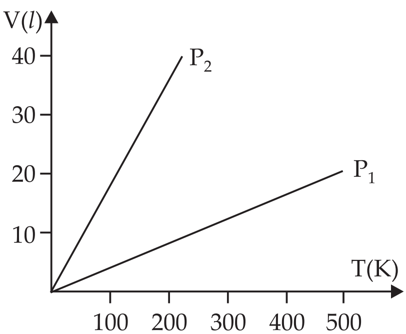
(*d*) P will change but not V.

Ans. (*c*): The pressure on gas does not change from initial to final position. As piston and cylinder is frictionless so by ideal gas equ.

$PV = nRT$ as P, R, n are constants so $V \propto T$.

So on increasing temperature of system it's volume increased but P will remain constant.

Q13.5. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relation between P_1 and P_2?



(*a*) $P_1 > P_2$

(*b*) $P_1 = P_2$

(*c*) $P_1 < P_2$

(*d*) Data is insufficient

Ans. (*a*): As pressure and quantity of gas in system are constant. So by gas equation $PV = nRT$

$$V \propto T \quad \text{as } n, R, P \text{ are constant}$$

$$\therefore \quad \frac{V_1}{T_1} = \text{constant} \quad \text{or slope of graph is constant}$$

$$V = \frac{nRT}{P}$$

$$\frac{dV}{dT} = \frac{nR}{P} \quad \text{so} \quad \frac{dV}{dt} \text{ increase when P decreases}$$

$$\frac{dV}{dT} \propto \frac{1}{P}$$

as slope of P_1 is smaller than P_2 Hence $P_1 > P_2$ verifies option (*a*).

Q13.6. 1 mole of H_2 gas is contained in a box of volume V = 1.00 m^3 at T = 300 K. The gas is heated to a temperature of T = 3000 K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal.)

(*a*) same as the pressure initially. (*b*) 2 times the pressure initially

(*c*) 10 times the pressure initially (*d*) 20 times the pressure initially.

Ans. (*c*): Pressure exerted by gas is due to rate of change of momentum (p) imparted by particles to wall and p depends on m and u of particle. When H_2 molecules at high temperature breaks up into atoms then number of particle become double but mass becomes half, as the velocity of gas molecule depends only on temperature. So the velocity of H_2 molecule and H atom at 3000 K remain same.

If the speed of H_2 molecule is u and mass of each atom is m_H so momentum imparted becomes $2(2m_H)u = 4m_H u$ but when H_2 molecule changes H atom then momentum imparted by 2, H atom $2(2m_H u) = 4m_H u$ so pressure does not changes due to change into atomic form at same temperature

But $$u \propto T \Rightarrow P \propto T$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{3000}{300}$$

$$P_2 = 10\, P_1$$

So pressure must be 10 times. According to law of conservation p does not change at same temperature. So option (*c*) verified not (*d*).

Q13.7. A vessel of volume V contains a mixture of 1 mole of hydrogen and 1 mole of oxygen (both considered as ideal). Let $f_1(v)\, dv$, denote the fraction of molecules with speed between v and $(v + dv)$, with $f_2(v)\, dv$, similarly for oxygen. Then

(*a*) $f_1(v) + f_2(v) = f(v)$ obeys the Maxwell's Distribution Law.

(*b*) $f_1(v), f_2(v)$ will obey the Maxwell's Distribution Law separately.

(*c*) neither $f_1(v)$ nor $f_2(v)$ will obey the Maxwell's distribution law.

(*d*) $f_2(v)$ and $f_1(v)$ will be the same.

Ans. (*c*): For function $f_1(v)$ the number of molecules n will have their speed $(v + dv)$.

For $f_1(\theta)$ and $f_2(v)$ number of molecules remains same (1 mole each) but due to mass difference their speed will be different, so both gases will obey the Maxwell's distribution law separately.

Q13.8. An inflated rubber balloon contains one mole of an ideal gas, has a pressure p, volume V and temperature T. If the temperature rises to 1.1 T, and the volume is increased to 1.05 V, the final pressure will be:

(*a*) 1.1 p (*b*) p

(*c*) less than p (*d*) between p and 1.1

Ans. (*d*): By gas equation

$$PV = nRT \qquad n, \text{R are constant for the system here}$$

$$\therefore \qquad \frac{PV}{T} = \text{constant} \quad \text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

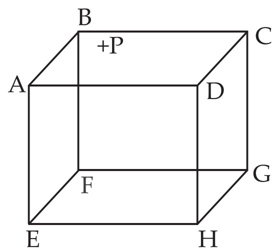
$$P_2 = \frac{P_1V_1}{T_1} \times \frac{T_2}{V_2} = \frac{pV\,1.1T}{T\,1.05\,V} = \frac{1.1}{1.05}p$$

$$= (1.0476)\,p$$

i.e. P_2 is between p and $1.1p$. So option (*d*) is verifies.

MULTIPLE CHOICE QUESTIONS-II

Q13.9. ABCDEFGH is a hollow cube made of an insulator (figure). Face ABCD has positive charge on it. Inside the cube, we have ionized hydrogen.



The usual kinetic theory expression for pressure

(*a*) will be valid.

(*b*) will not be valid since the ions would experience forces other than due to collisions with the walls.

(*c*) will not be valid since collisions with walls would not be elastic.

(*d*) will not be valid because isotropy is lost.

Ans. (*b, d*): Due to the presence of hydrogen ions, +ive charged wall ABCD there will be electrostatic force will acts apart of collision, so kinetic theory of gas will not be valid.

Due to the presence of ions in place of hydrogen molecules the isotropy will also be lost.

Q13.10. Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory $pV = \frac{2}{3}E$, E is

(*a*) the total energy per unit volume.

(*b*) only the translational part of energy because rotational energy is very small compared to the translational energy.

(*c*) only the translational part of the energy because during collisions with the wall pressure relates to change in linear momentum.

(*d*) the translational part of the energy because rotational energies of molecules can be of either sign and its average over all the molecules is zero.

Ans. (*c*): According to the postulate of kinetic theory of gases it assumed that pressure due to gas molecule is due to only perpendicular forces on wall due to motion of molecule, i.e. molecule stricking at other than at 90° will not be exert pressure.

So the pressure on the wall is due to only change in translational motion.

Hence $pV = \frac{2}{3}E$ represents the only translation part of energy.

Q13.11. In a diatomic molecule, the rotational energy at a given temperature.

(*a*) obeys Maxwell's distribution

(*b*) have the same value for all molecules

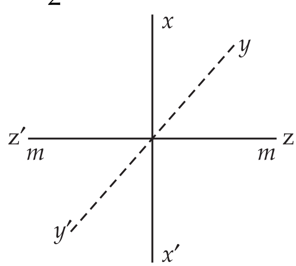
(*c*) equals the translational kinetic energy for each molecule

(*d*) is $\left(\frac{2}{3}\right)$rd the translational kinetic energy for each molecule

Ans. (*a, d*): Consider a diatomic molecule along *z*-axis so its rotational energy about *z*-axis is zero. So energy of diatomic molecule

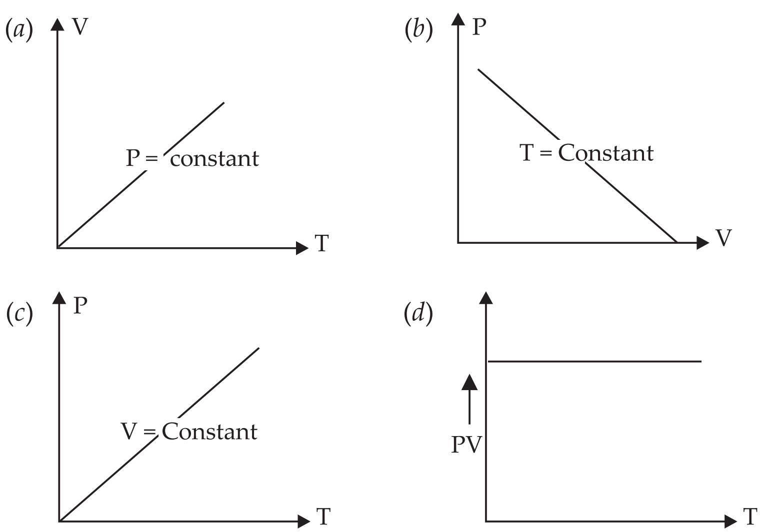
$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 \qquad (\because \quad I_z = 0)$$

The independent terms in the above expression is 5. As we can predict velocities of molecules by Maxwell's distribution, hence the above expression also obeys Maxwell's distribution. As 2 rotational and 3 translational energies are associated with each molecule.



So the rotational energy at given temperature is 2/3 of its translational KE of each molecule.

Q13.12. Which of the following diagrams (figure) depicts ideal gas behaviour?



Ans. (*a, c*): Gas equation for ideal gas

$$PV = nRT \quad n \text{ and R are constant for a system} \qquad (I)$$

(*a*) as P is constant

$\therefore \qquad V \propto T$ verifies the diagram (*a*).

(*b*) at constant temperature T = constant

PV = constant, graph must be parabola (rectangular). Hence rejects (*b*).

(*c*) at V = constant from (I)

$P \propto T$ i.e. straight line

Hence verifies the option (*c*).

(*d*) from graph $\frac{PV}{T}$ (slope) is constant

at $\qquad P = 0 \quad$ Constant K = 0

and $\qquad V = 0 \quad \Rightarrow \quad K = 0$

So graph must pass through origin.

Hence rejects option (*d*).

Q13.13. When an ideal gas compressed adiabatically, its temperature rises, the molecules on the average have more kinetic energy than before. The kinetic energy increases.

(*a*) because of collisions of moving parts with the wall only.

(*b*) because of collisions with the entire wall.

(*c*) because the molecules gets accelerated in their motion inside the volume.

(*d*) because of redistribution of energy amongst the molecules.

Ans. (*a*): As the ideal gas compress, then the mean free path becomes smaller so the number of collisions per second between the molecules and walls increases which increase the temperature of gas in turn KE of gas molecule increases.

VERY SHORT ANSWER TYPE QUESTIONS

Q13.14. Calculate the number of atoms in 39.4 g gold. Molar mass of gold is 197 gm mole^{-1}.

Ans. 197 gm gold has number of atoms = 6.023×10^{23}

1 gm gold will have number of gold atoms = $\dfrac{6.023 \times 10^{23}}{197}$

39.4 gm gold has number of Au atoms = $\dfrac{39.4 \times 6.023 \times 10^{23}}{197}$

$$= \frac{12.046 \times 10^{23}}{10} = 1.2046 \times 10^{23} \text{ atoms}$$

Q13.15. The volume of a given mass of a gas at 27°C and 1 atm is 100 cc. What will be its volume at 327°C?

Ans. By gas equation of ideal gas

$P_1 = 1$ atm $\qquad P_2 = 1$ atm $\quad V_1 = 100$ cc $\quad V_2 = ?$

$$T_1 = 273 + 27 = 300\text{ K} \quad T_2 = 327 + 273 = 600\text{ K}$$

$$\therefore \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{1 \times 100}{300} = \frac{1 \times V_2}{600} \Rightarrow V_2 = \frac{100 \times 600}{300} = 200\text{ cc}$$

Units of (P_1, P_2) and (V_1, V_2) must same separately but unit of T must be in only on Kelvin scale.

Q13.16. The molecules of a given mass of a gas have root mean square speeds of 100 ms^{-1} at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure?

Ans. $$v_{rms} = \sqrt{\frac{3RT}{M}}$$

M = Molar mass of gas, for a gas M is constant

$$\therefore \quad v_{rms} \propto \sqrt{T}$$

$$\frac{v_{1rms}}{v_{2rms}} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{100}{v_{2rms}} = \sqrt{\frac{300}{400}}$$

$v_{rms} = 100$ m/s

$T_1 = 27 + 273 = 300$ K

$v_{2rms} = ?$

$T_2 = 127 + 273 = 400$ K

$$v_{2rms} = \frac{100 \times \sqrt{400}}{\sqrt{300}} = \frac{100 \times 2 \times 10}{10\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{200 \times \sqrt{3}}{3} = \frac{200 \times 1.732}{3}$$

$$v_{2rms} = 115.4\text{ ms}^{-1}.$$

Q13.17. Two molecules of a gas have speeds of 9×10^6 ms^{-1} and 1×10^6 ms^{-1} respectively. What is the root mean square speed of these molecules?

Ans. $$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots v_n^2}{n}} \qquad \begin{pmatrix} v_1 = 9 \times 10^6\text{ ms}^{-1} \\ v_2 = 1 \times 10^6\text{ ms}^{-1} \end{pmatrix}$$

$$\therefore \quad v_{rms} = \sqrt{\frac{(9 \times 10^6)^2 + (1 \times 10^6)^2}{2}}$$

$$v_{rms} = \sqrt{\frac{81 \times 10^{12} + 1 \times 10^{12}}{2}} = \sqrt{\frac{10^{12}(81+1)}{2}}$$

$$= 10^6\sqrt{\frac{82}{2}} = 10^6\sqrt{41} = 10^6 \times 6.4$$

$$v_{rms} = 6.4 \times 10^6\text{ m/s.}$$

Q13.18. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T. Neglecting all vibrational modes, calculate the total internal energy of the system (oxygen has two rotational modes).

Ans. To find total energy of a given molecule of a gas we must find its degree of freedom. In molecule of oxygen it has 2 atom. So it has degree of freedom 3T + 2R = 5, so total internal energy = 5/2 RT per mole as gas O_2 is 2 mole so total internal energy of 2 mole oxygen = $\frac{2 \times 5}{2}$ RT = 5 RT

Neon gas is mono atomic so its degree of freedom is only 3 hence total internal energy = 3/2 RT per mole

So, total internal energy of 4 mole Ne = $\frac{4 \times 3}{2}$ RT = 6 RT

Total internal energy of 2 mole oxygen and 4 mole Ne

$$= 5RT + 6RT = 11\ RT$$

Q13.19. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1 Å and 2 Å. The gases may be considered under identical conditions of temperature, pressure and volume.

Ans. Mean free path $\boxed{\lambda = \frac{1}{\sqrt{2}\pi d^2 n}}$

n = number of molecules per unit volume, d = diameter of molecule as the condition for both gases are identical so n will be constant

$$\text{or} \quad \lambda \,\alpha\, \frac{1}{d^2} \quad \text{or} \quad \frac{\lambda_1}{\lambda_2} = \frac{d_2^2}{d_1^2} = \frac{(2)^2}{(1)^2}$$

$$\lambda_1 : \lambda_2 = 4 : 1.$$

SHORT ANSWER TYPE QUESTIONS

Q13.20. The container shown in figure has two chambers, separated by a partition, of volumes V_1 = 2.0 L and V_2 = 3.0 L. The chambers contains μ_1 = 4.0 and μ_2 = 5.0 moles of a gas at pressures P_1 = 1.00 atm and P_2 = 2.00 atm. Calculate the pressure after the partition is removed and mixture attains equilibrium.

Ans. For ideal gas equation

$$PV = \mu RT$$

For gases in chamber 1 and 2

$$P_1V_1 = \mu_1 RT_1$$
$$P_2V_2 = \mu_2 RT_2$$

P_1 = 1 atm	P_2 = 2 atm
V_1 = 2L	V_2 = 3L
T_1 = T	T_2 = T
μ_1 = 4	μ_2 = 5

When partition between gases removed then

$$\mu = \mu_1 + \mu_2 \quad \text{and} \quad V = V_1 + V_2$$

V_1	V_2
μ_1	μ_2
P_1	P_2

By the kinetic theory of gas

the kinetic translational energy = PV = 2/3 E per mole

So the KE (translational) by gas of μ_1 moles

$$P_1V_1 = \frac{2}{3}\mu_1 E_1, \quad \text{and} \quad P_2V_2 = \frac{2}{3}\mu_2 E_2$$

Adding both above

$$P_1V_1 + P_2V_2 = \frac{2}{3}\mu_1 E_1 + \frac{2}{3}\mu_2 E_2$$

or $$\mu_1 E_1 + \mu_2 E_2 = \frac{3}{2}(P_1V_1 + P_2V_2)$$

Combined effect, $PV = \frac{2}{3} E_{Total}$ per mole $= \frac{2}{3}\mu E$ per mole

$$P(V_1 + V_2) = \frac{2}{3}\left[\frac{3}{2}(P_1V_1 + P_2V_2)\right]$$

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2} = \frac{1.00 \times 2.0 + 2.00 \times 3.0}{2.0 + 3.0} \text{ atm}$$

$$P = \frac{2+6}{5} = \frac{8}{5} = 1.6 \text{ atm}.$$

Q13.21. A gas mixture consists of molecules of types A, B and C with masses $m_A > m_B > m_C$ at constant temperature and pressure. Rank the three types of molecules in decreasing order of (*a*) average K.E. (*b*) rms speeds.

Ans. (*a*)

$$v_{av} = \bar{v} = \sqrt{\frac{8K_BT}{\pi m}} = \sqrt{\frac{8RT}{\pi m}} = \sqrt{\frac{8PV}{\pi m}}$$

In this ques. as the temperature and pressure are same

$$v_{av} = \vec{v} \,\alpha\, \frac{1}{\sqrt{m}}$$

$\therefore$ $$v_C > v_B > v_A$$

$\therefore$ as $KE \,\alpha\, v^2$ and $KE \,\alpha\, m$

So, u will affect the KE more than mass of atom.

So KE of molecules in decreasing order is

$$KE_C > KE_B > KE_A$$

(b) Similarly the $v_{rms} = \sqrt{\frac{3K_BT}{m}}$

P, T are constant

$\therefore$ $$v_{rms} \,\alpha\, \frac{1}{\sqrt{m}}$$

$$m_A > m_B > m_C \quad \text{(given)}$$

$\therefore$ $$(v_{rms})_C > (v_{rms})_B > (v_{rms})_A.$$

Q13.22. We have 0.5 *g* of hydrogen gas in a cubic chamber of size 3 cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law, in the final state? (Hydrogen molecules can be consider as spheres of radius 1 Å)

Ans. Volume of 1 molecule $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (10^{-10})^3$

$$r = 1\ \text{Å} = 10^{-10}\ \text{m} \qquad \text{(Given)}$$

$\therefore$ Volume of 1 molecule $= 4 \times 1.05 \times 10^{-30}\ \text{m}^3 = 4.20 \times 10^{-30}\ \text{m}^3$

Number of mole is .5 g H_2 gas $= \frac{.5}{2} = .25$ mole [$\because$ H_2 has 2 mole]

$\therefore$ Number of H_2 molecules is .25 mole H_2 gas $N = .25 \times 6.023 \times 10^{23}$

$\therefore$ Volume of H_2 molecules in .25 mole

$$= .25 \times 6.023 \times 10^{23} \times 4.2 \times 10^{-30}\ \text{m}^3$$
$$= 1.05 \times 6.023 \times 10^{+23-30}$$
$$= 6.324 \times 10^{+23-30}$$

Volume of H_2 molecules $= 6.3 \times 10^{-7}\ \text{m}^3$

Now for ideal gas at constant temperature

$$P_i V_i = P_f V_f$$

$$V_f = \frac{P_i V_i}{P_f} = \frac{1}{100} \times (3 \times 10^{-2})^3$$ [$\because$ vol. of cube $V_i = (\text{side})^3$ and $P_i = 1$ atm at NTP]

$$V_f = \frac{27 \times 10^{-6}}{100} = 2.7 \times 10^{-5-2} = 2.7 \times 10^{-7}\ \text{m}^3$$

Hence on compression the volume of the gas of the order of nuclear force of interaction will play the role, as in kinetic theory of gas molecules do not interact each other so gas will not obey the ideal gas behaviour.

Q13.23. When air is pumped into a cycle tyre, both volume and pressure of the air in the tyre are increased. What about Boyle's law in this case?

Ans. According to Boyle's law; at a constant temperature the volume of gas is inversely proportional to pressure i.e. it is valid only for constant mass of gas. But in this case gas is pumped continuously in tyre, so number of moles of air increases. So Boyle's or gas laws does not valid in this case.

Q13.24. A balloon has 5.0 g mole of helium at 7°C. Calculate

(*a*) the number of atoms of helium in the balloon

(*b*) the total internal energy of the system.

Ans. For gas helium $n = 5$ mole

$$T = 7 + 273 = 280\ \text{K}$$

(*a*) Number of atoms of He in 5 mole $= 5 \times 6.023 \times 10^{23}$ atoms

$$= 30.115 \times 10^{23}\ \text{atoms}$$
$$= 3.0115 \times 10^{24}\ \text{He atoms.}$$

(*b*) He atom is mono atomic so degree of freedom is 3

So average kinetic energy $= \frac{3}{2} K_B T$ per molecule

$$= \frac{3}{2} K_B T \times \text{Number of He Atom}$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 280 \times 3.0115 \times 10^{24}$$

Total E of 15 mole of He = 1.74×10^4 J

Q13.25. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP.

Ans. Hydrogen molecule is diatomic so it has 3 translational degree of freedom and 2 rotational.

So degree of freedom in H_2 molecule = 3 + 2 = 5

For number of molecule in 1 cc

22.4 lit = 22400 cc H_2 gas at STP has = 6.023×10^{23} molecule

1 cc H_2 gas at STP has = $\frac{6.023}{22400} \times 10^{23}$ molecule = 2.688×10^{19}

So total degree of freedom = $5 \times 2.688 \times 10^{19}$

$= 13.440 \times 10^{19} = 1.344 \times 10^{20}$.

Q13.26. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.

Ans. As the gas is mono atomic so its degree of freedom, will be due to only translational motion, which is three

So if KE per molecule = $\frac{3}{2}$RT

When insulated container stops suddenly its KE is transferred to gas molecules in the form of translational KE, so increase in the absolute temperature of gas let it be ΔT if n = moles of gas

So increase in translational KE = $n \cdot \frac{3}{2} R\Delta T$

KE = increased of molecule due to velocity $v_0 = \frac{1}{2}(mn)v_0^2$

$$\therefore \quad \frac{mn}{2} v_0^2 = \frac{3}{2} nR\Delta T$$

$$\Delta T = \frac{mnv_0^2 \times 2}{2 \times 3 \times nR} = \frac{mv_0^2}{3R}.$$

LONG ANSWER TYPE QUESTIONS

Q13.27. Explain why

(*a*) there is no atmosphere on moon?

(*b*) there is a fall in temperature with increase in altitude?

Ans. (*a*) As acceleration due to gravity of moon is 1/6th of g on earth.

So the escape velocity on moon $V_{es} = \sqrt{2gR}$ = 2.38 km/s

m = Mass of hydrogen as H_2 is lightest gas = 1.67×10^{-24} kg

$$v_{rms} = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{1.67 \times 10^{-24}}} = 2.72 \text{ km/s}$$

Due small gravitational force and v_{rms} is greater than escape velocity so molecule of air can escape out.

As the distance of moon from sun is approximately equal to that of earth so the intensity of energy of sun reaches to moon is larger due to lower density of atmosphere, distance become smaller than earth when moon is towards sun during its rotation around earth.

Due to this (sun light), rms speed of molecule increase and some of them can speed up more than escape velocity and so probability of escaping out increased.

Hence over a long time moon has lost most of its atmosphere.

(*b*) The temperature of atmosphere is due to the KE kinetic energy of air molecule.

Due to lower atmospheric pressure at higher altitude molecules of air rises up so their potential energy increase in turn the kinetic energy decrease results the decrease in temperature.

Due to lower atmospheric pressure at higher altitude the gas expands and gives cooling effect and so decrease the temperature.

Q13.28. Consider an ideal gas with following distribution of speeds.

Speed m/s	200	400	600	800	1000
% of molecules	10	20	40	20	10

(*i*) Calculate V_{rms} and hence T. ($m = 3 \times 10^{-26}$ kg)

(*ii*) If all the molecules with speed 1000 m/s escape from the system. Calculate new V_{rms} and hence T.

Ans. $$v_{rms}^2 = \frac{n_1 v_1^2 + n_2 v_2^2 \cdots n_n v_n^2}{n_1 + n_2 + n_3 \cdots n_n}$$

$$v_{rms}^2 = \frac{10 \times (200)^2 + 20 \times (400)^2 + 40(600)^2 + 20 \times (800)^2 + 10(1000)^2}{10 + 20 + 40 + 20 + 10}$$

$$v_{rms}^2 = \frac{10^5[1 \times 2^2 + 2 \times (4)^2 + 4 \times 6^2 + 2 \times 8^2 + 1 \times 10^2]}{100}$$

$$v_{rms}^2 = \frac{10^5[4 + 32 + 144 + 128 + 100]}{100} = 10^3[408] \qquad \text{(I)}$$

$$v_{rms} = \sqrt{10^4 \times 40.8} = 10^2 \times 6.39 \text{ m/s}$$

$$\frac{1}{2} m\, v_{rms}^2 = \frac{3}{2} K_B T$$

$$T = \frac{m v_{rms}^2}{3\, K_B} = \frac{3 \times 10^{-26} \times 10^5 \times 4.08}{3 \times 1.38 \times 10^{-23}} = \frac{204 \times 10^{-23} \times 10^2}{69 \times 10^{-23}}$$

$$T = 2.96 \times 10^2 = 296 \text{ K}$$

(*ii*) When the molecules of 1000 m/s escape out then

$$v_{rms}^2 = \frac{10 \times (200)^2 + 20 \times (400)^2 + 40(600)^2 + 20(800)^2}{10 + 20 + 40 + 20}$$

$$v_{rms}^2 = \frac{10^5[1\times 2^2 + 2\times 4^2 + 4\times 6^2 + 2\times 8^2]}{90}$$

$$v_{rms}^2 = \frac{10^5[4+32+144+128]}{90}$$

$$v_{rms} = \sqrt{\frac{10^5[308]}{90}} = \sqrt{\frac{10^4}{9}\times 308} = \frac{100}{3}\sqrt{308}$$

$$= 33.33 \times 17.55 \cong 585 \text{ m/s}$$

$$\text{T} = \frac{1}{3}\frac{mv_{rms}^2}{\text{K}_\text{B}} = \frac{3\times 10^{-26}\times (585)^2}{3\times 1.38\times 10^{-23}} = \frac{(585)^2}{138}\times 10^{-24+23}$$

$$\text{T} = 4.24\times 10^{-1}\times 585 = 248.04 \text{ K}$$

Q13.29. Ten small planes are flying at the speed of 150 km/h in total darkness in an air space that is $20 \times 20 \times 1.5$ km^3 in volume. You are in one of the planes, flying at random within this space with no way of knowing where the other planes are. On the average about how long a time will elapse between near collision with your plane. Assume for this rough computation that safety region around the plane can be approximated by a sphere of radius 10 m.

Ans. Planes can be considered as the motion of molecules in confined space. Time of relaxation mean free path (λ) is distances between two planes travelled between the collision or just to avoid accident.

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{\lambda}{v} = \frac{1}{\sqrt{2}n\cdot \pi d^2\cdot v}$$

$$n = \text{number of particles per unit volume V} = \frac{\text{N}}{\text{Volume}}$$

$$n = \frac{10}{20\times 20\times 1.5 \text{ km}^3} = 0.0167 \text{ km}^{-3}$$

$$d = 2\times 10 \text{ m} = 20 \text{ m} = 20\times 10^{-3} \text{ km}, \; v = 150 \text{ km/hr}$$

$$\therefore \quad \text{time} = \frac{1}{\sqrt{2}n\pi d^2 v}$$

$$= \frac{1}{1.414\times 0.0167\times 3.14\times 20\times 20\times 10^{-6}\times 150}$$

$$t = \frac{10^6}{4448.8} = 225 \text{ hrs}.$$

Q13.30. A box of 1.00 m^3 is filled with nitrogen at 1.5 atm at 300 K. The box has a hole of an area 0.010 mm^2. How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm.

Ans. Volume of box = 1 m^3 = V_1

Initial pressure = 1.5 atm = P_1

Final pressure = 1.5 – 0.1 = 1.4 atm = P'_2

Air pressure out side box = P_2 = 1 atm

initial temperature T_1 = 300 K

final temperature T_2 = 300 K

a = area of hole = 0.01 mm^2 = 0.01×10^{-6} m^2 = 10^{-8} m^2

initial pressure difference between tyre and atmosphere $\Delta P = (1.5 - 1)$ atm

$$\text{mass of a } N_2 \text{ gas molecule} = \frac{0.028 \text{ Kg}}{6.023 \times 10^{23}} = 46.5 \times 10^{-27} \text{ Kg}$$

$$K_B = 1.38 \times 10^{-23}$$

Let ρ_{n1} is the initial number of N_2 gas molecule per unit volume in time Δt

Let v_{ix} is the speed of molecules along x axis

Number of molecule colliding in time Δt on a wall of cube

$$= \frac{1}{2}\rho_{ni}[(v_{ix})\Delta t]A$$

$\frac{1}{2}$ is multiplied as other $\frac{1}{2}$ molecule will strike to opposite wall

$$v_{rms}^2 \,(N_2 \text{ molecule}) = v_{ix}^2 + v_{1y}^2 + v_{1z}^2$$

$$\because \quad |v_x| = |v_{iy}| = |v_{iz}|$$

$$\text{then} \quad v_{rms}^2 = 3v_{ix}^2$$

$$\text{KE of gas molecule} = \frac{3}{2}K_B T$$

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}K_B T$$

$$m\,3v_{ix}^2 = 3K_B T$$

$$v_{ix} = \sqrt{\frac{K_B T}{m}} \qquad \text{(A)}$$

Number of N_2 gas molecule striking to a wall in Δt time

$$\text{Outward} = \frac{1}{2}\rho_{ni}\sqrt{\frac{K_B T}{m}}\,\Delta t.a \quad \text{out}$$

Temperature inside the box and air are equal to T

The number of air molecule striking to hole in Δt

$$\text{inward} = \frac{1}{2}\rho_{n2}\sqrt{\frac{K_B T}{m}}\,\Delta t \cdot a \text{ inward}$$

$$\rho_{n2} = \text{density of air molecule}$$

Net number of molecule (going outward)

$$= \frac{1}{2}\rho_{n1}\sqrt{\frac{K_B T}{m}}\,\Delta ta - \frac{1}{2}\rho_{n2}\sqrt{\frac{K_B T}{m}}\,\Delta t \cdot a$$

Net number of molecules going out from hole in Δt time

$$= \frac{1}{2}[\rho_{n1} - \rho_{n2}]\sqrt{\frac{K_B T}{m}}\,.\Delta t\,.a \qquad \text{(I)}$$

$$\text{Gas equation } P_1 V = \mu RT \quad \Rightarrow \quad \mu = \frac{P_1 V}{RT}$$

as for box $\frac{\mu}{V} = \frac{P_1}{RT}$ (μ = No. of moles of gas in box)

$$\rho_{n1} = \frac{\text{N (Total no. of molecule in box)}}{\text{Volume of box}} = \frac{\mu N_A}{V}$$

$$= \frac{P_1 N_A}{RT} \text{ per unit volume} \qquad \text{(II)}$$

Let after time T pressure reduced by 0.1 and becomes

$(1.5 - .1) = 1.4 \text{ atm} = P'_2$

then final new density of N_A molecule $= \rho'_{n1}$

$$\rho'_{n1} = \frac{P'_2 N_A}{RT} \quad \text{per unit volume} \qquad \text{(III)}$$

Net number of molecules going out from volume V

$$= (\rho_{n1} - \rho'_{n1})\, V = \frac{P_1 N_A}{RT} V - \frac{P'_2 N_A}{RT} V$$

$$= \frac{N_A V}{RT} [P_1 - P'_2] \qquad \text{(IV)} \quad \text{(from II, III)}$$

P'_2 = final pressure of box.

From (I) total number of molecule going out in time τ from hole

$$= \frac{1}{2}[\rho_{n1} - \rho_{n2}] \sqrt{\frac{K_B T}{m}}\, \tau . a$$

$$\rho_{n1} - \rho_{n2} = \frac{P_1 N_A}{RT} - \frac{P_2 N_A}{RT}$$

$$\therefore \qquad \rho_{n1} - \rho_{n2} = \frac{N_A}{RT}[P_1 - P_2] \qquad (P_2 = \text{Press of air out of box})$$

Net number of molecule going out in τ time from above

$$= \frac{1}{2} \frac{N_A}{RT} [P_1 - P_2] \sqrt{\frac{K_B T}{m}} \cdot \tau\, a \qquad \text{(V)}$$

From (V) and IV

$$\frac{N_A V}{RT}(P_1 - P'_2) = \frac{1}{2} \frac{N_A}{RT} (P_1 - P_2) \sqrt{\frac{K_B T}{m}} \cdot \tau a$$

$$\tau = \frac{N_A V}{RT}(P_1 - P'_2) \frac{2RT}{N_A} \frac{1}{(P_1 - P_2)} \sqrt{\frac{m}{K_B T}} \cdot \frac{1}{a}$$

$$\tau = \frac{2(P_1 - P'_2)}{(P_1 - P_2)} \cdot \frac{V}{a} \sqrt{\frac{m}{K_B T}}$$

$$= \frac{2[1.5 - 1.4]}{(1.5 - 1)} \frac{1}{10^{-8}} \sqrt{\frac{46.5 \times 10^{-27}}{1.38 \times 10^{-23} \times 300}}$$

$$= \frac{2 \times 0.1}{0.5 \times 10^{-8}} \sqrt{\frac{4650 \times 10^{-27+23-2}}{138 \times 3}}$$

$$= 0.4 \times 10^{+8} \sqrt{\frac{775 \times 10^{-6}}{69}}$$

$$= 0.4 \times 10^8 \times 10^{-3} \times \sqrt{11.23} = 0.4 \times 10^5 \times 3.35$$

$$\tau = 1.34 \times 10^5 \text{ seconds}$$

Q13.31. Consider a rectangular block of wood moving with a velocity v_0 in a gas at temperature T and mass density ρ. Assume the velocity is along x-axis and area of cross-section of the block perpendicular to v_0 is A. Show that the drag force on the block is $4\rho A v_0 \sqrt{\frac{kT}{m}}$, where m is the mass of the gas molecule.

Ans. Let ρ_m is the number of molecule per unit volume *i.e.* ρ_m is molecular density per unit volume.

$$v = v_{rms} \text{ is velocity of gas molecules}$$

When box moves in gas the molecules of gas strike to front face in opposite direction and on back face in same direction as $v >> v_0$ (box) so relative velocity of molecule in front side of block = $(v + v_0)$
and relative velocity on back face = $(v - v_0)$
Change in momentum by a molecule on front face = $2m(v + v_0)$
Change in momentum by a molecule on back side = $2m(v - v_0)$
Number of molecule striking on front face in Δt time

$$= \frac{1}{2} \text{Vol} \times \text{molecular density/vol.}$$

to front face $$= \frac{1}{2}[A \cdot (v + v_0)\Delta t]\rho_m$$

Number of molecules striking to front face

$$N_F = \frac{1}{2}(v + v_0) A \rho_m \Delta t$$

Similarly as the speed of molecule and block are same so number of molecule striking on backend face $N_B = \frac{1}{2}(v - v_0) A \rho_m \Delta t$

Total change in momentum due to striking the molecule on front face

$$p_F = 2m(v + v_0) \cdot N_F = 2m(v + v_0) \times \frac{1}{2}(v + v_0) A \cdot \rho_m \Delta t$$

$$p_F = -m(v + v_0)^2 A\rho_m \Delta t \text{ (Backward direction)}$$

So rate of change of momentum on front face is equal to the force

$$F_F = -m(v + v_0)^2 A\rho_m \text{ in Backward direction}$$

Similarly force on back end $F_B = +m(v - v_0)^2 A\rho_m$

$$\text{Net dragging force} = -m(v + v_0)^2 A\rho_m + m(v - v_0)^2 A\rho_m$$
$$= -mA\rho_m[(v + v_0)^2 - (v - v_0)^2]$$
$$= -m A\rho_m[v^2 + v_0^2 + 2vv_0 - (v^2 + v_0^2 - 2v \cdot v_0)]$$
$$= -mA\rho_m 4v \cdot v_0$$

So magnitude of dragging force due to gas molecule = $4 m v v_0 A \rho_m$

KE of gas molecule = $\frac{1}{2} mv^2 = \frac{3}{2} K_B T$

$\therefore$ $$v = \sqrt{\frac{K_B T}{m}}$$ [using equ. (A) of Q.13.30]

$\therefore$ Dragging force becomes = $4m A \rho_m v_0 \sqrt{\frac{K_B T}{m}}$.

❑❑❑

14 Oscillations

MULTIPLE CHOICE QUESTIONS-I

Q14.1. The displacement of a particle is represented by the equation $y = 3\cos\left[\frac{\pi}{4} - 2\omega t\right]$ the motion of the particle is

(*a*) Simple harmonic with period $\frac{2\pi}{\omega}$

(*b*) Simple harmonic with period $\frac{\pi}{\omega}$

(*c*) Periodic but not simple harmonic.

(*d*) Non-periodic

Ans. (*b*): In simple harmonic motion acceleration (or force) is directly proportional to the negative of displacement of particle i.e., $a \propto -y$

$$y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right) \qquad ...(i)$$

$$v = \frac{dy}{dt} = -3(-2\omega)\sin\left(\frac{\pi}{4} - 2\omega t\right)$$

$$a = \frac{dv}{dt} = (+6\omega)(-2\omega)\cos\left(\frac{\pi}{4} - 2\omega t\right)$$

$$a = -4\omega^2\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$a = -4\omega^2 y$$

$$a \propto -y$$

Hence the motion is simple harmonic motion.

A simple harmonic motion is always periodic. So the motion is periodic simple harmonic motion.

$$\text{Time period} = T' = \frac{2\pi}{\omega'}$$

$$\therefore \qquad T' = \frac{2\pi}{2\omega} = \frac{\pi}{\omega} \qquad \omega' = 2\omega \text{ from } (i)$$

Hence the motion is simple harmonic with time period $\frac{\pi}{\omega}$.

Verifies the option (*b*).

Q14.2. The displacement of a particle is represented by the equation $y = \sin^3 \omega t$ the motion is

(*a*) non-periodic

(*b*) periodic but not simple harmonic

(*c*) Simple harmonic with period $\frac{2\pi}{\omega}$

(*d*) Simple harmonic with period $\frac{\pi}{\omega}$

Ans. (*b*): A motion will be harmonic if $a \propto$ displacement and a simple harmonic motion is always periodic but all simple harmonic motion are periodic but all periodic are not harmonic.

$$y = \sin^3 \omega t$$

$$\because \quad \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$4 \sin^3 \theta = 3 \sin \theta - \sin 3\theta$$

$$\sin^3 \theta = \frac{3 \sin \theta - \sin 3\theta}{4}$$

$$\therefore \quad y = \frac{3 \sin \omega t - \sin 3\omega t}{4}$$

$$v = \frac{dy}{dt} = \frac{1}{4}[3\omega \cos \omega t - 3\omega \cos 3\omega t]$$

$$v = \frac{3\omega}{4}[\cos \omega t - \cos 3\omega t]$$

$$a = \frac{3\omega}{4}[-\omega \sin \omega t + 3\omega \sin 3\omega t]$$

a is not directly proportional to y. So motion is **not harmonic**.

$$y(t) = \sin^3 \omega t$$

$$y(t + \mathrm{T}) = \sin^3[\omega(t + \mathrm{T})] = \sin^3 [(\omega t + \omega \mathrm{T})]$$

$$= \sin^3 \left[\frac{2\pi}{\mathrm{T}}\mathrm{T} + \omega t\right] = \sin^3 (2\pi + \omega t)$$

$$= \sin^3 \omega t$$

$y(t + \mathrm{T}) = y(t)$ so function is periodic.

Hence given function of motion is periodic but not harmonic. Verifies the option (*b*).

Q14.3. The relation between acceleration and displacement of four particles are given below:

(*a*) $a_x = +2x$ (*b*) $a_x = 2x^2$ (*c*) $a_x = -2x^2$ (*d*) $a_x = -2x$

Which one of the particle is exempting simple harmonic motion?

Ans. (*d*): The acceleration of particle must be opposite (negative) of restoring force. So

$$\mathrm{F} = ma$$

$$\mathrm{F} = m(-2x) = -2mx, \text{ so } \mathrm{F} \propto -x \text{ so } a_x = -2x \text{ is}$$

Verified i.e., option (*d*).

Q14.4. Motion of an oscillating liquid column in a U-tube is

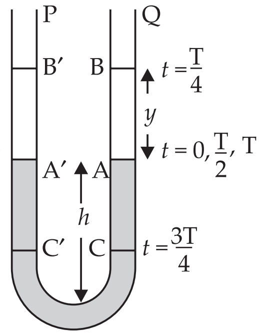
(*a*) periodic but not simple harmonic.
(*b*) non periodic.
(*c*) simple harmonic and time period is independent of the density of the liquid.
(*d*) simple harmonic and time period is directly proportional to the density of the liquid.

Ans. (*c*): Consider a U-tube filled with a liquid of density ρ upto height h as shown in figure. When liquid column lifted upto height

y from A to B in arm Q. The liquid level in arm P becomes at C′. So the difference between the height of two columns are

$$= AB + A'C = y + y = 2y$$

restoring force due to gravity on liquid column $= -mg$



$$F = -V.\rho.g = -A.2y\,\rho\,g$$

A = Area of cross section of tube

$$F = -A\,2y\,\rho\,g$$

as restoring force at A opposite to gravitational force as liquid is lifted against ωt (mg) $\mu k = 2A\,\rho\,g$ then $F \propto -y$, so motion is simple harmonic.

$$T = 2\pi\sqrt{\frac{m\,(\text{inertia})}{k\,(\text{spring})}} = 2\pi\sqrt{\frac{A\,(2h)\,\rho}{2\,A\,\rho\,g}}$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

Time period is independent of density and motion is harmonic. So verifies the option (*c*).

Q14.5. A particle is acted simultaneously perpendicular simple harmonic motions $x = a\cos\omega t$ and $y = a\sin\omega t$. The trajectory of motion of the particle will be

(*a*) an ellipse (*b*) a parabola (*c*) a circle (*d*) a straight line

Ans. (*c*): Resultant displacement is $x + y$

$$x = a\cos\omega t \quad ...(i)$$

$$y = a\sin\omega t \quad ...(ii)$$

$$\text{Displacement} = a\cos\omega t + a\sin\omega t$$

$$y' = a(\cos\omega t + \sin\omega t)$$

$$y' = a\sqrt{2}\left[\frac{\cos\omega t}{\sqrt{2}} + \frac{\sin\omega t}{\sqrt{2}}\right]$$

$$y' = a\sqrt{2}\,[\cos\omega t\cos 45° + \sin\omega t\sin 45°]$$

$$y' = a\sqrt{2}\cos(\omega t - 45°)$$

So the displacement is not straight line, not parabola.

Now, squaring and adding (*i*), (*ii*)

$$x^2 + y^2 = a^2\cos^2\omega t + a^2\sin^2\omega t = a^2[\cos^2\omega t + \sin^2\omega t]$$

$$x^2 + y^2 = a^2$$

It is the equation of circle, so motion is circular or a circle of radius a, independent of time. Verifies the option (*c*).

Q14.6. The displacement of particle varies with time according to the relation $y = a\sin\omega t + b\cos\omega t$

(*a*) The motion is oscillatory but not SHM

(*b*) The motion is SHM with amplitude $(a + b)$

(*c*) The motion is SHM with amplitude $(a^2 + b^2)$

(*d*) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

Ans. (*d*): $$y = a \sin \omega t + b \cos \omega t \quad \ldots(i)$$

Let $$a = A \sin \theta \quad \ldots(ii) \quad \text{and } b = A \cos \theta \quad \ldots(iii)$$

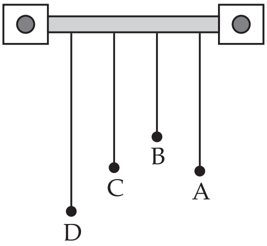
$$y = A \sin \theta \sin \omega t + A \cos \omega t \cos \theta$$
$$= A [\sin \omega t \sin \theta + \cos \omega t \cos \theta]$$
$$y = A \cos (\omega t - \theta) \quad \ldots(iv)$$
$$\frac{dy}{dt} = - A\omega \sin (\omega t - \theta)$$
$$\frac{d^2y}{dt^2} = - A\omega^2 \cos (\omega t - \theta)$$
$$\frac{d^2y}{dt^2} = - \omega^2 y \quad \text{from } (iv)$$
$$\frac{d^2y}{dt^2} \propto - y. \text{ So motion is SHM}$$

Squaring and adding (*ii*) and (*iii*)

$$a^2 + b^2 = A^2 \sin^2 \theta + A^2 \cos^2 \theta$$
$$= A^2 (\sin^2 \theta + \cos^2 \theta)$$
$$a^2 + b^2 = A^2$$
$$A = \sqrt{a^2 + b^2} \quad \text{(amplitude)}$$

Hence, verifies the option (*d*).

Q14.7. Four pendulums A, B, C and D are suspended from the same elastic support as shown in figure. A and C are of the same length, while B is smaller than A and D is larger than A. If A is given a transverse displacement.



(*a*) D will vibrate with maximum amplitude.

(*b*) C will vibrate with maximum amplitude.

(*c*) B will vibrate with maximum amplitude.

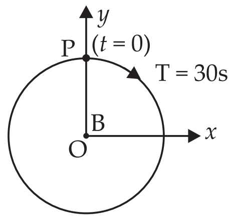
(*d*) All the four will oscillate with equal amplitude.

Ans. (*b*): When pendulum vibrate with transverse vibration then

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad l = \text{length of pendulum A and C.}$$

The disturbance produce in elastic rigid support of time period T, which is transmitted by support to all pendulum B, C, D, but the frequency or time period of C is same as of A. So a periodic force of period T produces resonance in C and C will vibrate with maximum as in resonance. Hence, verifies the option (*b*).

Q14.8. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution, and initial position are indicated on the figure. The simple harmonic motion of the X-projection of the radius vector of the rotating particle P is

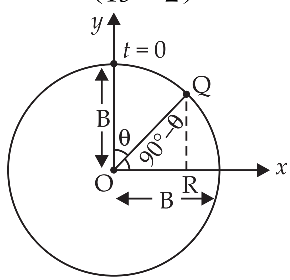


(*a*) $x(t) = \text{B}\sin\left(\dfrac{2\pi t}{30}\right)$ (*b*) $x(t) = \text{B}\cos\left(\dfrac{\pi t}{15}\right)$

(*c*) $x(t) = \text{B}\sin\left(\dfrac{\pi t}{15} + \dfrac{\pi}{2}\right)$ (*d*) $x(t) = \text{B}\cos\left(\dfrac{\pi t}{15} + \dfrac{\pi}{2}\right)$

Ans. (*a*): As the particle P is executing circular motion with radius B.

Let particle P is at Q at instant t, foot of perpendicular on X-axis is at R vector OQ makes < θ with its zero position not P displacement of particle for O to R



$$x = \text{OQ}\cos(90° - \theta)$$

$$x = \text{OQ}\sin\theta = \text{B}\sin\omega t \qquad \because \theta = \omega t$$

$$x = \text{B}\sin\frac{2\pi}{\text{T}}t \qquad \text{T} = 30 \text{ seconds (given)}$$

$$\therefore \quad x = \text{B}\sin\left(\frac{2\pi}{30}t\right) \text{ Verifies option } (a).$$

Q14.9. The equation of motion of a particle is $x = a\cos(\alpha t)^2$. The motion is

(*a*) periodic but not oscillatory (*b*) periodic and oscillatory

(*c*) oscillatory but not periodic (*d*) neither periodic nor oscillatory

Ans. (*c*) oscillatory but not periodic

Q14.10. A particle executing SHM has a maximum speed 30 cm s^{-1} and a maximum acceleration of 60 cm s^{-2} the period of oscillation is

(*a*) π sec (*b*) $\dfrac{\pi}{2}$ sec (*c*) 2π sec (*d*) $\dfrac{\pi}{t}$ sec

Ans. (*a*): Consider a SHM equation of motion

$$y = a\sin\omega t$$

$$v = \frac{dy}{dt} = a\omega\cos\omega t \qquad ...(i)$$

$$a = \frac{dv}{dt} = -a\omega\sin\omega t$$

$$a = -a\omega^2\sin\omega t \qquad ...(ii)$$

$$v_{max} = 30 \text{ cm/s} \quad \text{(given)}$$

v_{max} from (*i*)

$$v_{max} = a\omega$$

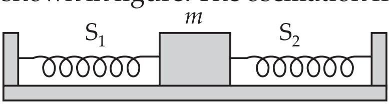
$$\therefore \quad a\omega = 30 \qquad ...(iii)$$

$$a_{max} = a\omega^2 \quad \text{from } (ii)$$

$$60 = a\omega^2 \quad ...(iv) \text{ (given)}$$
$$60 = \omega \times 30$$
$$\omega = 2 \text{ rad/s}$$
$$\frac{2\pi}{T} = 2$$
$$T = \pi. \text{ Verifies option } (a).$$

MULTIPLE CHOICE QUESTIONS-II MORE THAN ONE OPTION

Q14.11. When a mass is connected individually to two springs S_1 and S_2, the oscillation frequencies are ν_1 and ν_2. If the same is attached to the two springs as shown in figure. The oscillation frequency would be



(*a*) $\nu_1 + \nu_2$ (*b*) $\sqrt{\nu_1^2 + \nu_2^2}$ (*c*) $\left[\frac{1}{\nu_1} + \frac{1}{\nu_2}\right]^{-1}$ (*d*) $\sqrt{\nu_1^2 - \nu_2^2}$

Ans. (*b*): When a mass (*m*) is connected to a spring on a horizontal frictionless surface then their frequencies are

$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \quad ...(i) \text{ and } \nu_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}}$$

Now, the springs are in parallel because one end of both spring are connected with rigid body and other end to body. So their equivalent spring constant becomes $k_p = k_1 + k_2$

and frequency $$\nu_p = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$$

$$\nu_p = \frac{1}{2\pi}\left[\frac{k_1}{m} + \frac{k_2}{m}\right]^{1/2}$$

From (*i*) $$\frac{k_1}{m} = (2\pi\nu_1)^2 = 4\pi^2\nu_1^2 \text{ and } \frac{k_2}{m} = 4\pi^2\nu_2^2$$

$$\nu_p = \frac{1}{2\pi}[4\pi^2\nu_1^2 + 4\pi^2\nu_2^2]^{1/2} = \frac{2\pi}{2\pi}[\nu_1^2 + \nu_2^2]^{1/2}$$

$$\nu_p = \sqrt{\nu_1^2 + \nu_2^2}$$

Hence, verifies the option (*b*).

Q14.12. The rotation of earth about it's axis is

(*a*) periodic motion
(*b*) simple harmonic motion
(*c*) periodic but not simple harmonic motion
(*d*) non-periodic motion

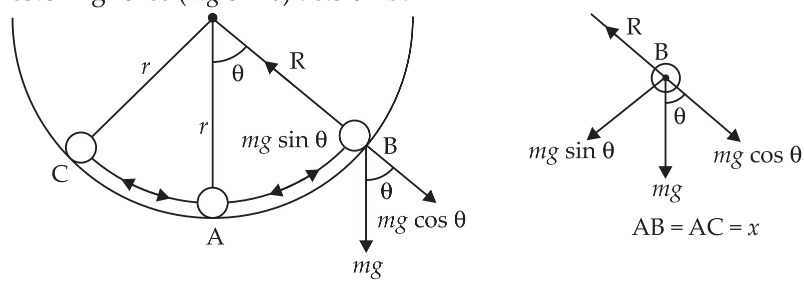
Ans. (*a, c*): The motion of earth about its own axis is circular and complete its one complete revolution in regular interval of time. So **it is periodic**. But motion is not about a fixed point from which we can measure it's displacement or about which it moves both side so it is **not simple harmonic motion**. So verifies the option (*a*) and (*c*).

Q14.13. The motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower point is

(*a*) simple harmonic motion (*b*) non periodic motion

(*c*) periodic motion (*d*) periodic but not SHM

Ans. (*a, c*): Let the ball is lifted from A to B inside the smooth bowl and released then it moves B to A then A to C, and comeback to A from C and finally to B always, so motion is periodic just like simple pendulum. Reaction force R (by bowl) is balanced by mg sin θ and a restoring force (mg sin θ) acts on ball



So $$ma = mg \sin\theta$$

$$a = g \sin\theta$$

or $$\frac{d^2x}{dt^2} = -g \sin\theta \text{ (when ball moves up)}$$

$$\frac{d^2x}{dt^2} = -g\frac{x}{r}$$

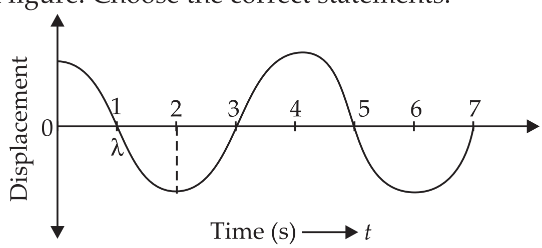
(–) sign is because when ball moves upward displacement and force (or *a*) are opposite

∴ $$\frac{d^2x}{dt^2} \propto (-x) \text{ So motion is SHM.}$$

$$\omega = \sqrt{\frac{g}{r}} \quad \text{or} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g}}$$

So motion is periodic and SHM, verifies the options (*a*) and (*c*).

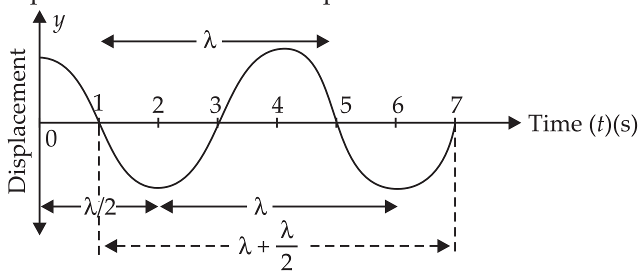
Q14.14. Displacement versus time curve for a particle executing SHM is shown in figure. Choose the correct statements.



(*a*) Phase of the oscillator is same at $t = 0$ s and $t = 2$ s
(*b*) Phase of oscillator is same at $t = 2$ s and $t = 6$ s
(*c*) Phase of oscillator is same at $t = 1$ s and $t = 7$ s
(*d*) Phase of oscillator is same at $t = 1$ s and $t = 5$ s.

Ans. (*b, d*): Two particles are said to be in same phase if the mode of vibration is same i.e., their distance will be $n\lambda$ ($n = 1, 2, 3, ...$)

(*a*) Distance between particles at $t = 0$ and $t = 2$ is $\frac{\lambda}{2}$.
So particles are not in same phase.



(*b*) As from figure the particles at $t = 2$ sec and 6 sec are at distance λ, so are in same phase.

(*c*) Particles at $t = 1$, $t = 7$ are the distance $\lambda + \frac{\lambda}{2} = \frac{3\lambda}{2}$ so are not in phase.

(*d*) Particles at $t = 1$ sec and 5 sec are at distance $= \lambda$ so are in same phase.
Verifies the options (*b, d*).

Q14.15. Which of the following statements is/are true for a simple harmonic motion oscillator?

(*a*) Force acting is directly proportional to displacement from mean position and opposite to it.
(*b*) Motion is periodic.
(*c*) Acceleration of the oscillator is constant.
(*d*) The velocity is periodic.

Ans. (*a, b, d*): Consider a SHM $x = a \sin \omega t$. ...(*i*)

$$v = \frac{dx}{dt} = a\omega \cos \omega t \quad ...(ii)$$

Acceleration
$$A = \frac{dv}{dt} = \frac{d}{dt}(a\omega \cos \omega t) = a\omega\,(-\,\omega \sin \omega t)$$

$\therefore$
$$A = -\,a\omega^2 \sin \omega t \quad \text{or} \quad A = -\,\omega^2 x$$
$$mA = -\,m\omega^2 x$$
$$F \propto -\,(x)$$

Hence, force is directly proportional to displacement opposite the direction of displacement. We can prove

$$x(t) = x(t + T)$$

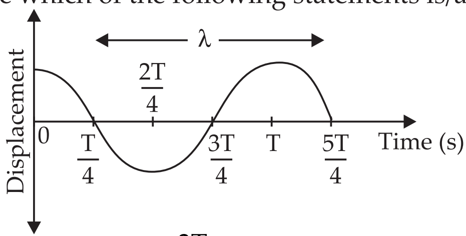
So motion is periodic and SHM.

From (*ii*) v is also periodic as we can prove

$$v(t) = v(t + T)$$

Hence, verifies the option (*a*, *b*, *d*).

Q14.16. The displacement-time graph of the particle executing SHM is shown in figure which of the following statements is/are true.



(*a*) The force is zero at $t = \frac{3T}{4}$

(*b*) The acceleration is maximum at $t = \frac{4T}{4}$

(*c*) The velocity is maximum at $t = \frac{T}{4}$

(*d*) The P.E. is equal to K.E. of oscillator at $t = \frac{T}{2}$

Ans. (*a, b, c*): (*a*) At $t = \frac{3T}{4}$ particle is at it's mean position so force acting on it is zero, but it continue the motion due to inertia of mass, here $a = 0$, so F = 0.

(*b*) At $t = \frac{4T}{4} = T$, particle's velocity changes increasing to decreases so maximum change in velocity occur at T. As acceleration = $\frac{\text{Change in velocity}}{\text{Time}}$, so acceleration is maximum here.

(*c*) At $t = \frac{T}{4}$ it is at its mean position where the velocity is maximum as no retarding force on it.

(*d*) $t = \frac{T}{2} = \frac{2T}{4}$, the particle has K.E. = 0. So KE ≠ PE.

Hence, verifies the options (*a, b, c*).

Q14.17. A body is performing SHM, then its

(*a*) Average total energy per cycle is equal to it's maximum kinetic energy.

(*b*) Average kinetic energy per cycle is equal to half of it's maximum kinetic energy.

(*c*) Mean velocity over a complete cycle is equal to $\frac{2}{\pi}$ times of its maximum velocity.

(*d*) Root mean square velocity is $\frac{1}{\sqrt{2}}$ times of it's maximum velocity.

Ans. (*a*, *b*, *d*): (*a*) Let us consider a periodic SHM,

$$x = a \sin \omega t$$

Let mass m is executing the SHM

$$v = \frac{dx}{dt} = a\omega \cos \omega t$$

$$v_{max} = a\omega \qquad (\because \cos \omega t = 1)$$

$\therefore$ Total mechanical energy = K.E.$_{max}$ or P.E.$_{max}$

$$\therefore \quad \text{T.E.} = \frac{1}{2} m a^2 \omega^2$$

$$\text{K.E.}_{max} = \frac{1}{2} m a^2 \omega^2$$

or average Total energy is K.E.$_{max}$.

(*b*) If amplitude is a and angular frequency is ω, then maximum velocity of particle will be equal to $a\omega$ and it varies according to Sine law. Hence, *r.m.s.* value of velocity of the particle over a complete cycle will be equal to $\frac{1}{\sqrt{2}} a\omega$.

$$\therefore \text{ Average K.E.} = \frac{1}{2} m v_{rms}^2$$

$$= \frac{1}{2} m \left(\frac{1}{\sqrt{2}} a\omega \right)^2 = \frac{1}{2} m \frac{1}{2} a^2 \omega^2$$

$$\therefore \text{ Average K.E.} = \frac{1}{2} \left\{ \frac{1}{2} m a^2 \omega^2 \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{2} m v_{max}^2 \right\} \qquad [\because v_{max} = a\omega]$$

$$= \frac{1}{2} \text{K.E.}_{max}$$

(*c*)

$$v = a\omega \cos \omega t$$

$$v_{mean} = \frac{v_{max} + v_{min}}{2} = \frac{a\omega + (-a\omega)}{2}$$

$$v_{mean} = 0 \quad \therefore v_{max} \neq v_{mean}$$

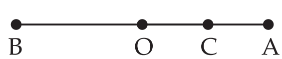
(*d*)

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2}{2}} = \sqrt{\frac{0 + a^2\omega^2}{2}} = \frac{a\omega}{\sqrt{2}}$$

$$v_{rms} = \frac{v_{max}}{\sqrt{2}}$$

Hence, verifies the options (*a*, *b*, *d*).

Q14.18. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart (figure) take the direction from A to B as the positive direction and choose the correct statements.



AO = OB = 5 cm

BC = 8 cm

(*a*) The sign of velocity, acceleration and force on the particle when it is 3 cm away from A going towards B are positive.

(*b*) The sign of velocity of the particle at C going towards B is negative.

(*c*) The sign of velocity, acceleration and force on the particle when it is 4 cm away from B going towards A are negative.

(*d*) The sign of acceleration and force on the particle when it is at point B is negative.

Ans. (*a, c, d*): (*a*) When the particle is going from A to B (+ve direction) and it is 3 cm from A velocity increases upto O so velocity is positive.

Acceleration in SHM is towards O so it is also +ve. So both v and a are +ve.

(*b*) As the particle is going towards B so velocity is positive not negative.

(*c*) As the particle is at 4 cm from B and going towards A i.e., (–)ve side, so velocity and acceleration towards mean position at O. So both are negative.

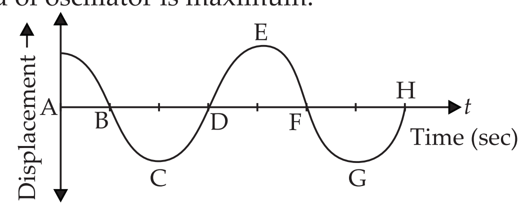
(*d*) When particle is at B force and acceleration both are towards 'O', so both are negative.

VERY SHORT ANSWER TYPE QUESTIONS

Q14.19. Displacement versus time curve for a particle executing simple harmonic motion is shown in figure. Identify the points marked at which:

(*i*) velocity of the oscillator is zero.

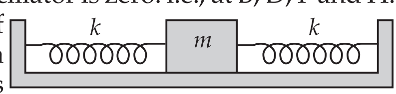
(*ii*) speed of oscillator is maximum.



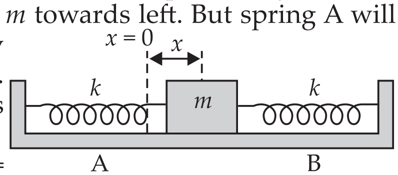
Ans. (*i*) Velocity of oscillator is maximum where the displacement is maximum i.e., at A, C, E and G.

(*ii*) Speed of oscillator will be maximum when no restoring force acts i.e., where displacement of oscillator is zero. i.e., at B, D, F and H.

Q14.20. Two identical springs of spring constant k are attached to a block of mass m and fixed supports as shown in figure. When the mass is displaced from equilibrium position by a distance x towards right. Find the restoring force.



Ans. When the mass m is displaced from equilibrium position by a distance x towards right then spring B will be compressed by distance x, and apply the force (kx) on mass m towards left. But spring A will get extend by distance x and apply the force kx on mass towards left. So net force acting on block towards left side is restoring force



$$F = kx + kx = 2kx.$$

$$F = 2kx \text{ (restoring force towards left)}$$

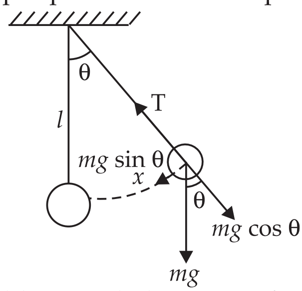
Q14.21. What are the two basic characteristics of simple harmonic motion?

Ans. Two basic characteristics of a simple harmonic motion are:

(*i*) Acceleration is directly proportional to displacement from mean position, and the direction of acceleration is towards mean position.

(*ii*) Restoring force is directly proportional to displacement, the direction of force and displacement are opposite i.e., $F = -kx$.

Q14.22. When will be the motion of a simple pendulum be simple harmonic motion?



Ans. Consider a pendulum of length l and mass of bob m is displaced by angle θ as shown in figure.

The restoring force $= F = -mg \sin\theta$

If θ is small then $\sin\theta = \theta = \dfrac{\text{arc}}{\text{radius}} = \dfrac{x}{l}$

$\therefore \quad F = -mg\dfrac{x}{l}$ or $F \propto (-x)$

($\because$ m, g, l are constants)

Hence the motion of simple pendulum will be simple harmonic for small angle θ.

Q14.23. What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic motion?

Ans. Consider a SHM. $x = A\sin\omega t$

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

For v_{max} $\quad \cos\omega t = 1$

$\therefore \quad v_{max} = A\omega$

$$a = \frac{dv}{dt} = -A\omega^2\sin\omega t$$

For a_{max} $\quad \sin\omega t = -1$

$$a_{max} = A\omega^2$$

$$\therefore \quad \frac{a_{max}}{v_{max}} = \frac{A\omega^2}{A\omega} = \frac{\omega}{1}$$

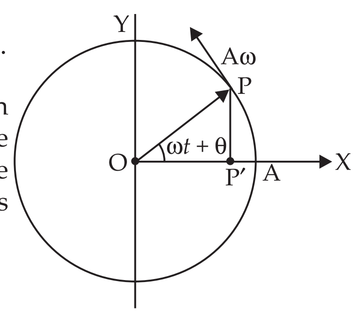
Q14.24. What is the ratio between the distance travelled by the oscillator in one time-period and amplitude?

Ans. Distance travelled by oscillator in one time-period = 4A

where A = amplitude of the oscillation

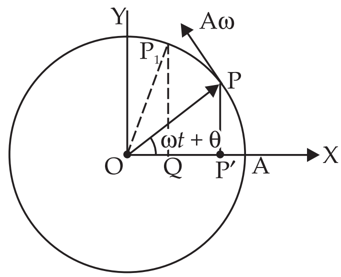
$\therefore \quad$ Required ratio $= \dfrac{4A}{A} = \dfrac{4}{1}$ is 4 : 1.

Q14.25. In figure what will be the sign of velocity of the point P′ which is the projection of the velocity of the reference particle P. P is moving in a circle of radius R in anti-clockwise direction.



Ans. P′ is foot of perpendicular of velocity vector of particle P at any time t.

Now particle moves from P to P_1 then its foot shifts from P′ to Q i.e., towards negative axis. Hence the sign of θ motion of P′ is negative.



Q14.26. Show that for a particle executing SHM velocity and displacement have a phase difference $\frac{\pi}{2}$.

Ans. Consider a SHM $x = A \sin \omega t$ ...(*i*)

$$v = \frac{dx}{dt} = A\omega \cos \omega t = A\omega \sin (90° + \omega t)$$

$$(\because \sin (90° + \theta) = \cos \theta)$$

$$\therefore \quad v = A\omega \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(ii)$$

Phase of displacement from (*i*) is (ωt)

Phase of velocity from (*ii*) is $\left(\omega t + \frac{\pi}{2}\right)$.

Hence, the phase difference $= \omega t + \frac{\pi}{2} - \omega t = \frac{\pi}{2}$.

Q14.27. Draw a graph to show the variation of P.E. and K.E. and total energy of a simple harmonic oscillator with displacement.

Ans. Consider a spring of spring constant k in attached to a mass lying on a horizontal frictionless surface.

Now the mass m is displaced through a distance A from its mean position. Then it will execute simple harmonic motion. P.E. of mass m at this **stretched position** $= \frac{1}{2}kA^2$

At maximum stretch K.E. = 0 at $x = A$

$$PE = \text{Total Energy} = \frac{1}{2}kA^2 \quad \text{at } x = \pm A$$

Now let mass is at its x displacement from mean position then restoring force acts on particle

$$\text{P.E.} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

where $k = m\omega^2$ called restoring force constant of oscillator when $x = \pm A$ then

$$\text{P.E.}_{max} = \frac{1}{2}m\omega^2 A^2$$

$$\text{K.E. of SHM} = \frac{1}{2}mv^2 \quad (\because v^2 = \omega^2(A^2 - x^2))$$

$$\therefore \quad \text{K.E.} = \frac{1}{2}m\omega^2 (A^2 - x^2)$$

$\therefore$ When $x = \pm$ A K.E. = 0

x	K.E.	P.E.	T.E.
0	$\frac{1}{2}m\omega^2A^2$	0	$\frac{1}{2}m\omega^2A^2$
+ A	0	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$
− A	0	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$

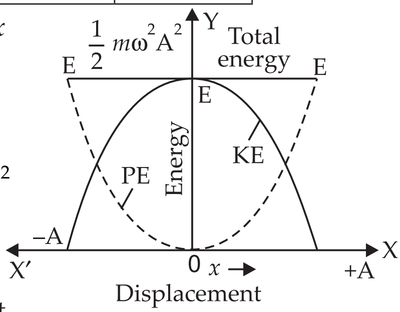
Total Energy E at any displacement x

E = P.E. + K.E.

$$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2$$

$$= \frac{1}{2}m\omega^2A^2$$



As E, not have x in it so it is constant with displacement x.

Q14.28. The length of a second's pendulum on the surface of earth is 1 m. What will be the length of a second's pendulum on moon.

Ans. A pendulum of time period (T) of 2 sec is called second pendulum.

$$T_e = 2\pi\sqrt{\frac{l_e}{g_e}} \Rightarrow T_e^2 = 4\pi^2\frac{l_e}{g_e} \quad ...(i)$$

$$T_m = 2\pi\sqrt{\frac{l_m}{g_m}} \quad \because g_m = \frac{g_e}{6}$$

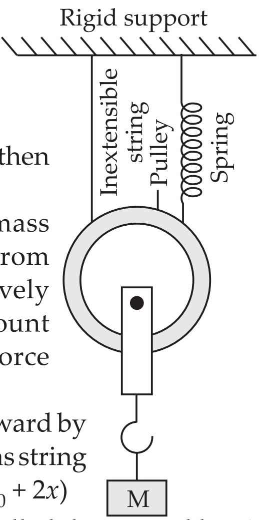
$$\therefore \quad T_m^2 = 4\pi^2\frac{l_m \times 6}{g_e} \quad ...(ii)$$

For second pendulum $T_e = T_m = 2$ sec

$$\frac{T_m^2}{T_e^2} = \frac{\frac{4\pi^2\, 6l_m}{g_e}}{4\pi^2\frac{l_e}{g_e}} \text{ or } \frac{(2)^2}{(2)^2} = \frac{6l_m}{l_e} \quad l_e = 1\text{ m}$$

$$\frac{1}{1} = \frac{6l_m}{1\text{ m}} \Rightarrow l_m = \frac{1}{6}\text{ m.}$$

SHORT ANSWER TYPE QUESTIONS



Q14.29. Find the time period of mass M when displaced from it's equilibrium position and then released for the system shown.

Ans. When mass M is pulled and released then mass M oscillates up down along with pulley.

Let the spring extends by x_0 when loaded by mass M. The extension and compression of spring from initial position is larger and smaller respectively due to acceleration due to gravity by same amount of forces always. So effect of gravitational force can be neglected.

Now let the mass 'M' is pulled by force 'F' downward by displacement x. Then extension in spring will $2x$ as string can not extend. So total extension in spring = $(x_0 + 2x)$

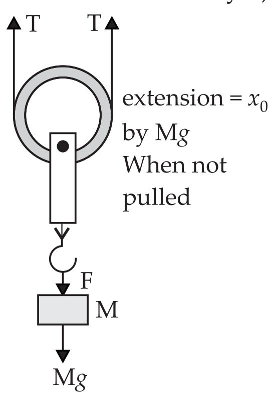
$$T' = k(x_0 + 2x) \quad \text{(when pulled downward by } x\text{)}$$

$$T = kx_0 \quad \text{(when no pulling)}$$

$$F = 2T \Rightarrow F = 2kx_0$$

and $\quad F' = 2T'$

$$F' = 2k(x_0 + 2x)$$



Restoring force

$$F_{rest.} = -(F' - F)$$

$$F_{rest.} = -[2k(x_0 + 2x) - 2kx_0)]$$

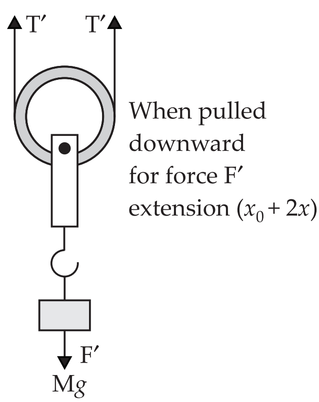
$$F_{rest.} = -2k.2x$$

$$Ma = -4kx$$

$$a = \frac{-4k}{M}x$$

$\Rightarrow \quad a \propto -x$

Hence, motion is simple harmonic motion in SHM.



$$a = -\omega^2 x$$

$\therefore$
$$\omega^2 = \frac{-a}{x} = \frac{+4k}{M}$$

$$\omega = 2\sqrt{\frac{k}{M}}$$

$\Rightarrow$
$$\frac{2\pi}{T} = 2\sqrt{\frac{k}{M}}$$

$$T = \pi\sqrt{\frac{M}{k}}.$$

Q14.30. Show that the motion of a particle represented by $y = \sin \omega t - \cos \omega t$ is simple harmonic with period of $\frac{2\pi}{\omega}$.

Ans. A function will represents S.H.M. if it can be written uniquely in the form of $a \cos\left(\frac{2\pi}{T}t + \phi\right)$ or $a \sin\left(\frac{2\pi}{T}t + \phi\right)$

Now $\quad y = \sin \omega t - \cos \omega t \quad$ (given)

$$y = \sqrt{2}\left[\sin \omega t \frac{1}{\sqrt{2}} - \cos \omega t \frac{1}{\sqrt{2}}\right]$$

$$y = \sqrt{2}\left[\sin \omega t \cos \frac{\pi}{4} - \cos \omega t \sin \frac{\pi}{4}\right]$$

$$y = \sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right)$$

Comparing with standard SHM $y = a \sin\left(\frac{2\pi}{T} t + \phi\right)$

we get, $\quad \omega = \frac{2\pi}{T}$ or $T = \frac{2\pi}{\omega}$.

Q14.31. Find the displacement of a simple harmonic oscillator at which its P.E. is half of the maximum energy of the oscillator?

Ans. Consider an oscillator is at displacement x from its mean position then P.E. $= \frac{1}{2}kx^2$, where $k = m\omega^2$ = Force constant of oscillator of mass m

$$\therefore \quad \text{P.E.} = -m \;\; {}^2x^2$$

Maximum P.E. when K.E. = 0 is at x = A it will be total energy of the oscillator.

$$E = \frac{1}{2}m\omega^2A^2$$

P.E. is half of total energy E at displacement x

$$\therefore \quad \text{P.E.} = \frac{1}{2}\text{Total energy}$$

$$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}.\frac{1}{2}m\omega^2A^2$$

$$x^2 = \frac{1}{2}A^2 \quad \text{or} \quad x = \pm\frac{A}{\sqrt{2}}$$

i.e., P.E. will be half of total energy when its displacement is $\pm\frac{1}{\sqrt{2}}$ amplitude from mean position.

Q14.32. A body of mass m is situated in a potential field

$U(x) = U_0(1 - \cos \alpha x)$, where U_0 and α are constants.

Find the time period of small oscillations.

Ans. $\because \quad dW = F.dx$ if $W = U$, then

$$dU = F.dx \text{ or } F = \frac{-dU_x}{dx}$$ (here restoring force is opposite to displacement)

$$F = \frac{-d}{dx}[U_0(1 - \cos \alpha x)] = \frac{-d}{dx}[U_0 + U_0 \cos \alpha x]$$
$$F = -[0 - U_0(-\sin \alpha x) . \alpha]$$
$$F = -\alpha U_0 \sin \alpha x$$

For SHM. αx is small

So $\sin \alpha x$ becomes αx ...(*i*)

$\therefore$ $F = -\alpha . U_0 \alpha x = -\alpha^2 U_0 x$ ...(*ii*)

α, U_0 are constants.

$\therefore$ $F \propto -x$. So motion is SHM.

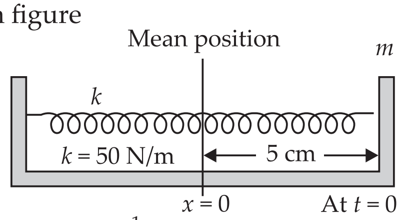
Here from (*ii*) $k = \alpha^2 U_0$

$$m\omega^2 = \alpha^2 U_0 \Rightarrow \omega^2 = \alpha^2 \frac{U_0}{m}$$
$$\left(\frac{2\pi}{T}\right)^2 = \alpha^2 \frac{U_0}{m} \Rightarrow T^2 = 4\pi^2 \frac{m}{U_0 \alpha^2} \text{ or}$$
$$T = \frac{2\pi}{\alpha}\sqrt{\frac{m}{U_0}}.$$

From (*i*) this time period is valid for small angle αx.

Q14.33. A mass of 2 kg is attached to the spring of spring constant 50 $N\text{-}m^{-1}$. The block is pulled to a distance of 5 cm from its equilibrium position at $x = 0$ on a horizontal frictionless surface from rest at $t = 0$. Write the expression for its displacement at anytime t.

Ans. The mass m attached to spring oscillate SHM with amplitude 5 cm as shown in figure



$m = 2$ kg $\quad k = 50\ Nm^{-1}$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}} = \sqrt{25} = 5 \text{ radian per second.}$$

Assume the displacement function is $y(t) = A \sin(\omega t + \phi)$

ϕ = initial phase

at $t = 0$ $\quad y(t) = +A$ (given)

$\therefore$ $A \sin(\omega \times 0 + \phi) = +A$ or $\sin(0 + \phi) = 1$

$$\sin \phi = 1 \text{ or } \sin \phi = \sin \frac{\pi}{2}$$

$\therefore$ $\phi = \frac{\pi}{2}$. So desired equation becomes

$$y(t) = A \sin\left(\omega t + \frac{\pi}{2}\right)$$

Put $A = 5$ $\quad \omega = 5$ radian per second

$$F = \frac{-d}{dx}[U_0(1 - \cos \alpha x)] = \frac{-d}{dx}[U_0 + U_0 \cos \alpha x]$$
$$F = -[0 - U_0(-\sin \alpha x) . \alpha]$$
$$F = -\alpha U_0 \sin \alpha x$$

For SHM. αx is small

So $\sin \alpha x$ becomes αx ...(*i*)

$\therefore$ $$F = -\alpha . U_0 \alpha x = -\alpha^2 U_0 x$$ ...(*ii*)

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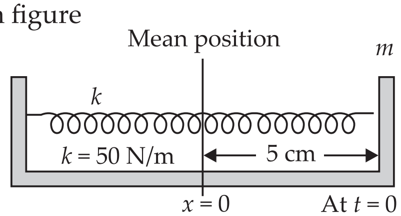

$m = 2$ kg $\quad k = 50 \text{ Nm}^{-1}$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}} = \sqrt{25} = 5 \text{ radian per second.}$$

Assume the displacement function is $y(t) = A \sin(\omega t + \phi)$

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at $t = 0$ $\quad y(t) = +A$ (given)

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$$\sin \phi = 1 \text{ or } \sin \phi = \sin \frac{\pi}{2}$$

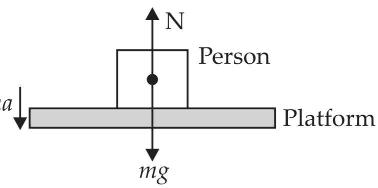
$\therefore$ $\phi = \frac{\pi}{2}$. So desired equation becomes

$$y(t) = A \sin\left(\omega t + \frac{\pi}{2}\right)$$

Put A = 5 $\quad \omega = 5$ radian per second of 2.0 s^{-1} and an amplitude 5.0 cm. A weighing machine on the platform gives the person's weight against time.

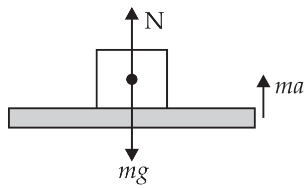
(*a*) Will there be any change in weight of the body, during the oscillation?

(*b*) If answer to part (*a*) is yes what will be the maximum and minimum reading in the machine and at which position?



Ans. (*a*) Weight in weight machine will be due to the normal reaction (N) by platform. Consider the top position of platform, two forces due to weight of person and oscillator acts both downward.

So motion is downward. Let with acceleration *a* then



$$ma = mg - N \qquad ...(i)$$

When platform lifts from its lowest position to upward

$$ma = N - mg \qquad ...(ii)$$

$$a = \omega^2 A \text{ acceleration of oscillator}$$

(*i*) $\therefore$ From (*i*) equation

$$N = mg - m\omega^2 A$$

where A is amplitude, ω angular frequency, m mass of oscillator.

$$\omega = 2\pi\nu = 2\pi \times 2 = 4\pi \text{ rad/sec.}$$

$$A = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}, m = 50 \text{ kg}$$

$$N = 50 \times 9.8 - 50 \times 4\pi \times 4\pi \times 5 \times 10^{-2}$$

$$= 50 [9.8 - 16\pi^2 \times 5 \times 10^{-2}]$$

$$= 50 [9.8 - 80 \times 3.14 \times 3.14 \times 10^{-2}]$$

$$N = 50 [9.8 - 7.89] = 50 \times 1.91 = 95.50 \text{ N}$$

So minimum weight is 95.50 N.

(*ii*) From (*ii*) $N - mg = ma$

For upward motion from lowest point of oscillator.

$$N = mg + ma = m(a + g)$$

$$= m [9.81 + \omega^2 A] \qquad a = \omega^2 A$$

$$= 50 [9.81 + 16\pi^2 \times 5 \times 10^{-2}]$$

$$= 50 [9.81 + 7.89] = 50 [17.70]$$

$$N = 885.00 \text{ N.}$$

(*a*) Hence, there is a change in weight of the body during oscillation.

(*b*) The maximum weight is 885 N, when platform moves from lowest to upward direction.

And the minimum is 95.5 N, when platform moves from highest point to downward direction.

Q14.36. A body of mass *m* is attached to one end of massless string which is suspended vertically from a fixed point. The mass is held in hand, so that the spring is neither stretched nor compressed. Suddenly the support of hand is removed. The lowest position attain by the mass is 4 cm below the point where it was held in hand

(*a*) What is the amplitude of oscillation?

(*b*) Find the frequency of oscillation.

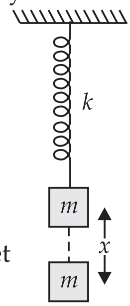
Ans. (*a*) When mass *m* is held in support by hand the extension in spring will be zero as no deforming force acts on spring.

Let the mass reaches at its new position *x* unit displacement from previous.

Then P.E. of spring or mass = gravitational P.E. lost by man

$$\text{P.E.} = mg\,x$$

But P.E. due to spring is $\frac{1}{2}kx^2$ $\qquad k = \omega^2 A$



$$\therefore \qquad \frac{1}{2}kx^2 = mg\,x$$

$$x = \frac{2mg}{k} \qquad \text{...}(i)$$

Mean position of spring by block will be when let extension is x_0 then

$$F = +kx_0$$

$$F = mg \quad \therefore mg = +kx_0 \quad \text{or} \quad x_0 = \frac{mg}{k} \qquad \text{...}(ii)$$

From (*i*) and (*ii*)

$$x = 2\left(\frac{mg}{k}\right) = 2x_0$$

$$x = 4 \text{ cm} \quad \therefore 4 = 2x_0$$

$$x_0 = 2 \text{ cm.}$$

The amplitude of oscillator is the maximum distance from mean position i.e., $x - x_0 = 4 - 2 = 2$ cm.

(*b*) Time period $T = 2\pi\sqrt{\frac{m}{k}}$ which does not depend on amplitude

$$\frac{2mg}{k} = x \qquad \text{from } (i)$$

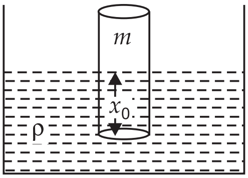
$$\frac{m}{k} = \frac{x}{2g} = \frac{4\times 10^{-2}}{2\times 9.8} \quad \text{or} \quad \frac{k}{m} = \frac{2\times 9.8}{4\times 10^{-2}}$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\times 3.14}\sqrt{\frac{2\times 9.8}{4\times 10^{-2}}} = \frac{\sqrt{4.9\times 10^2}}{6.28}$$

$$\nu = \frac{10\times 2.21}{6.28} = 3.52 \text{ Hz.}$$

Oscillator will not rise above the position from where it was released because total extension in spring is 4 cm when released and amplitude is 2 cm. So it oscillates below the released position.

Q14.37. A cylindrical log of wood of height h and area of cross-section A floats in water. It is pressed and then released, show that the log would execute SHM with a time period $T = 2\pi\sqrt{\frac{m}{A\rho g}}$. Where m is mass of the body and ρ is density of liquid.



Ans. When log is pressed downward into the liquid then an upward Buoyant force (B.F.) action it which moves the block upward and due to inertia it moves upward from its mean position due to inertia and then again come down due to gravity. So net restoring force on block = Buoyant force – mg

V = volume of liquid displaced by block

Let when block floats then

$$mg = \text{B.F.} \quad \text{or} \quad mg = V\rho g$$

$$mg = Ax_0\rho g \qquad ...(i)$$

A = area of cross-section

x_0 = height of block liquid

Let x height again dip in liquid when pressed into water total height of block in water = $(x + x_0)$

So net restoring force = $[A(x + x_0)]\rho.g - mg$

$$F_{restoring} = Ax_0\rho g + Ax\rho g - Ax_0\rho g \qquad \text{from } (i)$$

$$F_{restoring} = -Ax\rho g$$

(as Buoyant force is upward and x is downward)

$$F_{restoring} \propto -x$$

So motion is SHM here $k = A\rho g$

$$a = -\omega^2 x \quad \omega^2 = \frac{k}{m} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}}$$

$$F_{restoring} = -A\rho g x$$

$$ma = -A\rho g x$$

$$a = \frac{-A\rho g x}{m} \quad \Rightarrow \quad -\omega^2 x = \frac{-A\rho g x}{m}$$

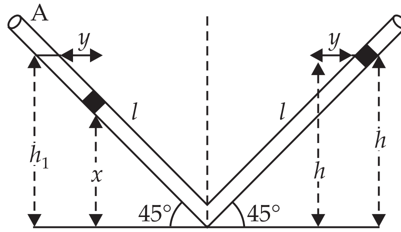
$$\omega^2 = \frac{A\rho g}{m}$$

$$k = A\rho g$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{A\rho g}{m} \quad \Rightarrow \quad \frac{T}{2\pi} = \sqrt{\frac{m}{A\rho g}} \quad \Rightarrow T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Q14.38. One end of a V tube containing mercury is connected to a suction pump and other end to atmosphere. The two arms of the tube are inclined to horizontal at an angle of 45° each. A small pressure difference is created between two columns when the suction pump is removed. Will the column of mercury in V tube execute simple harmonic motion? Neglect capillary as viscous forces. Find the time period of oscillation.

Ans. Let the liquid column in both columns are at h_0 heights initial.



Now due to pressure difference the liquid columns in A arm pressed by x and in arm B lifts by x. (so difference in vertical heights between two levels = $2x$)

Consider an element of liquid of height dx (in tube)

Then it's mass $dm = A.dx\ \rho$.   A = area of cross-section of tube

P.E. of the left dm elements column = $(dm)\ g\ h$

P.E. of dm element in left column = $A\rho g x\ dx$

$$\text{Total P.E. in left column} = \int_0^{h_1} A\rho g x\, dx = A\rho g\left[\frac{x^2}{2}\right]_0^{h_1}$$

$$= A\rho g \frac{h_1^2}{2}$$

From figure $$\sin 45° = \frac{h_1}{l}$$

$$h_1 = h_2 = l \sin 45° = \frac{l}{\sqrt{2}}$$

$$\therefore \quad h_1^2 = h_2^2 = \frac{l^2}{2}$$

$$\therefore \quad \text{P.E. in left column} = A\rho g \frac{l^2}{4}$$

$$\text{Similarly P.E. in right column} = A\rho g \frac{l^2}{4}$$

$$\therefore \text{ Total Potential energy} = A\rho g \frac{l^2}{4} + A\rho g \frac{l^2}{4} = \frac{A\rho g l^2}{2}$$

Due to pressure difference, let element moves towards right side is y unit.

Then the liquid column is left arm = $(l - y)$

and the liquid column in right arm = $(l + y)$

P.E. of liquid column in left arm = $A\rho g\ (l - y)^2 \sin^2 45°$

P.E. of liquid column in right arm = $A\rho g\ (l + y)^2 \sin^2 45°$

$\therefore$ Total P.E. due liquid column = $A\rho g\left(\frac{1}{\sqrt{2}}\right)^2[(l-y)^2+(l+y)^2]$

Final P.E. due to difference in liquid columns

$$= \frac{A\rho g}{2}[l^2+y^2-2ly+l^2+y^2+2ly]$$

$$\text{Final P.E.} = \frac{A\rho g}{2}(2l^2+2y^2)$$

$$\text{Change P.E.} = \text{Final P.E.} - \text{Initial P.E.}$$

$$= \frac{A\rho g}{2}(2l^2+2y^2) - \frac{A\rho g\, l^2}{2}$$

$$= \frac{A\rho g}{2}[2l^2+2y^2-l^2]$$

$$\text{Change in P.E.} = \frac{A\rho g}{2}(l^2+2y^2)$$

If change in velocity (v) of total liquid column

$$\Delta KE = \frac{1}{2}mv^2$$

$$m = (A.2l)\rho$$

$$\Delta KE = \frac{1}{2}(A.2l\rho)\,v^2 = A\rho l\, v^2$$

$$\therefore \text{Chance in Total energy} = \frac{A\rho g}{2}(l^2+2y^2) + A\rho l\, v^2$$

Total change in energy $\Delta PE + \Delta KE = 0$

$$\therefore \frac{A\rho g}{2}[l^2+2y^2] + A\rho l\, v^2 = 0$$

$$\frac{A\rho}{2}[g(l^2+2y^2)+2l\,v^2] = 0$$

$$\frac{A\rho}{2} \neq 0$$

$$\therefore \quad g(l^2+2y^2)+2lv^2 = 0$$

Differentiating w.r.t. t, $g\left[0+2\times 2y\frac{dy}{dt}\right]+2l.2v.\frac{dv}{dt} = 0$

$$4gy\frac{dy}{dt}+4v.l\frac{d^2y}{dt^2} = 0 \qquad \left[\because a = \frac{dv}{dt} = \frac{d^2y}{dt^2}\right]$$

$$4gy.v+4vl\frac{d^2y}{dt^2} = 0 \Rightarrow 4v\left[gy+l.\frac{d^2y}{dt^2}\right] = 0$$

$$\frac{d^2y}{dt^2}+\frac{g}{l}y = 0 \quad \because 4v \neq 0$$

It is the equation of SHM oscillator and standard equation of SHM is

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

$$\therefore \qquad \omega^2 = \frac{g}{l}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$$

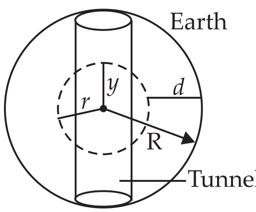
Q14.39. A tunnel is dug through centre of earth. Show that a body of mass *m* when dropped from rest from one end of the tunnel will execute simple harmonic motion.

Ans. As the acceleration due to gravity of earth inside the earth is g'

$$g' = g\left(1 - \frac{d}{R}\right) = g\left[\frac{R-d}{R}\right]$$

$$R - d = y$$

$$\therefore \qquad g' = g\frac{y}{R}$$



Force on body at depth d is

$$F = -mg' = -mg\frac{y}{R}$$

$$F \propto (-y)$$

So motion of body in tunnel is SHM. For a period we can write

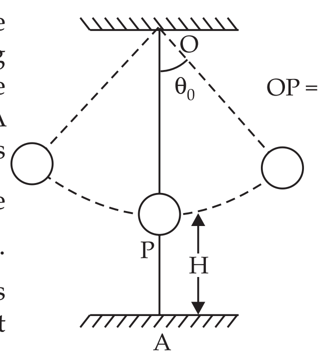
$$ma = -mg'$$

$$a = \frac{-g}{R}y$$

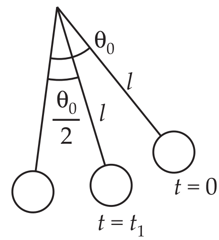
$$-\omega^2 y = \frac{-g}{R}y \qquad (\because a = -\omega^2 y)$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{R}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{R}{g}}.$$

Q14.40. A simple pendulum of time period 1 second and length l is hung from a fixed support at O, such that the bob is at a distance H vertically above A on the ground (figure). The amplitude is θ_0. The string snaps at $\theta = \frac{\theta_0}{2}$. Find the time taken by the bob to hit the ground. Also find distance from A where bob hits the ground. Assume θ_0 to be small so that $\sin\theta_0 \simeq \theta_0$ and $\cos\theta_0 \simeq 1$.



Ans. Consider the diagram at $t = 0$, $\theta = \frac{\theta}{2}$



At $t = t_1$ $\qquad \theta = \frac{\theta_0}{2}$

$$\theta = \theta_0 \cos \omega t$$

at $t = t_1$, $\theta = \frac{\theta_0}{2}$

$$\therefore \qquad \frac{\theta_0}{2} = \theta_0 \cos \frac{2\pi}{T} t_1$$

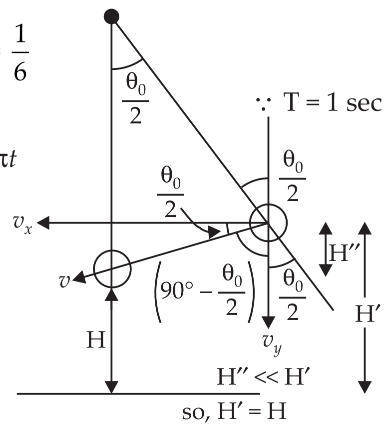
$$T = 1 \text{ sec} \qquad \text{(given)}$$

$$\therefore \qquad \cos 2\pi\, t_1 = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$2\pi\, t_1 = \frac{\pi}{3} \quad \text{or} \quad t_1 = \frac{1}{6}$$

$$\theta = \theta_0 \cos 2\pi t$$

$$\frac{d\theta}{dt} = -\theta_0\, 2\pi \sin 2\pi t$$



At $t = \frac{1}{6}$ $\quad$ i.e., at $\theta = \frac{\theta_0}{2}$

$$\frac{d\theta}{dt} = -\theta_0\, 2\pi \sin 2\pi \frac{1}{6}$$

$$\frac{d\theta}{dt} = -\theta_0\, 2\pi \sin \frac{\pi}{3}$$

$$= -\theta_0\, 2\pi \frac{\sqrt{3}}{2}$$

$$\frac{d\theta}{dt} = -\theta_0 \pi \sqrt{3}$$

$$\omega = -\theta_0 \pi \sqrt{3} \qquad \left(\because \frac{d\theta}{dt} = \omega\right)$$

$$\frac{v}{l} = -\theta_0 \pi \sqrt{3}$$

$$v = -\sqrt{3}\pi \theta_0\, l$$

(–)ve shows that bob's motion is towards left.

$$v_x = v \cos \frac{\theta_0}{2} = -\sqrt{3}\pi \theta_0\, l \cos \frac{\theta_0}{2}$$

$$v_y = v \sin \frac{\theta_0}{2} = -\sqrt{3}\pi \theta_0\, l \sin \frac{\theta_0}{2}$$

Let vertical distance covered by v_y is H′ (downward)

$$s = ut + \frac{1}{2} g t^2$$

$$H' = v_y t + \frac{1}{2} g t^2$$

$$\frac{1}{2}gt^2 + \sqrt{3}\pi\theta_0 \, l \sin\frac{\theta_0}{2} \times t - H' = 0$$

$$\sin\frac{\theta_0}{2} = \frac{\theta_0}{2} \qquad \text{(given)}$$

$$\therefore \qquad \frac{1}{2}gt^2 + \sqrt{3}\pi\theta_0 . l\frac{\theta_0}{2}t - H' = 0$$

$$\text{or} \qquad \frac{1}{2}gt^2 + \sqrt{3}\pi\frac{\theta_0^2}{2}lt - H' = 0$$

By quadratic formula $t = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$t = \frac{-(\sqrt{3}\pi\,\theta_0^2\, l)/4 \pm \sqrt{(3\pi^2\,\theta_0^4\, l^2)/4 + 4.\frac{1}{2}gH'}}{2g/2}$$

As θ_0 is very small so by neglecting θ_0^4 and θ_0^2

$$t = \frac{+\sqrt{2gH'}}{g} \qquad H' = H + H'' \text{ from fig}$$

$$\therefore \qquad t = \sqrt{\frac{2H}{g}} \qquad H'' << H' \text{ as } \frac{\theta_0}{2} \text{ is very small}$$

$$\therefore \; H = H'$$

Distance covered in horizontal = $v_x.t$

$$X = \sqrt{3}\pi\theta_0\, l \cos\frac{\theta_0}{2}.\sqrt{\frac{2H}{g}}$$

$$\therefore \qquad X = \sqrt{3}\pi\theta_0\, l.\sqrt{\frac{2H}{g}} \qquad \cos\frac{\theta_0}{2} = 1 \text{ (given)}$$

$$X = \theta_0\, l\,\pi\sqrt{\frac{6H}{g}}$$

At the time of snapping, the bob was at distance $l\sin\dfrac{\theta_0}{2} = l\dfrac{\theta_0}{2}$ from A

Thus the distance of bob from A where it meet the ground is

$$= \frac{l\theta_0}{2} - X = \frac{l\theta_0}{2} - \theta_0 l\pi\sqrt{\frac{6H}{g}}$$

$$= \theta_0\, l\left[\frac{1}{2} - \pi\sqrt{\frac{6H}{g}}\right]$$

❑❑❑

15 Waves

MULTIPLE CHOICE QUESTIONS-I

Q15.1. Water waves produced by a motor boat sailing in water are
(*a*) neither longitudinal nor transverse
(*b*) both longitudinal and transverse
(*c*) only longitudinal
(*d*) only transverse

Ans. (*b*): As the waves are produce by motor boat on surfaces as well as inside water. So the waves are transverse as well as longitudinal both.

Q15.2. Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where its speed is $2v$ m/s wavelength of sound waves in the second medium is:

(*a*) λ (*b*) $\frac{\lambda}{2}$ (*c*) 2λ (*d*) 4λ

Ans. (*c*): When wave passes from one medium to another its frequency (ν) does not change, but its velocity and wavelength changes.

$$v = \nu\lambda \quad \text{or} \quad \nu = \frac{v}{\lambda}$$

$$\frac{v}{\lambda} = \frac{2v}{\lambda_2} \Rightarrow \lambda_2 = 2\lambda.$$ Hence verifies the option (*c*).

Q15.3. Speed of the sound wave in air
(*a*) is independent of temperature
(*b*) increases with pressure
(*c*) increase on increasing humidity
(*d*) decreases with increase in humidity

Ans. (*c*): Speed of sound (longitudinal) wave in air is $v = \sqrt{\frac{\gamma P}{\rho}}$. The density of water vapours is small (rises up) than the air so on increasing humidity the density of medium decrease in turn increases the speed of sound in air by $v \propto \frac{1}{\sqrt{\rho}}$ (relation). Hence verifies the option (*c*).

Q15.4. Change in temperature of the medium changes
(*a*) frequency of sound waves
(*b*) amplitude of sound waves
(*c*) wavelength of sound waves
(*d*) loudness of sound waves

Ans. (*c*): We know that $v_t = v_0\,(1 + .61t)$ or $v_t \propto \sqrt{T}$. So on increasing temperature the speed also increases as frequency does not change during propagation of wave by formula $v = \nu\lambda$. So velocity v and wavelength λ both increases. Hence, verifies the option (*c*).

Q15.5. With propagation of longitudinal wave through a medium, the quantity transmitted is

(*a*) a matter
(*b*) energy
(*c*) energy and matter
(*d*) energy matter and momentum

Ans. (*b*): During propagation of any wave in a medium only energy is transmitted from one point to another. Matter does not change its own position it vibrates about its mean position only.

Q15.6. Which of the following statements are true for a wave motion?

(*a*) Mechanical transverse waves can propagate through all medium.
(*b*) Longitudinal wave can propagate through solid only
(*c*) Mechanical transverse wave can propagate through solid only
(*d*) Longitudinal wave can propagate through vacuum.

Ans. (*c*): Mechanical transverse wave can propagate through solid, and on surface of liquid also, so best option is (*c*).

Transverse wave can not propagate in gases rejects option (*a*).

Longitudinal can propagate through gases rejects option (*b*).

Longitudinal waves are not e.m. waves and can mechanical wave can never propagate in vacuum rejects option (*d*).

Q15.7. Sound wave is passing through air column in the form of compression and rarefaction. In consecutive compressions and rarefactions.

(*a*) density remains constant
(*b*) Boyle's law is obeyed
(*c*) bulk modulus of air oscillates
(*d*) there is no transfer of heat

Ans. (*d*): (*i*) The density of medium particles are maximum and minimum at compression and rarefaction points, so rejects option (*a*).

(*ii*) Also density changes very rapidly, so temperature of medium increases. So rejects option (*b*).
(*iii*) Bulk modulus of air remains constant, rejects option (*c*).
(*iv*) The time of compressions and rarefaction is very small so heat does not transfer.

Q15.8. The equation of plane progressive wave is given by $y = 0.6 \sin 2\pi\left[t + \frac{x}{2}\right]$. On reflection from a denser medium its amplitude becomes $\frac{2}{3}$ of the amplitude of the incident wave. The equation of reflected wave is (*a*) $y = 0.6 \sin 2\pi\left[t + \frac{x}{2}\right]$ (*b*) $y = -0.4 \sin 2\pi\left[t + \frac{x}{2}\right]$

(*c*) $y = 0.4 \sin 2\pi\left[t + \frac{x}{2}\right]$ (*d*) $y = -0.4 \sin 2\pi\left[t - \frac{x}{2}\right]$

Ans. (*b*): After reflection of wave changes by phase 180°

$$y_i = 0.6 \sin 2\pi\left[t + \frac{x}{2}\right]$$

$$y_r = \left(\frac{2}{3} \times 0.6\right) \sin 2\pi\left[\pi + t + \frac{x}{2}\right]$$

$$y_r = -0.4 \sin 2\pi\left(t + \frac{x}{2}\right).$$ Verifies the option (*b*).

Q15.9. A string of mass 2.5 kg is under Tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at the one end of the string. The disturbance will reach the other end in

(*a*) one second (*b*) 0.5 second

(*c*) 2 second (*d*) data given insufficient

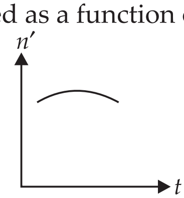
Ans. (*b*): M = mass string = 2.5 kg, $l = 20$ m

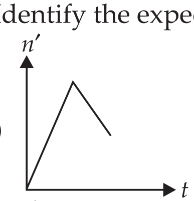
$$m = \text{mass per unit length} = \frac{\text{M}}{l} = \frac{2.5}{20} = 0.125 \text{ kg/m}$$

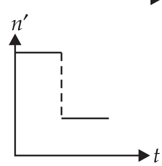
$$v = \sqrt{\frac{\text{T}}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/s}$$

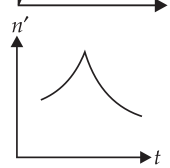
$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{20 \text{ m}}{40 \text{ m/s}} = \frac{1}{2} \text{ sec} = 0.5 \text{ sec}.$$

Q15.10. A train whistling at constant frequency is moving towards a station at a constant speed v. The train goes past a stationary observer on station. The frequency n' of the sound as heard by observer is plotted as a function of time (t) figure. Identify the expected curve.

(*a*)


(*b*)


(*c*)


(*d*)


Ans. (*c*): When observer is at rest and source of sound is moving towards observer then observed frequency n'.

$$n' = \left(\frac{v}{v - v_s}\right) n_o$$

where n_o = original frequency of source of sound

v = speed of sound in medium

$\therefore\ n' > n_o$ v_s = speed of source

When source is moving away from observer

$$n'' = \frac{v}{(v + v_s)} n_o \qquad n'' < n_o$$

Hence, the frequencies in both cases are same and $n' > n''$. So graph (c) verifies the answer.

MULTIPLE CHOICE QUESTIONS-II MORE THAN ONE OPTION

Q15.11. A transverse harmonic wave on a string is described by $y(x,\ t) = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right)$, where x, y are in cm and t is in second. The positive direction of x is from left to right.

(*a*) The wave is travelling right to left

(*b*) The speed of the wave is 20 m/s

(*c*) Frequency of the wave is 5.7 Hz.

(*d*) The least distance between the two successive crests in the wave is 2.5 cm.

Ans. (*a, b, c*): The standard form of a wave propagated from left to right i.e., in +ve direction

$$y = a \sin(\omega t - kx + \phi) \text{ and}$$

$$y = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right) \qquad \text{(given)}$$

(*a*) As in given equation x is in positive sign so given wave travelling from right to left. Verifies option (*a*).

(*b*),(*c*) $\omega = 36$ or $2\pi n = 36$ or $\nu = \frac{36}{2\pi} = \frac{18}{3.14} = 5.7$ Hz

Verifies option (*c*) $n = 5.7$ Hz

$k = 0.018 \quad \frac{2\pi}{\lambda} = 0.018 \Rightarrow \lambda = \frac{2\pi}{0.018}$

$$\therefore \qquad v = \nu\lambda = \frac{18}{\pi} \times \frac{2\pi}{0.018} = 2000 \text{ cm/s} = 20 \text{ m/s}$$

Verifies the option (*b*)

(*d*) Distance between two successive crests $= \lambda = \frac{2\pi}{0.018}$

$$\lambda = \frac{\pi}{0.009} = \frac{3.14 \times 1000}{9} = \frac{3140}{9} \text{ cm}$$

$$\lambda = 348.8 \text{ cm} = 3.48 \text{ m} \neq 2.5 \text{ cm}$$

Hence, rejects the option (*d*).

Q15.12. The displacement of a string is given by

$$y(x, t) = 0.06 \sin \frac{2\pi x}{3} \cos 120\pi t,$$

where x, y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.

(*a*) It represents the progressive wave of frequency 60 Hz

(*b*) It represents the stationary wave of frequency 60 Hz

(*c*) It is the result of superposition of two waves of wavelength 3 m, frequency 60 Hz each travelling with a speed of 180 m/s in opposite direction

(*d*) Amplitude of this wave is constant.

Ans. (*b, c*): We know that standard equation of stationary wave is

$$y(x, t) = a \sin (kx) \cos (\omega t)$$

and given equation is $y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right) \cos [(120\pi)t]$

(*a*) Clearly the given wave is stationary. Hence rejects the option (*a*).

(*b*) Comparing both equation $\omega = 120\pi$

$$2\pi\nu = 120\pi \quad \text{or} \quad \nu = \frac{120}{2} = 60 \text{ Hz}$$

Verifies the option (*b*).

(*c*) $\frac{2\pi}{\lambda} = k$ from eqn., $k = \frac{2\pi}{3}$

$\therefore \frac{2\pi}{\lambda} = \frac{2\pi}{3}$

$\Rightarrow \lambda = 3$ m, $\nu = 60$ Hz

Speed $v = \nu\lambda = 60 \times 3 = 180$ m/s. Hence, verifies the option (*c*).

(*d*) As waves are stationary so the amplitude of particles varies from 0 to $a = 0.06$ m from nodes to antinodes i.e., amplitude through out the wave are not constant. Rejects the option (*d*).

Q15.13. Speed of sound wave in a fluid depends upon

(*a*) directly on the density of the medium

(*b*) square of Bulk modulus of the medium

(*c*) inversely on the square root of density

(*d*) directly on the square root of Bulk modulus of the medium.

Ans. (*c, d*): Speed of sound wave in fluid of Bulk modulus k and density ρ is given by $v = \sqrt{\frac{k}{\rho}}$

So $\qquad v = \sqrt{k}$ (if ρ is constant)

and $\qquad v = \sqrt{\frac{1}{\rho}}$ (if k is constant)

So verifies the options (*c*) and (*d*).

Q15.14. During the propagation of plane progressive mechanical wave

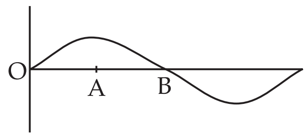
(*a*) all particles are vibrating in the same phase

(*b*) amplitude of all particles is equal

(*c*) particles of the medium executes SHM

(*d*) wave velocity depends upon the nature of medium.

Ans. (*b, c, d*): During propagation of mechanical wave each particles displaces from zero to maximum i.e., upto amplitude. So amplitude of each particle is equal. Verifies the option (*b*).



Each particle between any two successive crests and troughs are in different mode of vibration i.e., in different phase rejects the option (*a*).
For progressive wave medium particles oscillates about their mean position in which restoring force F $\propto$ $(-y)$. So motion of medium particles is simple harmonic motion. So verifies the option (*c*).
For progressive wave propagating in a medium of density (ρ) and Bulk modulus k the velocity (v).

$$v = \sqrt{\frac{k}{\rho}}$$

As the v depends on k and ρ and k, ρ are different for different medium so v of wave depends on nature of medium, hence, verifies the option (*d*).

Q15.15. The transverse displacement of a string (clamped at it's both ends) is given by

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

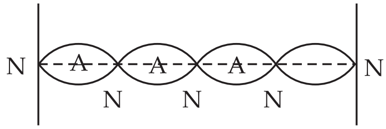
All the points on the string between two consecutive nodes vibrate with

(*a*) same frequency (*b*) same phase

(*c*) same energy (*d*) different amplitude

Ans. (*a, b, d*): Given $y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$ show the stationary waves, as the standard eqn. of stationary wave is

$$y(x, t) = a \sin kx \cos \omega t$$



(*a*) the frequencies of all particles are same, verifies the option (*a*).

(*b*) particles between any two consecutive nodes vibrates either upside or downside having same phase $120\pi t$ at a time, verifies the option (*b*).

(*c*) As the amplitude of different particles are different between two nodes and energy (E) $\propto$ A^2. So particles have different energies. So rejects the option (*c*) and verifies the option (*d*).

Q15.16. A train is standing in station yard, blows the whistle of frequency 400 Hz in still air. The wind start blowing in the direction from yard to the station with speed of 10 m/s. Given that the speed of sound in still air is 340 m/s, then

(*a*) the frequency of sound as heard by an observer standing on the platform is 400 Hz.

(*b*) the speed of sound for the observer standing on the platform is 350 m/s.

(*c*) the frequency of the sound as heard by the observer standing on the platform will increase.

(*d*) the frequency of sound as heard by the observer standing on the platform will decrease.

Ans. (*a*, *b*): $\nu_0 = 400$ Hz frequency of source of sound.

Velocity of wind $v_w = 10$ m/s from source of listener.

Speed of sound in still air $= v_s = 340$ m/s.

As the listener is standing on platform.

Speed of sound with respect to listener $= v_s + v_w = 340 + 10 = 350$ m/s.

Verifies the option (*b*).

As the distance between listener and source does not change so frequency of sound does not change as heard by listener. i.e., he heard $\nu_0 = 400$ Hz. Verifies option (*a*), rejects the option (*c*) and (*d*) as it is constant $\nu_0 = 400$ Hz.

Q15.17. Which of the following statements are true for stationary waves?

(*a*) Every particle has a fixed amplitude which is different from the amplitude of it's nearest particle.

(*b*) All the particles cross their mean position at the same time.

(*c*) All the particles are oscillating with same amplitude.

(*d*) There is no net transfer of energy across any plane.

(*e*) There are some particles which are always at rest.

Ans. (*a*, *b*, *d*, *e*): In stationary waves $[y(x, t) = a \sin kx \cos \omega t]$ the particles between two nodes vibrates with different amplitude which increases from node to antinodes from **zero to maximum**, and then decreases from **maximum to zero** from antinodes to nodes. The amplitude of a particle will remain constant $a \cos kx$, but varies with λ

$\because\ k = \dfrac{2\pi}{\lambda}$. Hence **verifies the option (*a*), rejects option (*c*)**, the amplitude of particles at nodes has amplitude zero verifies option (*e*). As the particles at nodes are rest so energy does not transfer verifies option (*d*).

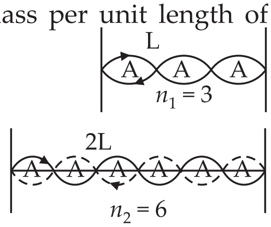
Particles between two nodes are in same phase i.e., motion of particles between two nodes will be either upward or downward and crosses the mean position at same time. Hence verifies option (*b*).

VERY SHORT ANSWER TYPE QUESTIONS

Q15.18. A sonometer wire is vibrating in resonance with a tuning fork keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be is resonance with the wire?

Ans. We know then when a wire of length L vibrates its resonant frequency in nth mode after stretching its by a tension T then frequency of nth harmonic is $\nu = \dfrac{n}{2L}\sqrt{\dfrac{T}{m}}$, here m is mass per unit length of stretched wire.

Let in given two cases



$$\nu_1 = \frac{n_1}{2L_1}\sqrt{\frac{T_1}{m_1}}$$

$$\nu_2 = \frac{n_2}{2L_2}\sqrt{\frac{T_1}{m_1}}$$

In given question $T_1 = T_2 = T$ (given)

$m_1 = m_2 = m$ as wire same

$L_2 = 2L_1$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{n_1\sqrt{T}\sqrt{m} \times 2 \times 2L_1}{n_2\sqrt{T}.\sqrt{m}\ 2L_1} = \frac{2n_1}{n_2}$$

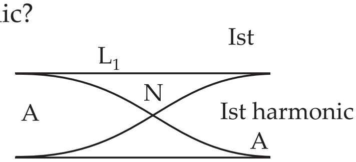
As tuning fork is same i.e., in both harmonics n_1 and n_2 frequency of resonance same $\therefore$ $\nu_1 = \nu_2$ or $\dfrac{2n_1}{n_2} = 1$

$$n_2 = 2n_1$$

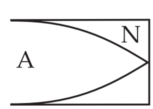
So when length of wire double the number of harmonics double for same resonant frequency, of tension and m.

Q15.19. An organ pipe of length L open at both ends is found to vibrate in its first harmonic when sounded with tuning fork 480 Hz. What should be the length of a pipe closed at one end so that it also vibrates in it's first harmonic with same harmonic?

Ans. As, the medium and frequency and number of harmonic in open and closed pipes are same, so number of nodes and λ (wave), in both cases will be same.



Both end open pipe



One end close pipe

In both end open pipe

$$L_1 = \frac{2 \times \lambda_1}{4} \quad \text{or} \quad \lambda_1 = 2L_1$$

and $\nu_1 = \dfrac{c}{\lambda_1}$, $\nu_1 = \dfrac{c}{2L_1}$

In one end open pipe

$$L_2 = \frac{1\lambda_2}{4} \quad \text{or} \quad \lambda_2 = \frac{c}{4L_2}$$

As medium and tuning fork in both cases are same $\nu_1 = \nu_2$ and $c_1 = c_2 = c$

$$\nu_2 = \frac{c}{4L_1} \quad \text{or} \quad \nu_1 = \nu_2$$

$$\frac{c}{2L_1} = \frac{c}{4L_2}$$

or $$4L_2 = 2L_1 \quad \text{or} \quad L_2 = \frac{L_1}{2}$$

So the length of one end closed pipe will be half of both end open pipe for resonant at Ist harmonic with same frequency.

Q15.20. A tuning fork 'A' marked 512 Hz produces 5 beats per second, when sounded with another unmarked tuning fork B. If B is loaded with wax. number of beats is again 5 per second. What is the frequency of tuning fork 'B' when unloaded.

Ans. $\nu_A = 512$, $\because$ $\nu_0 = \nu_A \sim \nu_B$ no. of frequency of beats

As on loading frequencies of B decreased

So in first case $\nu_1 = 5$

$$\nu_B = \nu_A \pm 5$$

When B is loads frequency of B is ν

$\therefore$ $\nu_B = 512 \pm 5$ i.e., ν_B either 507 or 517.

On load in frequency of B decreased 507 to lower value of number of beats will increase so $\nu_B \neq 507$. Now if $\nu_B = 517$ then its frequency decrease by 10 Hz then number of betas will also be same as $512 - 507 = 5$. So frequency of tuning fork when unloaded is **517**.

Q15.21. The displacement of elastic wave is given by the function $y = 3 \sin \omega t + 4 \cos \omega t$, where y is in cm and t is in second, calculate the resultant amplitude.

Ans. $\because$ $y = 3 \sin \omega t + 4 \cos \omega t$ ...(*i*)

Let $3 = a \cos \phi$ ...(*ii*) and $4 = a \sin \phi$ ...(*iii*)

Then $y = a \cos \phi \sin \omega t + a \sin \phi \cos \omega t$

$$y = a \sin (\omega t + \phi)$$

From (*ii*) and (*iii*)

$$\tan \phi = \frac{4}{3} \quad \text{or} \quad \phi = \tan^{-1}\frac{4}{3}$$

On squaring and adding (*ii*) and (*iii*) equations

$$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 3^2 + 4^2$$

$$a^2(\cos^2 \phi + \sin^2 \phi) = 9 + 16$$

$$a^2 = 25 \quad \Rightarrow \quad a = 5$$

$$y' = 5 \sin (\omega t + \phi) \text{ when } \phi = \tan^{-1}\frac{4}{3}$$

Hence, New amplitude is 5 cm.

Q15.22. A sitar wire is replaced by another wire of same length and material, but of three times their earlier radius. If the tension remains same, by what factor will the frequency change.

Ans. The wire is stretched both end so frequency of stretched wire is

$$\nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

As number of harmonic n, length L and tension (T) are kept same in both cases. $\therefore\ \nu \propto \frac{1}{\sqrt{m}}$

$$\frac{\nu_1}{\nu_2} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \quad ...(i)$$

$$\text{Mass per unit length} = \frac{\text{mass of wire}}{\text{length}} = \frac{(\pi r^2 l)\rho}{l}$$

$$m = \pi r^2 \rho$$

As material of wire is same.

$$\frac{m_2}{m_1} = \frac{\pi r_2^2 \rho}{\pi r_1^2 \rho} = \frac{(3r)^2}{r^2} = \frac{9}{1}$$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \sqrt{\frac{9}{1}} = \frac{3}{1} \quad \therefore\ \nu_2 = \frac{1}{3}\nu_1$$

So the frequency of sitar reduced by $\frac{1}{3}$ of previous value.

Q15.23. At what temperature (in °C) will the speed of sound in air be 3 times of its speed at 0°C?

Ans. $v \propto \sqrt{T}$ $\quad \frac{v_T}{v_0} = \sqrt{\frac{T}{T_0}}$

$$v_T = 3v_0 \text{ (given)}$$

$$\therefore \quad \frac{3v_0}{v_0} = \sqrt{\frac{T}{273+0}} \quad \text{or} \quad \sqrt{T} = 3\sqrt{273}$$

$$T = 9 \times 273 = 2457 \text{ K}$$

or $\quad T = 2457 - 273 = 2184°C.$

Q15.24. When two waves of almost equal frequencies n_1 and n_2 reached at a point simultaneously. What will be the time interval between successive maxima?

Ans. (*b*): Here frequencies of vibrations are nearly equal but exactly different $n_1 \simeq n_2$ so beats are form in the medium when they produces sound, the number of let $n_2 > n_1$.

Then number of beats per second i.e., frequency of maximas = $n = n_2 - n_1$.

So time period of maxima or beats = $\frac{1}{n} = \frac{1}{n_2 - n_1}$ second.

SHORT ANSWER TYPE QUESTIONS

Q15.25. A steel wire has a length of 12 m and mass of 2.10 kg. What will be speed of transverse wave in this wire? When tension is 2.06×10^4 N is applied.

Ans. $l = 12$ m M (Total mass) = 2.10 kg

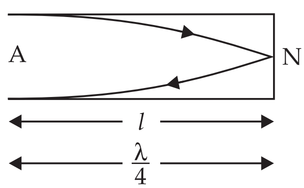
$$m = \frac{M}{l} = \frac{2.1}{12} \qquad T = 2.06 \times 10^4 \text{ N}$$

$$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{2.06 \times 10^4 \times 12}{2.10}} = \sqrt{\frac{1236 \times 10^4}{105}}$$

$$= \sqrt{11.77} \times 10^2 = 3.43 \times 10^2$$

$$v = 343.0 \text{ m/s.}$$

Q15.26. A pipe 20 cm is closed at one end. Which harmonic mode of the pipe is resonantly excited by source 1237.5 Hz (velocity of sound in air = 330 m/s)?



Ans. $l = 20$ cm = 0.2 m, $\nu = 1237.5$ Hz

$v = 330$ m/s

$l = \frac{\lambda}{4}$ or $\lambda = 4l$

for fundamental frequency

$$\therefore \quad \nu_1 = \frac{v}{\lambda} = \frac{v}{4l}$$

$$= \frac{330}{4 \times 20 \times 10^{-2}} = 412.5 \text{ Hz}$$

$$\nu \text{ (given)} = 1237.5 \text{ Hz}$$

$$\therefore \quad \frac{\nu(g)}{\nu_1} = \frac{1237.5}{412.5} = \frac{3}{1}$$

∵ The frequency in Ist, IInd, IIIrd ... harmonic are in the ratio 1 : 2 : 3 : 4 ... in one end open pipe. So there is IIIrd harmonic excited by 1237.5 Hz frequency.

Q15.27. A train standing at the outer signal of a railway station blows a whistle of 400 Hz in still air. The train begins to move with a speed of 10 m/s towards platform. What is the frequency of sound for an observer standing on the platform? (Sound velocity in air = 330 m/s)

Ans. $\nu_0 = 400$ Hz $v_s = 10$ m/s

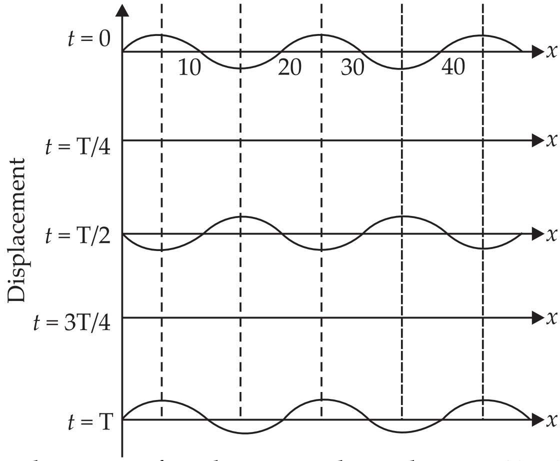
Velocity of sound in air $v_a = 330$ m/s

Apparent frequency by observer standing on platform

$$\nu' = \frac{v_a}{(v_a - v_s)}\nu_0 = \frac{330 \times 400}{(330 - 10)}$$

$$\nu' = \frac{330 \times 400}{320} = \frac{825}{2} = 412.5 \text{ Hz.}$$

Q15.28. The wave pattern on the stretched string is shown in figure. Interpret what kind of wave this is and find its wavelength.



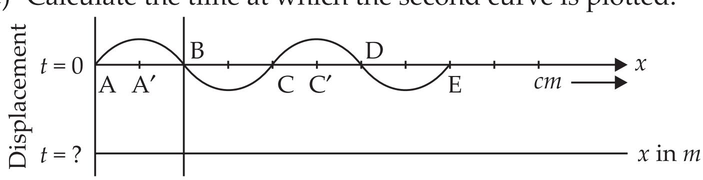
Ans. The displacement of medium particles at distance 10, 20, 30, 40 and 50 cm are always rest which is the property of nodes in stationary wave. At $t = \frac{T}{4}$ and $\frac{3T}{4}$ all particles are at rest which is in stationary wave when the particle crosses its mean position.

So the graph of wave shows stationary wave. The wave at $x = 10, 20, 30, 40$ cm there are nodes and distance between successive nodes is $\frac{\lambda}{2}$

$$\therefore \qquad \frac{\lambda}{2} = (30 - 20) \quad \text{or} \quad \lambda = 20 \text{ cm.}$$

Q15.29. The pattern of standing waves formed on a stretched string at two instant of time are shown in figure. The velocity of the two waves super-imposing to form stationary wave is 360 ms^{-1} and their frequencies are 256 Hz.

(*a*) Calculate the time at which the second curve is plotted.



(*b*) Mark nodes and anti-nodes on the curve.

(*c*) Calculate the distance between A′ and C′.

Ans. Given frequency of the wave $\nu = 256$ Hz

$$\therefore \qquad T = \frac{1}{\nu} = \frac{1}{256} \text{ second} = 0.00390$$

$$T = 3.9 \times 10^{-3} \text{ seconds.}$$

(*a*) In stationary wave a particle passes through it's mean position after every $\frac{T}{4}$ time

$\therefore$ In IInd curve displacement of all medium particle, are zero so

$$t = \frac{T}{4} = \frac{3.9 \times 10^{-3}}{4} = .975 \times 10^{-3} \text{ sec}$$

$$t = 9.8 \times 10^{-4} \text{ second.}$$

(*b*) Points does not vibrate i.e., their displacement is zero always so nodes are at A, B, C, D and E. The points A′ and C′ are at maximum displacements so there are anti-nodes at A′ and C′.

(*c*) At A′, C′ there are consecutive anti-nodes so the distance between A′ and C′ $= \lambda = \frac{v}{\nu} = \frac{360}{256} = \frac{90}{64} = 1.41$ m.

Q15.30. A tuning fork vibrating with a frequency of 512 Hz is kept close to open end of a tube filled with water (figure). The water level in the tube is gradually lowered.

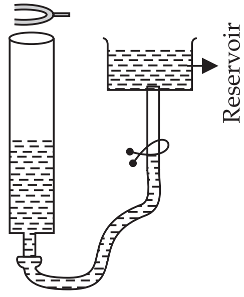
When the water level is 17 cm below the open end, maximum intensity of sound is heard. If the room temperature is 20°C. Calculate:

(*a*) speed of sound in air at room temperature.

(*b*) speed of sound in air at 0°C.

(*c*) if the water in the tube is replaced with mercury, will there be any difference in your observations?

Ans. (*a*) Pipe filled partially with water or mercury behave like an one end open organ pipe. In first harmonic, one anti-node and one node at water level formed so, first harmonic or at maximum intensity is heard at L = 17 cm so



$$L = \frac{\lambda}{4} \quad \therefore \quad \lambda = 4L = 4 \times 17$$

$$\lambda = 68 \text{ cm} = 0.68 \text{ m}$$

$$v = \nu\lambda = 512 \times 0.68 \text{ m/s}$$

$$v = 348.16 \text{ m/s}$$

So velocity of sound in air at room temperature 20°C is 348.16 m/s (v_{20})

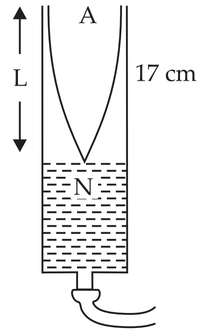
(*b*) $\because v \propto \sqrt{T} \quad \therefore \quad \frac{v_0}{v_T} = \sqrt{\frac{T_0}{T}}$

$T_0 = 273$ K

$v_T = v_{20} = 348.16 \qquad T = 20 + 273 = 293$ K

$$v_0 = v_T\sqrt{\frac{273}{293}} = 348.16\sqrt{0.9317}$$

$$v_0 = 348.16 \times 0.96526 = 336 \text{ m/s}$$

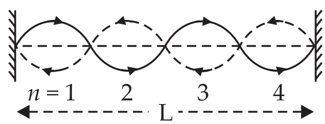


(*c*) Water and mercury in tube reflects the sound into air column to form stationary wave and reflection is more in mercury than water as mercury is more denser than water. So intensity of sound heard will be larger but reading does not change as medium in tube (air) and tuning fork are same.

Q15.31. Show that when a string fixed at its ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops the frequencies are in the ratio 1 : 2 : 3 : 4.

Ans. let n be the number of loops in the string.

The length of each loop is $\frac{\lambda}{2}$



$$\therefore \qquad L = \frac{n\lambda}{2} \quad \text{or} \quad \lambda = \frac{2L}{n}$$

$$v = \nu\lambda \quad \text{and} \quad \lambda = \frac{v}{\nu}.$$

$$\text{So} \qquad \frac{v}{\nu} = \frac{2L}{n}$$

$$\nu = \frac{n}{2L}.v$$

v in stretch string = $\sqrt{\frac{T}{m}}$

$$\therefore \qquad \nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

For $n = 1$, $\quad \nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu_0$

If $n = 2$ then $\quad \nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu_0$

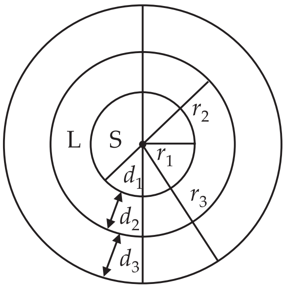
$n = 3$ then $\quad \nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu_0$

$\therefore\ \nu_1 : \nu_2 : \nu_3 : \nu_4 = n_1 : n_2 : n_3 : n_4 = 1 : 2 : 3 : 4$

LONG ANSWER TYPE QUESTIONS

Q15.32. The earth has the radius 6400 km. The inner core of 1000 km radius is solid. Outside it there is a region from 1000 km to a radius 3500 km which is in molten state. Then again 3500 km to 6400 km the earth is solid. Only longitudinal (P) waves can travel inside a liquid.

Assume that P waves have a speed of 8 km/second in solid part and of 5 km/second in liquid part of earth. An earthquake occurs at some place close to the surface of earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter.



Ans. $r_1 = 1000$ km

$r_2 = 3500$ km

$r_3 = 6400$ km

$d_1 = 1000$ km

$d_2 = 3500 - 1000 = 2500$ km

$d_3 = 6400 - 3500 = 2900$ km

Solid distance diametrically

$$= 2(d_1 + d_3) = 2(1000 + 2900)$$

$$= 2 \times 3900 \text{ km}$$

Time taken by wave produced by earthquake in solid part

$$= \frac{3900 \times 2}{8} \text{ sec}$$

Liquid part along diametrically $= 2d_2 = 2 \times 2500$

$\therefore$ Time taken by seismic wave in liquid part $= \dfrac{2 \times 2500}{5}$

$$\text{Total time} = \frac{2 \times 3900}{8} + \frac{2 \times 2500}{5} = 2\left[\frac{3900}{8} + \frac{2500}{5}\right]$$

$$= 2[487.5 + 500] = 2 \times 987.5 = 1975 \text{ sec.}$$

$$= 32 \text{ min } 55 \text{ sec.}$$

Q15.33. If c is the r.m.s. speed of molecules in a gas and v is the speed of sound wave in the gas. show that $\dfrac{c}{v}$ is constant and independent of temperature for all diatomic gases.

Ans. We know that $c = \sqrt{\dfrac{3P}{\rho}}$ for molecules.

$$c = \sqrt{\frac{3RT}{M}}$$

$\therefore$ $\dfrac{P}{\rho} = \dfrac{RT}{M}$ $\quad \because \dfrac{P}{\rho} = \dfrac{RT/V}{M/V}$ $\quad \because PV = nRT$, $n = 1$, $P = \dfrac{RT}{V}$

M = molar mass of gas

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{c}{v} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{\gamma RT}{M}}} = \sqrt{\frac{3}{\gamma}}$$

$$\gamma = \frac{C_p}{C_v} = \text{adiabatic constant for diatomic gas}$$

$$\gamma = \frac{7}{5}$$

$$\therefore \quad \frac{c}{v} = \sqrt{\frac{3}{7/5}} = \sqrt{\frac{15}{7}} = \text{constant}.$$

Q15.34. Given below are some functions of x and t to represent the displacement of an elastic wave.

(*i*) $y = 5 \cos (4x) \sin 20\, t$

(*ii*) $y = 4 \sin \left(5x - \frac{t}{2}\right) + 3 \cos \left(5x - \frac{t}{2}\right)$

(*iii*) $y = 10 \cos [(252 - 250)\pi t] \cos [(252 + 250)\pi t]$

(*iv*) $y = 100 \cos [100\pi t + 0.5x)$

State which of these represent

(*a*) a travelling wave along (– x) direction

(*b*) a stationary wave (*c*) beats

(*c*) a travelling wave along (+ x) direction.

Give reasons for the answers.

Ans. (*a*) A travelling wave along (– x) direction must have + kx i.e., in (*iv*) $y = 100 \cos (100\pi t + 0.5x)$ so (*a*) (*iv*).

(*b*) A stationary wave of the for $y = 5 \cos (4x) \sin 20t$ is a stationary wave so (*b*) (*i*).

(*c*) Beats involve $(\nu_1 + \nu_2)$ and $(\nu_1 - \nu_2)$ so beats can be represented by $y = 10 \cos [(252 - 250)\pi t]$ represents beat so (*c*) (*iii*).

(*d*) $y = 4 \sin \left(5x - \frac{t}{2}\right) + 3 \cos \left(5x - \frac{t}{2}\right)$ ...(*i*)

Let $4 = a \cos \phi$ ...(*ii*) and $3 = a \sin \phi$ ...(*iii*)

$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 4^2 + 3^2$ squaring and adding (*ii*), (*iii*)

$a^2 = 25 \Rightarrow a = 5$

Substituting (*ii*), (*iii*) in (*i*)

$$y = a \cos \phi \sin \left(5x - \frac{t}{2}\right) + a \sin \phi \cos \left(5x - \frac{t}{2}\right)$$

$$y = a \sin \left(5x - \frac{t}{2} + \phi\right)$$

$$y = 5\sin\left(5x - \frac{t}{2} + \phi\right)$$

Which represents the progressive wave in + x direction as the sign of kx (or $5x$) and $\omega t\left(\frac{1}{2}t\right)$ are opposite so it travels in + x direction. So (d) (ii)

Q15.35. In given progressive wave $y = 5\sin(100\pi t - 0.4\pi x)$, where x, y is in m and t in seconds, what is the

(a) amplitude
(b) wavelength
(c) frequency
(d) wave velocity
(e) particle velocity amplitude

Ans. Standard form of progressive wave travelling in + x direction (kx and ωt have opposite sign is given)

eqn. is $$y = a\sin(\omega t - kx + \phi)$$
$$y = 5\sin(100\pi t - 0.4\pi x + 0)$$

(a) Amplitude $a = 5$ m

(b) Wavelength λ, $k = \frac{2\pi}{\lambda}$
$$k = 0.4\pi$$
$$\lambda = \frac{2\pi}{k} = \frac{2\times\pi}{0.4\pi} = 5 \text{ m}$$

(c) Frequency ν, $\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi}$ $\quad\because \omega = 100\pi$

$$\therefore \quad \nu = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

(d) Wave velocity $v = \nu\lambda = 50 \times 5 = 250$ m/s

(e) Particle (medium) velocity in the direction of amplitude at a distance x from source.
$$y = 5\sin(100\pi t - 0.4\pi x)$$
$$\frac{dy}{dt} = 5 \times 100\pi\cos(100\pi t - 0.4\pi x)$$
For maximum velocity of particle is at its mean position
$\cos(100\pi t - 0.4\pi x) = 1$
$\Rightarrow 100\pi t - 0.4\pi x = 0$
$$\therefore \left(\frac{dy}{dt}\right)_{max} = 5 \times 100\pi \times 1$$

v_{max} of medium particle = 500π m/s.

Q15.36. For the harmonic travelling wave
$$y = 2\cos 2\pi(10t - 0.0080x + 3.5)$$
where x and y are in cm and t is in second. What is the phase difference between the oscillatory motion at two points separated by a distance of

(*a*) 4 m (*b*) 0.5 m

(*c*) $\frac{\lambda}{2}$ (*d*) $\frac{3\lambda}{4}$ at given instant of time

(*e*) What is the phase difference between the oscillation of the particle located at $x = 100$ cm at $t = \text{T}$ second and $t = 5$ s?

Ans.
$$y = 2 \cos 2\pi\,(10t - 0.0080x + 3.5)$$
$$y = 2 \cos (20\pi t - 0.016\pi x + 7.0\pi)$$

Wave is propagated in + x direction because ωt and kx are in with opposite sign standard equation $y = a \cos (\omega t - kx + \phi)$

$a = 2,\quad \omega = 20\pi,\quad k = 0.016\pi \quad$ and $\quad \phi = 7\pi$

(*a*) Path difference $p = 4$ m (given) $= 400$ cm

phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times p = \frac{2\pi}{\lambda} \times 400$

$$\Delta\phi = k \times 400 = 0.016\pi \times 400$$

phase difference $\Delta\phi = 6.4\pi$ rad.

(*b*) Path difference $p = 0.5$ m $= 50$ cm

$$\Delta\phi = kp = 0.016\pi \times 50 = 0.8\pi \text{ rad.}$$

(*c*) Path difference $p = \frac{\lambda}{2}$

$$\Delta\phi = \frac{2\pi}{\lambda} p = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ radian}$$

(*d*)
$$\Delta\phi = \frac{2\pi}{\lambda} p = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3}{2}\pi \text{ radian}$$

(*e*)
$$\text{T} = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} \text{ sec}$$

$$x = 100 \text{ cm}$$
$$t = \text{T}$$

At $x = 100$, $t = \text{T}$

$\phi_1 = 20\pi\text{T} - 0.016\pi\,(100) + 7\pi = 20\pi \times \frac{1}{10} - 1.6\pi + 7\pi = 7.4\pi$

At $t = 5$ s

$\phi_2 = 20\pi(5) - 0.016\pi\,(100) + 7\pi = 100\pi - 1.6\pi + 7\pi = 105.4\pi$

$\phi_2 - \phi_1 = 105.4\pi - 7.4\pi = 98\pi$ radian.

❑❑❑

Sample Question Paper-I

Q1. If momentum (P), Area (A) and time (T) are taken to be fundamental quantities, then energy has the dimension at formula

(*a*) $[P^1A^{-1}T^1]$ (*b*) $[P^2A^1T^1]$ (*c*) $[P^1A^{-1/2}T^1]$ (*d*) $[P^1A^{1/2}T^{-1}]$

Sol. (*a*):

$$E = [P]^a [A]^b [T]^c$$

$$[ML^2T^{-2}] = [MLT^{-1}]^a [L^2]^b [T]^c$$

$$[ML^2T^{-2}] = [M^a L^{a+2b} T^{-a+c}]$$

Comparing the powers of M, L and T

$a = 1$

$a + 2b = 2$, $1 + 2b = 2$, $2b = 2 - 1$, $b = \frac{1}{2}$

$-a + c = -2$, $-1 + c = -2$, $c = -2 + 1$, $c = -1$

$\therefore$ $[E] = [P^1 A^{1/2} T^{-1}]$ verifies the option (*d*).

Q2. The average velocity of particle is equal to instantaneous velocity what is the nature of the motion.

Sol. A motion in which velocity (magnitude and direction) remains always constant and magnitude of velocity constant always so the magnitude of average velocity is equal to the magnitude of velocity of any time (instantaneous) and direction does not change.

So, the average velocity of particle is equal to the instantaneous velocity in **Uniform Motion**.

Q3. A force $F = (6\hat{i} - 3\hat{j})$ N acts on a mass of 2 kg. Find the magnitude of acceleration.

Sol. $\because$

$$\vec{F} = m\vec{a}$$

$$(6\hat{i} - 3\hat{j}) = 2\vec{a}$$

$$\vec{a} = 3\hat{i} - \frac{3}{2}\hat{j} = 3\hat{i} - 1.5\hat{j}$$

$$|\vec{a}| = \sqrt{3^2 + (-1.5)^2} = \sqrt{9 + 2.25} = \sqrt{11.25}$$

$$|\vec{a}| = 3.35 \text{ m/s}^2$$

Q4. The work done by body against friction always results

(*a*) loss of kinetic energy (*b*) loss of potential energy

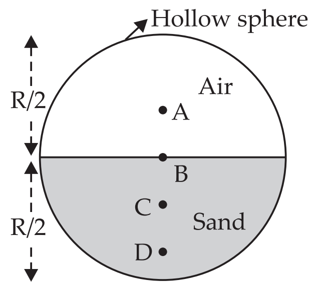
(*c*) gain in kinetic energy (*d*) gain in potential energy

Sol. (*a*): As friction force always opposes the relative motion of body (not the position) so it always decreases the velocity *i.e.* The work done by a body against friction always results loss in kinetic energy. Hence, verified the option (*a*).

Q5. Which of the following points is the likely position of the centre of mass of the system shown in figure:

(*a*) A (*b*) B

(*c*) C (*d*) D



Sol. (*c*): The sand is filled inside hollow sphere upto half *i.e.*, upto centre of hollow sphere *i.e.*, in hollow sphere half-half portion are filled with air and sand.

As the density of air is much smaller than sand and density is symmetric to vertical diameter. So centre of mass will shift along diametrically near the centre B. Hence, new centre of mass of the system will be at C, verifies the option (*c*).

Q6. Two molecules of a gas have speed 9×10^6 m/s, 1.0×10^6 m/s respectively, what is its r.m.s. speed?

Sol.
$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2}{2}} = \sqrt{\frac{(9 \times 10^6)^2 + (1.0 \times 10^6)^2}{2}}$$

$$v_{rms} = 10^6\sqrt{\frac{81+1}{2}} = 10^6 \times \sqrt{\frac{82}{2}} = \sqrt{41} \times 10^6$$

$$v_{rms} = 6.4 \times 10^6 \text{ m/s}$$

Q7. A particle in S.H.M. has displacement $x = 3\cos(5\pi t + \pi)$, where x is in metres and t in seconds. Where is particle at $t = 0$, $t = \frac{1}{2}$ sec?

Sol. At $t = 0$,
$$x = 3\cos(0 + \pi) = 3\cos\pi = 3(-1)$$
$$x = -3 \text{ m}$$

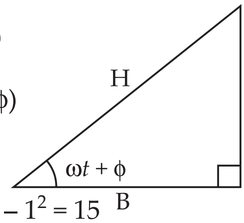
At $t = \frac{1}{2}$ sec,
$$x = 3\cos\left[5\pi\left(\frac{1}{2}\right) + \pi\right] = 3\cos\cos(3.5)\pi$$
$$x = 3\cos(2\pi + 1.5\pi) = 3\cos(1.5\pi)$$
$$= 3\cos\left(\pi + \frac{\pi}{2}\right) = 3\left(-\cos\frac{\pi}{2}\right)$$
$$= 3(-0)$$
$$x = 0 \text{ m}$$

Q8. When the displacement of a particle in S.H.M. is one fourth of the amplitude, what fraction of the total energy is the kinetic energy?

Sol. Let S.H.M.
$$x = A\sin(\omega t + \phi) \quad ...(i)$$
$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi) \quad ...(ii)$$

When $x = \frac{1}{4}$ A (given)

From (i) $\frac{A}{4} = A \sin(\omega t + \phi)$



$$\sin(\omega t + \phi) = \frac{1}{4}$$

$$B^2 = H^2 - P^2 = 4^2 - 1^2 = 15$$

$$B = \sqrt{15}$$

v = velocity (v) when amplitude is $\frac{A}{4} = A\omega \times \frac{\sqrt{15}}{4}$ [using (ii)]

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m A^2 \omega^2 \frac{15}{16}$$

Total Energy (T.E.) = $\frac{1}{2}mA^2\omega^2 \left[\text{WD} = \int F\, dx = \int kx\, dx = \frac{kx^2}{2} \right]$

$$\text{Required ratio} = \frac{\text{K.E.}}{\text{T.E.}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mA^2\omega^2}$$

$$= \frac{\frac{1}{2}m A^2 \omega^2}{\frac{1}{2}mA^2\omega^2}\frac{15}{16} = \frac{15}{16}$$

Q9. The displacement of a progressive wave is represented by $y = A \sin(\omega t - kx)$ where x is distance and time t write the dimensional formula of (i) ω (ii) k.

Sol. The angle or trigonometrical ratio has no unit or no dimensional formula so $\omega t - kx$ has no dimensional formula or dimensions of ωt or kx must be $[M^0L^0T^0]$ *i.e.*,

$$[\omega]\,[t] = [M^0L^0T^0]$$

$$[M^aL^bT^c][T^1] = [M^0L^0T^0]$$

$$[M^aL^bT^{c+1}] = [M^0L^0T^0]$$

c comparing the powers $a = 0$, $b = 0$, $c + 1 = 0$ or $c = -1$

$\therefore$ $[\omega] = [M^0L^0T^{-1}]$

Similarly $[k]\,[x] = [M^0L^0T^0]$

$$[M^aL^bT^c]\,[L^1] = [M^0L^0T^0]$$

$$[M^aL^{b+1}T^c] = [M^0L^0T^0]$$

$a = 0$, $b + 1 = 0$ or $b = -1$, $c = 0$

So $[k] = [M^0L^{-1}T^0]$

Q10. 100 g water is super cooled to – 10°C. At this point due to some disturbance mechanised or otherwise some of it suddenly freezes to ice. What will be the temperature of the resultant mixture and how much mass would freeze?

Sol. As the super cooled water converts into mixture of ice and water. So the final temperature of mixture will be 0°C and temperature of water and ice will be at 0°C.

By the principle of calorimetry.

Heat gain by m gm of – 10°C to °C water

$$mc\,\Delta t = m'L = \text{heat lost by } x \text{ gm } 0°C \text{ water to } 0°C \text{ ice}$$

$$100 \times 1 \times [0 - (-10)] = x \times 80$$

$$x = \frac{\overset{25}{\cancel{1000}}}{\underset{2}{\cancel{80}}} = 12.5 \text{ gm ice in the mixture}$$

Final temperature of mix. °C ice and 0°C water = 0°C

Q11. One day in the morning to take bath, I filled up $\frac{1}{3}$ bucket of hot water from geyser. Remaining two third was to be filled by cold water (at room temperature) to bring the mixture to a comfortable temperature. Suddenly I had to attend to some work which would take say 5–10 minutes before I can take bath. Now I had two options (*i*) fill the remaining bucket completely by cold water and then attend to the work (*ii*) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer? Explain.

Sol. We know that loss of heat from hot body by Newton's law of cooling is that "the rate of loss of heat from hot body is directly proportional to the difference between the hot body and surrounding."

In first case of the bucket of hot water is filled with cold water its temperature lowered, so temperature difference between the surrounding temperature and water in bucket becomes smaller as compared to the difference in temperature of surrounding and hot water from geyser, so loss of heat in same time (to do work) **will be smaller in (*i*) case** than of (*ii*) case.

Q12. Prove the following:

For two angles of projection θ and (90° – θ) with horizontal with same velocity V.

(*a*) range is the same

(*b*) heights are in the ratio $\tan^2\theta : 1$

Sol. (*a*): We know that horizontal range of projectile, with velocity V and at an angle θ with horizontal is

$$R = \frac{V^2 \sin 2\theta}{g}$$

Now, if the angle of projection θ is changed from θ to (90° – θ) then

$$R' = \frac{V^2 \sin 2(90° - \theta)}{g}$$

$$R' = \frac{V^2 \sin(180° - 2\theta)}{g}$$

$$R' = \frac{V^2 \sin 2\theta}{g}$$

$R' = R$ Hence to be proved.

(*b*) Height attain by projectile

$$H = \frac{V^2 \sin^2 \theta}{2g}$$

Now, again $\theta \to 90° - \theta$, $H' = \dfrac{V^2 \sin^2 (90° - \theta)}{2g}$

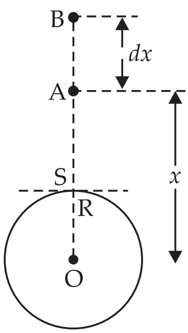
$$\Rightarrow \quad H' = \frac{V^2 \cos^2\theta}{2g}$$

$$\frac{H}{H'} = \frac{\dfrac{V^2 \sin^2 \theta}{2g}}{\dfrac{V^2 \cos^2 \theta}{2g}} \quad \Rightarrow \quad \frac{H}{H'} = \frac{\tan^2 \theta}{1}$$

Hence, required ratio = $\tan^2 \theta : 1$.

Q13. What is meant by escape velocity? Obtain an expression for escape velocity of an object projected from the surface of the earth.

Sol. **Escape Velocity:** The minimum upward velocity given to an object so that it may just escape out from the gravitational force of earth.



Expression for escape velocity: 'O' is the centre of earth (spherical) from the centre consider a point A. Consider body of mass m at height x from centre of earth. Gravitational force of attraction due to earth.

$$F = \frac{GMm}{x^2}$$

W.D. to raise the body from A to B by distance dx is

$$dW = F.dx = \frac{GMm}{x^2} dx$$

$$\int_0^W dW = \int_R^\infty \frac{GMm}{x^2} dx = GMm \int \frac{dx}{x^2}$$

$$W = GMm\left[\frac{-1}{x}\right]_R^\infty = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$W = \frac{GMm}{R}$$

Let v_e is escape velocity on the surface of earth then

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\because \quad g = \frac{GM}{R^2} \quad \text{or} \quad GM = gR^2$$

$$\therefore \quad v_e = \sqrt{\frac{2.gR^2}{R}} = \sqrt{2gR}$$

$$g = 9.8, \quad R = 6.4 \times 10^6 \text{ m}$$

$$v = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

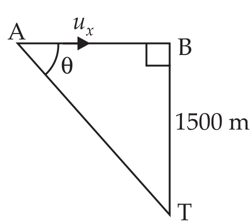
$$v_e = \sqrt{2 \times 2 \times 7 \times 7 \times 8 \times 8 \times 10^4}$$

$$v = 2 \times 7 \times 8 \times 10^2 \text{ m/s}$$

$$= 11200 \text{ m/s} = 11.2 \text{ km/s}$$

Q14. A fighter plane is flying horizontally at an altitude of 1.5 km with speed 720 km/hr. At what angle of sight (w.r.t. horizontal) when the target is seen, should the pilot drop the bomb in order to hit the target?

Sol. $u_x = 720 \text{ km/hr} = \overset{40}{\cancel{720}} \times \frac{5}{\cancel{18}} \frac{m}{s} = 200 \text{ m/s}$



Height from target (T) = 1.5 km = 1500 m

A = Position of fighter from where packet should be drop. So that bomb hit the target T.

$\theta = ?$

Time taken by bomb moving with $u_x = 200$ m/s is t then

$$s_y = u_y t + \frac{1}{2}at^2 \quad \text{(Vertical motion of bomb)}$$

$$1500 = 0 \times t + \frac{1}{2} \times 10t^2$$

$$t^2 = \frac{1500}{5} = 300$$

$$t = \sqrt{300} \text{ sec} = 10\sqrt{3} \text{ sec}$$

$\therefore$ Horizontal distance

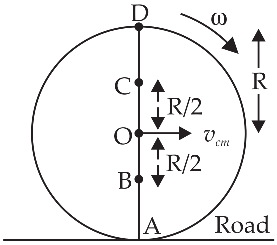
$$AB = u_x \times t = 200 \times 10\sqrt{3} = 2000\sqrt{3} \text{ m}$$

$$AB = 2000 \times 1.732 = 3464 \text{ m}$$

$$\tan\theta = \frac{1500}{2000\sqrt{3}} = \frac{3}{4 \times \sqrt{3}} = \frac{\sqrt{3}\ \cancel{\sqrt{3}}}{4\cancel{\sqrt{3}}} = \frac{\overset{0.433}{\cancel{1.732}}}{\cancel{4}}$$

$$\theta = \tan^{-1} 0.433$$

Q15. A sphere of radius R rolls without slipping on horizontal road. A, B, C and D are the four points on the vertical line through the point of contact 'A' (figure). What are the translational velocity of particles at point A, B, C and D?



Sol. The velocity of the centre of mass is v_{cm}.

(*i*) As the motion in without slipping *i.e.*, point A does not slip on road so the velocity of point A w.r.t. road is zero or $v_A = 0$.

(*ii*) v_0: centre 'O' of sphere remains always at a vertical distance of R. So it has not rotational motion, it has only translational motion v_0. But $v_0 = \omega R$

$\therefore$ $v_0 = \omega R$, where ω is angular velocity

$$v_A = v_{cm} - \omega R \quad \text{but} \quad v_A = 0$$
$$v_{cm} = \omega R \quad \text{or} \quad v_0 = \omega R = v_{cm}$$

Point C moves in forward and O in backward, while sphere rolls, so $v_c = v_o$ + velocity of C w.r.t. O $= \omega R + \omega\left(\frac{R}{2}\right)$

$$v_c = \frac{3}{2}\omega R = \frac{3}{2}v_{cm}$$

$$\therefore \qquad v_c = \frac{3}{2}v_{cm}$$

Similarly point D and O moves in opposite direction, while the sphere rolls.

So $\qquad v_D = v_o + \omega R = \omega R + \omega R$

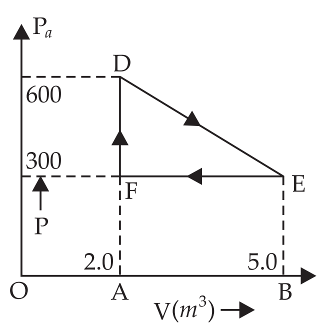
$$v_D = 2\omega R = 2v_{cm} \quad \text{or} \quad v_D = 2v_{cm}$$

Point B is at a distance $\frac{R}{2}$ from the point A which always in rest so its velocity w.r.t. A or Road $= v_B = \omega\frac{R}{2} = \frac{\omega R}{2}$

$$v_B = \frac{v_{cm}}{2}$$

Q16. A thermodynamic system is taken from an original state D to an intermediate state E by the linear process shown in figure.

Its volume then reduced to an original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F.



Sol. Net W.D. from D to E and then E to F

$= \text{Area of graph (projection) DE} - \text{ar of projection EF}$
$= \text{ar Trap. ABED} - \text{ar Rectangle ABEF}$
$= \text{Area of } \Delta\text{DEF} = \frac{1}{2}(\text{EF}) \times \text{DF}$
$= \frac{1}{2}(5-2)\,(600-300)\text{ m}^3\text{ Pa}$
$= \frac{1}{2} \times 3 \times 300 = 450\text{ N-m}^{-2}\text{ m}^3 = 450\text{ N-m} = 450\text{ J}$

Q17. A flask contain argon and chlorine in the ratio 2 : 1 by mass. The temperature of the mixture of 37°C. Obtain the ratio of (*i*) average kinetic energy per molecule and (*ii*) root mean square speed (v_{rms}) of the molecules of the gases. Atomic mass of argon **39.9 amu**, molecular mass of chlorine **70.9 u**.

Sol. (*i*) As the temperature of argon and chlorine in flask is 37°C. So their velocity (v) remains equal.

$$\frac{\text{K.E. argon per molecule}}{\text{K.E. of chlorine per molecule}} = \frac{\cancel{1/2}\, M_A v^2}{\cancel{1/2}\, (M)_{Cl} v^2} = \frac{M_A}{M_{Cl}}$$

$$= \frac{39.9}{70.9} = \frac{1}{709/399} = \frac{1}{1.78}$$

$$= 1 : 1.78$$

(*ii*) $\because$ $$\text{K.E. per molecule} = \frac{3}{2}K_B T$$

(As K.E. depends on v_{rms})

$$\frac{1}{2}M\, v_{rms}^2 = \frac{3}{2}K_B T$$

$$\therefore \quad \frac{\text{K.E. of argon per molecule}}{\text{K.E. of chlorine per molecule}} = \frac{3/2\, K_B T_a}{3/2\, K_B T_{ch}}$$

As temperature in a flask is same for both gases and k_B Boltzman constt. are same

$$\therefore \quad \frac{\frac{1}{2}M_a\, v_{rms}^2 \text{ (of argon)}}{\frac{1}{2}M_c\, v_{rms}^2 \text{ (of chlorine)}} = \frac{1}{1}$$

$$\frac{v_{rms}(\text{Ar})}{v_{rms}\,(\text{Cl})} = \sqrt{\frac{M_c}{M_a}} = \sqrt{\frac{70.9}{39.9}} = \sqrt{1.78}$$

$$\frac{v_{rms}(\text{Ar})}{v_{rms}\,(\text{Cl})} = \frac{1.33}{1} \quad \text{or} \quad 1.33 : 1$$

Q18. Calculate the root mean square speed of smoke particle of mass 5×10^{-17} kg in brownian motion in air at NTP.

Sol. Pressure exerted by gas on the walls of container

$$P = \frac{1}{3}\rho v_{rms}^2 \quad \text{or} \quad P = \frac{1}{3}\frac{M}{V} v_{rms}^2$$

or $$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3nRT}{M}} \qquad n = 1 \text{ (let)}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3N_A k_B T}{M}} \qquad k_B = \frac{R}{N_A} \text{ or } R = N_A k_B$$

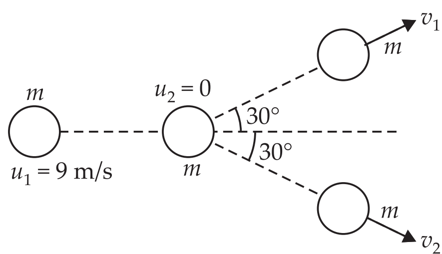
$$= \sqrt{\frac{3k_B T}{M/N_A}} = \sqrt{\frac{3k_B T}{\mu}}$$

N_A = Avogadro No.
μ = mass of 1 molecule

$$= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 273}{5 \times 10^{-17}}} = 15 \times 10^{-3} \text{ m/s}$$

$$v_{rms} = 1.5 \text{ cm s}^{-1}$$

Q19. A ball with speed of 9 m/s strikes another identical ball at rest such that after collision the direction of each ball makes an angle 30° with original direction. Find the speed of two balls after collision. Is kinetic energy conserved in this collision process?



Sol. By the law of conservation of momentum

$m_1 = m_2 = m$

$u_1 = 9$ m/s

$u_2 = 0$ m/s

Net vertical components of momentum become zero

Horizontal direction

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m \times 9 + m(0) = m v_1 \cos 30° + m v_2 \cos 30°$$

$$9 = v_1 \frac{\sqrt{3}}{2} + v_2 \frac{\sqrt{3}}{2}$$

$$v_1 + v_2 = \frac{9 \times 2}{\sqrt{3}} = 6\sqrt{3}$$

As the masses $m_1 = m_2 = m$ and angle formed by final velocities are equal $\therefore$ $v_1 = v_2 = 3\sqrt{3}$ m/s

$$KE_i = \frac{1}{2} m u_1^2 + \frac{1}{2} m u_2^2$$

$$= \frac{1}{2}m(9)^2 = \frac{81}{2}m = 40.5 \text{ m J}$$

$$KE_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$= \frac{1}{2}m(3\sqrt{3})^2 + \frac{1}{2}m(3\sqrt{3})^2$$

$$= \frac{1}{2}m(3\sqrt{3})^2 \times 2 = m \times 27$$

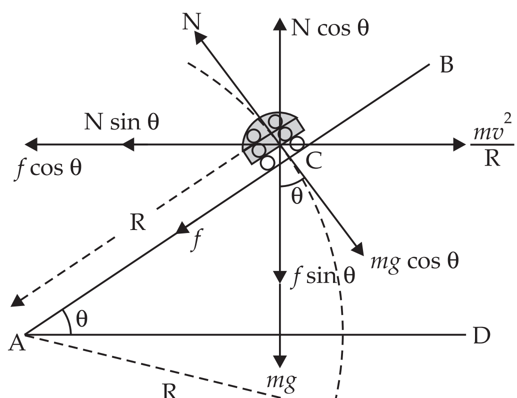
$$K_f = 27m\text{J}$$

$$\therefore \quad KE_f - KE_i = 27m - 40.5m = -13.5\ m\text{J}$$

So, there is loss of K.E. or law of conservation of Kinetic energy is not conserved.

Q20. Derive an relation for maximum velocity with a car can safely negotiate a circular turn of radius (r) on a road banked at an angle θ, given that the coefficient of friction between the car tyre and road is μ.

Sol. **Circular Motion of a Car on Banked Road:** Consider a car of mass m on a banked road AB, banked with an angle θ with horizontal AD.



The road AB and motion of car is perpendicular to the plane of paper.

Car is moving on a curved path of radius R from A.

The forces acting on car (vehicle) are

(*i*) Weight mg downward vertically.

(*ii*) Normal reaction N upward to the plane of road AB.

(*iii*) Force of friction f along the road down ward (because car slip outward on over speeding it skidding is prevented by force of friction.)

As car is moving in balanced (without skidding)

Resolving the horizontal components of N, mg, f and balancing then

$$N \sin \theta + f \cos \theta = \frac{mv^2}{R} \quad ...(i)$$

Balancing the vertical components also

$$N \cos \theta = mg + f \sin \theta$$

$$N \cos \theta - f \sin \theta = mg \quad ...(ii)$$

Dividing (i) by (ii)

$$\frac{N \sin \theta + f \cos \theta}{N \cos \theta - f \sin \theta} = \frac{mv^2/R}{mg}$$

$$\frac{N \cos \theta \left[\frac{\sin \theta}{\cos \theta} + \frac{f}{N}\frac{\cos \theta}{\cos \theta}\right]}{N \cos \theta \left[\frac{\cos \theta}{\cos \theta} - \frac{f}{N}\frac{\sin \theta}{\cos \theta}\right]} = \frac{v^2}{Rg}$$

$$\because \qquad f = \mu N \quad \text{or} \quad \mu = \frac{f}{N}$$

$$\frac{\tan \theta + \mu}{1 - \mu \tan \theta} = \frac{v^2}{Rg}$$

$$v^2 = Rg\left[\frac{\mu + \tan \theta}{1 - \mu \tan \theta}\right] \text{or } v_{max} = \sqrt{\frac{Rg\,(\mu + \tan \theta)}{(1 - \mu \tan \theta)}}$$

Q21. Give reason for the following:

(a) A cricketer move his hand backwards while, holding catch.

(b) It is easier to pull a lawn mower than to push it.

(c) A carpet is beaten with a stick to remove the dust from it.

Sol. (a) A cricketer moves his hand backward while holding catch to decrease the impact of force caused by fast moving ball.

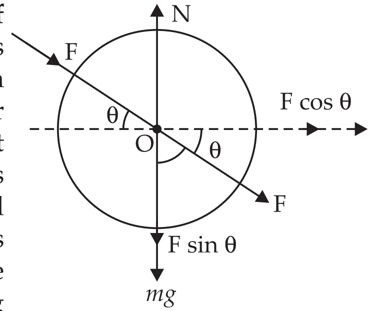
As $F = ma$ by Newton's second law and $a = \frac{dv}{dt}$

$\therefore F = m\frac{dv}{dt}$ to decrease the impact of force must be decrease it can be done by (i) to decrease m but we cannot decrease mass of moving ball.

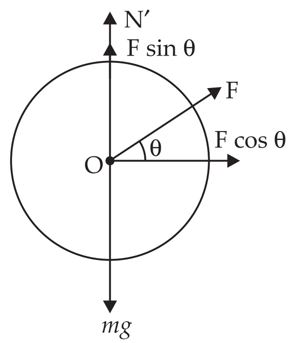
(ii) By decreasing dv, as the final velocity of ball is to be reduced to zero so change in $dv = v_f - u_i = 0 - u_i$, $\Rightarrow dv = -u_i$ velocity of coming ball cannot be reduce.

(iii) By increasing the time (dt) to catch the ball, which can be increased by moving hands backward.

(*b*) **Lawn mower: Case I: If pushed forward:** If a force F is applied at an angle θ with horizontal to push the roller then its horizontal component F cos θ gives motions (translatory) but vertical component F sin θ passes downward which increase the weight of roller. So pushing makes difficult in forward direction.



Case II: If pulled: If a force F is applied at an angle θ with horizontal to pull the mower in forward direction as shown in figure. Horizontal component F cos θ gives translatory motion, it same as Case I. But the vertical component F sin θ in upward direction decrease the effect of weight so makes the mower to move easier.



(*c*) When hanging carpet is beaten by stick, the dust is removed from carpet by Newton's law of inertia of rest.

When hanging carpet is beaten by stick, stick carried the carpet along it self but the dust particles which was already in rest remains in rest by law of inertia of rest. These dust particles are carried away by fast moving molecules of air. This process is repeated to remove dust.

Q22. A helicopter of mass 1000 kg rises with a vertical acceleration of 15 ms^{-2}. The crew and passengers weight 300 kg. Gives the magnitude and direction of the

(*a*) Force on the floor by crew and passengers.

(*b*) Action of the rotor of the helicopter on the surround air.

(c) Force on the helicopter due to surrounding air.

Sol. (*a*) Force exerted by crew and passenger on ground

$$= m(a + g) = 300(15 + 10)$$
$$= 300 \times 25 = 7500$$
$$= 7.5 \times 10^3 \text{ N downward on ground.}$$

(*b*) Action (downward) toward ground by rotor of helicopter on surround air = $(M + m)(a + g)$

Action downward an air $= (1000 + 300)[15 + 10]$

$$= 1300 \times 25$$
$$= 32500 = 3.25 \times 10^4 \text{ N downward.}$$

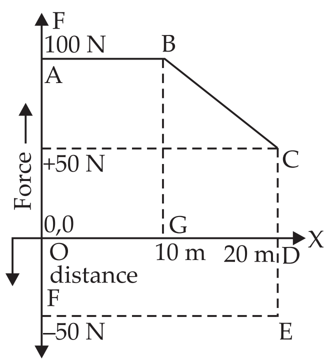
(*c*) Force on helicopter due to surrounding air is equal to the action force applied by helicopter but with opposite direction by Newton's third law of motion.
Hence, force on helicopter by surrounding air = 3.25×10^4 N upward.

Q23. A woman pushes a trunk on railway platform. Which has a rough surface. She applies a force of 100 N over a distance of 10 m. Therefore, she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance through which trunk has been moved is 20 m. Plot the force applied by woman and the frictional force which is 50 N. Calculate the work done by the two forces over 20 cm.

Sol. (*a*) For graph

Points	A	B	C
Distance *x* (m)	0	10	20
Force F (N)	100	100	50

(*b*) Net work done by woman is equal to area of F–*x* graph with *x* distance axis.



WD = ar. of rectangle ABGO + ar. of Trap. BGDC

$$= (100 \times 10) + \frac{(BG + CD)}{2} \times GD$$

$$= 1000 + \frac{(100 + 50)}{2} \times 10$$

$= 1000 + 750$

$= 1750$ Joule

(*c*) W.D. by frictional force = F . *x* = (ar. of graph ODEF)
$= - 50 \times 20 = - 1000$ J

(–)ive sign shows the displacement and frictional force are in opposite direction.

Q24. Derive equation of motion for a rigid body rotating with constant angular acceleration *a* and initial angular velocity ω_0.

Sol. Consider a body is rotating about an axis with its initial angular velocity ω_0.
Now a constant torque (τ) acts on this rotating body due to this constant torque will accelerate with constant angular acceleration α. So by definition of angular acceleration

$$\alpha = \frac{d\omega}{dt}$$

$$d\omega = \alpha \, . \, dt \qquad ...(I)$$

Let Torque acts upto time t and its angular velocity increases from ω_0 to ω after time t by integration (I) with proper limits

$$\int_{\omega_0}^{\omega} d\omega = \alpha.\int_0^t dt$$

$$[\omega]_{\omega_0}^{\omega} = \alpha[t]_0^t \quad \text{or} \quad \omega - \omega_0 = \alpha(t-0)$$

$\omega = \omega_0 + \alpha t$ is the (i) equation of motion.

(ii) **Derivation of second equation of motion:** Let θ angle is travelled by rotating body in time t then

$$\omega = \frac{d\theta}{dt} \quad \text{or} \quad d\theta = \omega dt$$

From (i) equation of motion

$$d\theta = (\omega_0 + \alpha t)\, dt$$

Integrating both side with proper limits

$$\int_0^{\theta} d\theta = \int_0^t \omega_0\, dt + \int_0^t \alpha.t\, dt$$

$$[\ ] = \omega_0 [t]_0^t + \alpha\left[\frac{t^2}{2}\right]_0^t \qquad (\because a \text{ is constant})$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \text{ is } (ii) \text{ equation of motion.}$$

(iii) Derivation of third equation of motion:

$$\alpha = \frac{d\omega}{dt} \times \frac{d\theta}{d\theta} = \frac{d\theta}{dt}.\frac{d\omega}{d\theta}$$

$$\alpha = \omega.\frac{d\omega}{d\theta} \quad \text{or} \quad \alpha.d\theta = \omega.d\omega$$

Integrating both side with proper limits

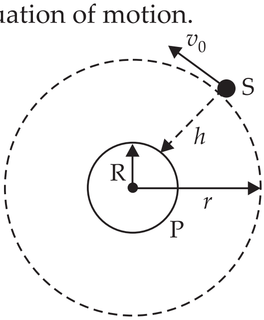
$$\int_0^{\theta} \alpha\, d\theta = \int_{\omega_0}^{\omega} \omega\, d\omega$$

$$\alpha[\theta]_0^{\theta} = \left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega}$$

$$\alpha\theta = \frac{1}{2}[\omega^2 - \omega_0^2]$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \text{ is } (iii) \text{ equation of motion.}$$

Q25. Derive an expression for the kinetic energy and potential energy of a satellite orbiting around a planet. A satellite of mass 200 kg revolves around a planet of mass 5×10^{30} kg in a circular orbit of 6.6×10^6 m radius. Calculate the B.E. of satellite. $G = 6.6 \times 10^{-11}$ N-m^2 kg^{-2}.



Sol. Consider a planet P and a satellite S.

S is revolving around P in circular orbit of radius r

$$r = R + h$$

R = radius of planet

h = height of satellite from surface of planet

M = mass of planet

m = mass of satellite

v_0 = orbital velocity of satellite around planet in stationary orbit.

(*i*) **K.E. of satellite:** According to the law of gravitational force of gravity on satellite

$$F_g = \frac{GMm}{r^2} \qquad ...(i)$$

This gravitational force F_g is balanced by centrifugal force

$$F_c = \frac{mv_0^2}{r} \qquad ...(ii)$$

Equating (*i*), (*ii*), $\frac{mv_0^2}{r} = \frac{GMm}{r^2} \quad \Rightarrow \quad v_0^2 = \frac{GM}{r}$

So, the K.E. of satellite $= \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$

$\therefore$ K.E. $= \frac{GMm}{2r}$ or K.E. $= \frac{GMm}{2(R+h)}$

(*ii*) **Potential energy of satellite:** We know the potential energy of mass m at distance r from the centre of planet is $U = -\frac{GMm}{r}$

Satellite is a distance r from the centre of planet

$$U = -\frac{GMm}{r} = \frac{-GMm}{(R+h)}$$

Q25. State and prove Bernoulli's theorem.

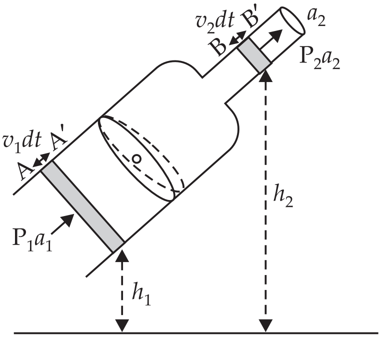
Sol. **Bernoulli's Theorem:** It state that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous flowing fluid in a stream lined, irrotational flow remains constant along a streamline.

Mathematically, it can be expressed:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant.}$$

Proof: Consider a non uniform pipe of area of cross-section a_1, a_2 at its two different point A and B. A non-viscous incompressible liquid of density ρ is flowing steadily from A to B, with velocities v_1 and v_2 are at A and B respectively.

Again let heights of A and B from ground are h_1 and h_2.
Mass m of the fluid entering in Δt time through A
= Volume × Density
m = Area of cross-section × Length × Density
$m_1 = a_1 (v_1 \Delta t) \times \rho$ ...(*i*)
Mass comes out from B in time Δt,
$m_2 = a_2 v_2 \rho \Delta t$ ...(*ii*)



From $m_1 = m_2$
$$a_1 v_1 \rho \Delta t = a_2 v_2 \Delta t \rho$$
$\Rightarrow$ $a_1 v_1 = a_2 v_2$ [Equation of continuity]

Change in K.E. of fluid = $KE_B - KE_A$
$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} m (v_2^2 - v_1^2)$$
$$\Delta K = \frac{1}{2}(a_1 v_1 \Delta t \rho)(v_2^2 - v_1^2)$$
Change in PE from A to B = $PE_B - PE_A$
$$\Delta U = mg h_2 - mg h_1 = mg(h_2 - h_1)$$
$$= a_1 v_1 \Delta t \rho g (h_2 - h_1)$$
Net work done by flowing fluid $\Delta W = WD_A - WD_B$ by fluid
$$\Delta W = P_1 V_1 - P_2 V_2$$
$$= P_1(a_1 v_1 \Delta t) - P_2(a_2 v_2 \Delta t)$$
$$\Delta W = a_1 v_1 \Delta t [P_1 - P_2] \quad \text{[By equ. of continuity]}$$
Net WD by fluid = $\Delta KE + \Delta U$
$$a_1 v_1 \Delta t (P_1 - P_2) = \frac{1}{2} a_1 v_1 \Delta t \rho (v_2^2 - v_1^2) + a_1 v_1 \Delta t \rho g (h_2 - h_1)$$
Dividing both side by $a_1 v_1 \Delta t$, we get
$$(P_1 - P_2) = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$
$\therefore$ $P + \frac{1}{2}\rho v^2 + \rho g h$ = Constant. Hence proved.

Dividing by ρg
$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constt.}$$

It is the another form of Bernoulli's principle according to which the sum of pressure head, velocity head and gravitational head remains constant in the stream line flow of an ideal fluid.

Q27. Consider a cycle tyre being filled with air by pump. Let V be the volume of tyre (fixed) and each stroke of the pump ΔV ($<<$ V) of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from P_1 to P_2?

Sol. For adiabatic change

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$P[V + \Delta V]^\gamma = [P + \Delta P]V^\gamma$$

$$PV^\gamma\left[1 + \frac{\Delta V}{V}\right]^\gamma = P\left[1 + \frac{\Delta P}{P}\right]V^\gamma$$

$$\left[1 + \frac{\Delta V}{V}\right]^\gamma = \left[1 + \frac{\Delta P}{P}\right]$$

$$\left[1 + \gamma\frac{\Delta V}{V}\right] = 1 + \frac{\Delta P}{P}$$

(Neglecting the higher terms as $\Delta V << V$)

$$\gamma\frac{\Delta V}{V} = \frac{\Delta P}{P}$$

$$\frac{\Delta V}{\Delta P} = \frac{V}{\gamma P} \quad \Rightarrow \quad \Delta V = \frac{V}{\gamma P}.\Delta P$$

$$WD = \int_{P_1}^{P_2} P.dV$$

$$\Rightarrow \quad WD = \int_{P_1}^{P_2} P\cdot\frac{V}{\gamma P}\Delta P \quad (\because \text{ V is constant vol. of tyre})$$

$$\therefore \quad WD = \frac{V}{\gamma}(P_2 - P_1)$$

$$WD = \frac{(P_2 - P_1)V}{\gamma}$$

Q28. In a refrigerator one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process mechanical work has to be done, which is provided by an electric motor. If the motor is of 1 kW power and heat is transferred from – 3°C to 27°C. Find the heat taken out of the refrigerator per second assuming its efficiency is 50% of perfect engine.

Sol. Efficiency of Carnot's engine $\eta = 1 - \dfrac{T_2}{T_1}$

$T_2 = -3°C = 273 - 3 = 270$ K
$T_1 = 27°C = 273 + 27 = 300$ K

$\therefore$ Efficiency $\eta = 1 - \frac{270}{300} = 1 - \frac{9}{10} = 1 - 0.9 = 0.1$

$\therefore$ Efficiency of refrigerator = 50% of efficiency of carnot's engine

$$= \frac{50}{100} \times \frac{1}{10}$$

$$\eta' = \frac{1}{20} = 0.05$$

If Q is the heat transferred per second to higher temperature then efficiency of performance

$$\frac{Q_2}{\text{W.D.}} = \frac{1-\eta'}{\eta'} = \frac{1-0.05}{0.05} = \frac{0.95}{0.05} = 19$$

$$\frac{Q_2}{\text{W.D}} = 19$$

$\therefore \quad Q_2 = 19 \times$ W.D. by motor

Heat removed from lower temperature = 19×1000 J/s

Heat removed from refrigerator per second = $Q_2 = 19$ kJ

Q29. Show that when a string fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops, and four loops. The frequencies are in ratio 1 : 2 : 3 : 4.

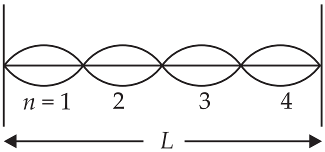
Sol. Let n be the number of the loops formed when resonance occur in stretched string. Then the length of each loop is $\frac{\lambda}{2}$.

For n loops

$L = n\left(\frac{\lambda}{2}\right)$ or $\lambda = \frac{2L}{n}$

$v = \nu\lambda$ or $\nu = \frac{v}{\lambda}$

$\nu = \frac{vn}{2L} = \frac{n}{2L}\sqrt{\frac{T}{m}}$



T = tension in string

m = mass per unit length of string

for $n = 1$

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu_0$$

If $n = 2$ loops, then,

$$\nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu_0$$

For $n = 3$ loops, then,

$$\nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu_0$$

For $n = 4$ loops, then,

$$\nu_4 = \frac{4}{2L}\sqrt{\frac{T}{m}} = 4\nu_0$$

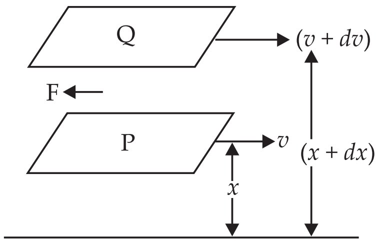
$$\nu_1 : \nu_2 : \nu_3 : \nu_4 = \nu_0 : 2\nu_0 : 3\nu_0 : 4\nu_0$$
$$= 1 : 2 : 3 : 4$$

Q30. (*a*) Define the coefficient of viscosity and write its SI units.

(*b*) Define terminal velocity and find an expression for the terminal velocity in case of a sphere falling through a viscous liquid.

Sol. (*a*) **Coefficient of Viscosity:** The velocity of fluid flow in a canal increases from bottom to top.

Consider a liquid is flowing steadily in the form of parallel layers on a fixed horizontal surface as shown in figure. Consider again two layers at P and Q at a distance or heights x and $(x + dx)$ from fixed horizontal surface, and their velocities are v and $(v + dv)$ respectively.



Fixed horizontal surface

According to Newton, a force of viscosity F acting tangentially between the two layer is

(*i*) Proportional to the area A of the layers in contact

$F \propto A$

(*ii*) Proportional to the velocity gradient $\frac{dv}{dx}$ between the two layers

$F \propto \frac{dv}{dx}$

$\therefore$ Combining both laws $F \propto A\frac{dv}{dx}$

$F = -\eta A\frac{dv}{dx}$ where η is coefficient of viscosity of the liquid.

It depends on the nature of liquid at that temperature and pressure. Negative sign shows that the viscous force acts in the opposite direction of flow of liquid.

Now if $A = 1\ m^2$, $\frac{dv}{dx} = 1$ per sec, then $F = \eta$

Hence, coefficient of viscosity of flowing liquid steadily can be defined as the tangential viscous force required to maintain a unit velocity gradient between its two parallel layers each of unit area.

S.I. Unit: $\eta = \frac{F}{A}\frac{dx}{dv} = \frac{[MLT^{-2}]}{[L^2]}\frac{[L]}{[LT^{-1}]} = [ML^{-1}T^{-1}]$ *i.e.*, $kgm^{-1}s^{-1}$

called decapoise or poiseuille

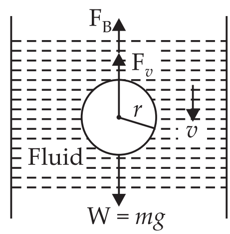
$$\eta = \frac{F}{A}\frac{dx}{dv} = \frac{1\,N}{1\,m^2}.\frac{1\,m}{1\,ms^{-1}} = Nm^{-2}\,s^1 \text{ or decapoise}$$

(*ii*) **Terminal Velocity:** When a body falls through a viscous fluid it accelerate due to gravity initially. As the velocity of body increases from zero to v a dragging force ($F_v = 6\pi\eta rv$) acts on the spherical body of radius r and mass m as $F \propto v$ so a stage is reached when the force due to weight (downward) becomes just equal to the dragging viscous force F_v (upward) and a buoyant force F_B (upward).

So at this stage net force on falling ball becomes zero and ball begins to move with constant velocity. This constant velocity is called terminal velocity.

So "the maximum constant velocity downward acquired by a freely falling body through a viscous fluid is called **terminal velocity**."

Expression for Terminal Velocity: Consider a spherical ball of radius r density ρ, is moving down and with velocity 'v' in a fluid of density σ and viscosity η then



(*i*) Weight (W) of spherical body acting downward,

$$W = mg = \frac{4}{3}\pi r^3 \rho g \downarrow$$

(*ii*) **Upward** thrust by Stokes' law is equal to the weight of displaced fluid

$$F_B = \text{Vol. of ball (V) } \sigma.g = \frac{4}{3}\pi r^3 \sigma g \uparrow$$

(*iii*) Upward and Force of viscosity F_v upward = $6\pi\eta r v_T$ when the ball attains the constant terminal velocity then net force on ball is zero.

$$F_v + F_B = W$$

$$6\pi\eta r v_T + \frac{4}{3}\pi r^3.\sigma g = \frac{4}{3}\pi r^3.\rho g$$

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

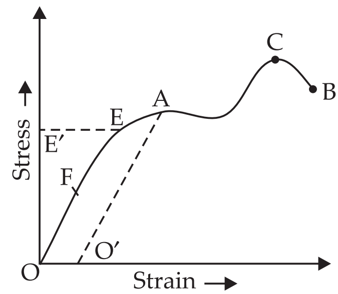
$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 g[\rho - \sigma]$$

$$v_T = \frac{4\pi r^3 g}{3 \times 6\pi\eta r}(\rho - \sigma)$$

$$v_T = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{\eta}$$

OR

Q31. The stress-strain graph for a metal is shown in figure. The wire returns to its original state O along the curve EFO, when it is gradually unloaded. Point corresponds to the breaking of the wire.



(*i*) Upto which point the curve obeyed Hooke's law?

(*ii*) Which point on the curve corresponds to the elastic limit or yield point of the wire?

(*iii*) Indicate the elastic and plastic regions of stress-strain graph.

(*iv*) Describe what happens when the wire is loaded upto a stress corresponding to the point A on the graph and then unloaded gradually. In particular explain the dotted curve.

(*v*) What is peculiar about the portion of stress-strain graph from C to B? Upto what stress can the wire be subjected without causing breaking?

Sol. (*i*) According to Hooke's law stress $\propto$ strain *i.e.*, stress-strain, graph is straight line which corresponds to OFE i.e., upto stress **point E**.

(*ii*) Point E corresponds to elastic limit, when wire is unloaded from point E, it comes to its original position. So value of stress at **E is called elastic limit**.

(*iii*) Beyond E strain increase more rapidly and on unloading, wire does not regain its original position, but attains a new position of stress.

So, E is elastic region and region E to B is plastic region.

(*iv*) Strain increases proportional to the load upto E beyond E, strain increases more rapidly, for a given increase in load (stress). Beyond E (elastic limit) wire does not retrace the original path upto O, but attains a new position AO′. Point O′ (strain) corresponding to zero load which implies a permanent set of strain in wire.

(*v*) Beyond point C to B, strain increases even if the wire is being unloaded at B, it breaks. Stress upto that corresponding to C can be applied without causing break.

Q32. It is a common observation that rain cloud can be at about a **kilometre** altitude above the ground.

(*a*) If a rain drop falls from such a height freely under gravity. What will be its speed? Also calculate in km/h ($g = 10\ \text{ms}^{-2}$)

(*b*) If a typical rain drop is about 4 mm diameter. Estimate the momentum if it hits you.

(*c*) Estimate the time required to flatten the drop *i.e.*, time between the first contact and the last contact.

(*d*) Estimate how much force such a drop exert on you.

(*e*) Estimate the order of magnitude of force on an umbrella of diameter 1 m. Typical lateral separation between the two raindrops is 5 cm. (Assume that umbrella cloth is not pierced through)

Sol. (*a*) $u = 0$, $s = h = 1000$ m, $g = 10$ ms^{-2}

$$v^2 = u^2 + 2gh$$
$$v^2 = 0^2 + 2 \times 10 \times 1000 = 2 \times 100 \times 100$$
$$v = 100\sqrt{2} \text{ m/s}$$
$$v = 100\sqrt{2} \times \frac{18}{5} \text{ km/h} = 360 \times \sqrt{2} = 360 \times 1.414$$
$$v = 509.04 \text{ km/h}$$

(*b*) Mass = Volume × Density $\Rightarrow$ $m = \frac{4}{3}\pi r^3 \rho$

Momentum imparted by drop to person = mv

$$p = \frac{4}{3}\pi r^3 \rho v$$
$$r = \frac{4}{2} \text{ mm} = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$
$$\rho = 10^3 \text{ kg/m}^3$$
$$v = 100\sqrt{2} \text{ m/s}$$

$\therefore$
$$p = \frac{4}{3} \times 3.14 \times 2 \times 2 \times 2 \times (10^{-3})^3 \times 10^3 \times 100\sqrt{2}$$
$$p = \frac{4 \times 314}{\cancel{3} \times \cancel{100}} \times 8 \times \cancel{100} \times \overset{0.471}{\cancel{1.414}} \times 10^{-9+3}$$
$$p = 32 \times 314 \times 0.471 \times 10^{-6} = 4732.6 \times 10^{-6}$$
$$p = 4.7 \times 10^{-3} \text{ kg m s}^{-1}$$

(*c*) The time to flatten the drop = time taken by drop to cover a distance of 4 mm diameter

$\therefore$
$$dt = \frac{ds}{v} = \frac{4 \times 10^{-3}}{141}$$
$$dt = 0.0284 \times 10^{-3}$$
$$= 28.4 \times 10^{-6} \text{ second}$$
$$= 28.4\ \mu\text{s} \cong 30\ \mu\text{s}$$

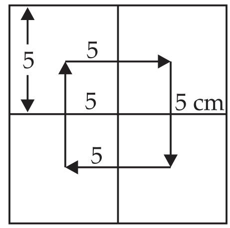
$$v = 100\sqrt{2}$$
$$= 100 \times 1.41$$
$$v = 141 \text{ m/s}$$
$$ds = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$$

(*d*) Force $F = \frac{dp}{dt} = \frac{4.7 \times 10^{-3}}{28 \times 10^{-6}}$ $\qquad p = 4.7 \times 10^{-3}$ kg ms^{-1}

$F = 0.168 \times 10^3 = 1.7 \times 10^2$ N $\qquad dt = 28 \times 10^{-6}$ sec

(e) Net force due to drops falling on umbrella of diameter 1 m.

$$r = \frac{1}{2}\text{ m}$$



drops are separated by 5 cm apart (average) so area occupied by each drop is area of square of 5 cm.

No. of drops falling on umbrella

$$= \frac{\text{Area of umbrella}}{\text{Area covered by 1 drop}}$$

$$= \frac{\pi r^2}{(5\times10^{-2})^2} = \frac{3.14\times0.5\times0.5}{5\times10^{-2}\times5\times10^{-2}}$$

No. of drops = $3.14 \times 10^{+4-2}$ = 314 drops

Average force exerted by one drop 1.7×10^2 N

$\therefore$ Total force exerted by rain drops on umbrella

$$= 314 \times 1.7 \times 10^2\text{ N} = 533.8 \times 10^2 = 5.34 \times 10^4\text{ N}$$

OR

Q33. A cricket fielder can throw a cricket ball with speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal.

(i) The effective angle to horizontal at which the ball is projected in air as seen by spectator.

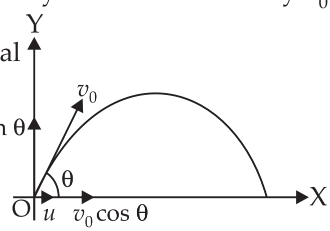
(ii) What will be the time of flight?

(iii) What is the distance (horizontal range) from the point of projection at which the ball will land?

(iv) Find θ at which he should throw the ball thats would maximise the horizontal range as found in (iii).

(v) How does θ for maximum range change if $u > v_0$, $u = v_0$, $u < v_0$?

Sol. The velocity of cricket ball will be equal to the resultant or sum of velocity of ball (v_0 at $\angle\theta$) thrown by bowler and velocity u_0 horizontally



(i) Velocity of ball with horizontal

$u_x = u + v_0 \cos\theta$

$u_y = u_0 \sin\theta$

$$\tan\theta = \frac{u_y}{u_x} = \frac{v_0 \sin\theta}{u + v_0 \cos\theta}$$

$$\theta = \tan^{-1}\left(\frac{v_0 \sin\theta}{u + v_0 \cos\theta}\right)$$

(ii) **Time of Flight:** Consider the vertical motion from ground to highest point of flight,

$$u_y = v_0 \sin\theta \qquad a = -g, \quad v_y = 0$$

$$v_y = u_y - gt$$
$$0 = v_0 \sin\theta - gt$$
$$t = \frac{v_0 \sin\theta}{g}$$

Hence, total time of flight $T = 2t = \dfrac{2v_0 \sin\theta}{g}$

(*iii*) Horizontal Range:

$$R = u_x.T = (u + v_0 \cos\theta)T$$
$$R = (u + v_0 \cos\theta)\frac{2v_0 \sin\theta}{g}$$
$$= \frac{v_0}{g}[2u \sin\theta + v_0\ 2 \sin\theta \cos\theta]$$
$$R = \frac{v_0}{g}[2u \sin\theta + v_0 \sin 2\theta]$$

(*iv*) For maximum range $\dfrac{dR}{d\theta} = 0$

$$\frac{d}{d\theta}\frac{v_0}{g}[2u \sin\theta + v_0 \sin 2\theta] = 0$$
$$\frac{v_0}{g}[2u \cos\theta + 2v_0 \cos 2\theta] = 0$$

$\dfrac{v_0}{g} \neq 0$ $\quad \therefore\ 2u \cos\theta + 2v_0 (2\cos^2\theta - 1) = 0$

$$2u \cos\theta + 4v_0 \cos^2\theta - 2v_0 = 0$$
$$2v_0 \cos^2\theta + u \cos\theta - v_0 = 0 \qquad a = 2v_0$$

By quadratic formula $\qquad b = u$

$c = -v_0$

$$\cos\theta = \frac{-u \pm \sqrt{u^2 + 4(2v_0)v_0}}{2.2v_0}$$
$$\cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}$$

or $\qquad \theta = \cos^{-1}\left(\dfrac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0}\right)$

(*v*) (*a*) If $u < v_0$. Then

$$\cos\theta = \frac{-u \pm \sqrt{u^2 + 8v_0^2}}{4v_0} \text{ from part } (iv)$$
$$= \frac{-u \pm \sqrt{u^2\left[1 + \dfrac{8v_0^2}{u^2}\right]}}{4v_0}$$

$\dfrac{8v_0^2}{u^2}$ can be neglecting as $u^2 >> v_0^2$

$$\therefore \quad \cos\theta = \frac{-u \pm u}{4v_0}$$

As θ is angle of projection can be only 0°→90° or cos θ can

$$\therefore \quad \cos\theta = \frac{-u + u}{4v_0} \quad \text{and} \quad \cos\theta \neq \frac{-u - u}{4v_0}$$

$$\therefore \quad \cos\theta = 0 \quad \text{or} \quad \cos\theta = \cos 90°$$

$$\theta = \frac{\pi}{2}$$

(*b*) If $u = v_0$, then cos θ from part (*iv*)

$$\cos\theta = \frac{-v_0 \pm \sqrt{v_0^2 + 8v_0^2}}{4v_0} = \frac{-v_0 \pm 3v_0}{4v_0}$$

θ is acute (or between 0°→90°) so cos θ cannot be negative

$$\therefore \quad \cos\theta = \frac{-v_0 + 3v_0}{4v_0} = \frac{2v_0}{4v_0} = \frac{1}{2} = \cos 60°$$

$$\theta = 60° \quad \text{or} \quad \theta = \frac{\pi}{3}$$

(c) $$\frac{1}{2} > -1$$

or $$\cos\theta_1 > \cos\theta_2$$

∴ For maximum value of cos θ and θ is acute angle

$$\cos\theta_1 = \frac{1}{2} \quad \text{and} \quad (\theta_2 = 180° \text{ not possible})$$

$$\cos\theta_1 = \cos 60° \quad \text{or} \quad \theta_1 = 60°$$

$$\theta_1 = \frac{\pi}{3}$$

$$u < v_0 \qquad \text{(Given)}$$

then $$\cos\theta = \frac{-u \pm \sqrt{0 + 8v_0^2}}{4v_0} \quad \text{from part } (iv)$$

(Neglecting u^2 as $u < v_0$, but not of u)

$$\cos\theta = \frac{-u \pm 2\sqrt{2}v_0}{4v_0}$$

As θ is acute angle so $\cos\theta \geq 0$ (Not negative)

$$\therefore \quad \cos\theta = \frac{-u + 2\sqrt{2}v_0}{4v_0} = \frac{-u}{4v_0} + \frac{2\sqrt{2}v_0}{4v_0}$$

$\frac{u}{4v_0}$ is very small as $u < v_0$ so neglecting $\frac{-u}{4v_0}$

$$\therefore \quad \cos\theta = \frac{\cancel{2}\sqrt{2}}{\cancel{4}_2} = \frac{1}{\sqrt{2}} = \cos 45°$$

$$\theta = \frac{\pi}{4}$$

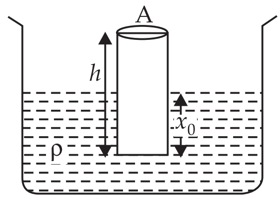
Q34. (*a*) Show that in S.H.M. acceleration is directly proportional to the displacement at a given instant.

(b) A cylindrical log of wood of height h and area of cross-section A floats in water. It is pressed and released. Show that the log would execute S.H.M. with a time period

$$T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

where m is mass of the body and
ρ is density of fluid.

Sol. When a vertically floating log is pressed downward into the liquid of density ρ. A buoyant force acts on log upward, due to inertia of log it rises beyond and crosses its floating level and then it come down due to gravity. Like this it oscillates in water.



Buoyant force F_B = Weight of displaced liquid.

Buoyant force, F_B = (Vol. of displaced liquid) × Density × g

If x_0 is the height or length of log inside water, then when log is floating, then

$$\text{Weight of log} = F_B$$
$$mg = (A.x_0) \times \rho.g$$
$$mg = A.x_0\,\rho g \qquad ...(i) \quad (m = \text{mass of log})$$

When block is pressed by length x, then total length of log inside water is $(x + x_0)$. Now new buoyant force F'_B

From (i) and (ii), we can conclude that $F'_B > mg$

$$F'_B = A(x + x_0)\rho\, g \qquad ...(ii)$$

From (i) and (ii), we can conclude that $F'_B > mg$

So net force acting upward on log of wood is restoring force

$$F_{restoring} = F'_B - \text{wt. of log}$$
$$F_{restoring} = A(x + x_0)\,\rho g\uparrow - mg\downarrow$$
$$= Ax\rho g\uparrow + Ax_0\rho g\uparrow - Ax_0\,\rho g\downarrow \text{ [From (i)]}$$
$$F_{restoring} = Ax\rho g\uparrow \qquad ...(iii)$$
$$F = kx \qquad \because k = A\rho g$$

$F_{restoring} \propto -x$ as restoring force gravity are in opposite direction

$$\therefore \quad ma = -Ax\rho g$$
$$a = \frac{-A\rho g}{m}x$$
$$\omega^2 = \frac{A\rho g}{m}$$
$$\frac{2\pi}{T} = \sqrt{\frac{A\rho g}{m}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

OR

Q35. A progressive wave represented by $y = 5 \sin (100\pi t - 0.4\pi x)$ where y, A and x are in m, t is in s. What is the

(*a*) amplitude (*b*) wavelength

(*c*) frequency (*d*) wave velocity

(*e*) magnitude of particle velocity?

Sol. Consider a standard equation

$$y = A \sin (\omega t - kx) \quad \text{(in X direction)}$$

$$y = 5 \sin (100\pi t - 0.4\pi x) \quad \text{(Given)}$$

Comparing the A, ω and k from both equations

(*a*) Amplitude (A): $A = 5$ m

(*b*) Wavelength (λ): $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k}$

$$\lambda = \frac{2\pi}{0.4\pi} = 5 \text{ m}$$

(*c*) Frequency (ν): $\omega = 2\pi\nu$

$$\nu = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

(*d*) Wave velocity (v): $v = \nu\lambda$

$$\nu = 50, \lambda = 5 \text{ m}$$

$$\therefore \quad v = 50 \times 5 = 250 \text{ m/s}$$

(*e*) Magnitude of particle velocity: $\frac{dy}{dt}$

$$y = 5 \sin (100\pi t - 0.4\pi x)$$

$$\left(\frac{dy}{dt}\right) = 5 \times 100\pi \cos (100\pi t - 0.4\pi x)$$

$$\frac{dy}{dt} = 500\pi \cos (100\pi t - 0.4\pi x)$$

Magnitude of particle velocity at x at any time t is

$$\frac{dy}{dt} = 500\pi \cos (100\pi t - 0.4\pi x)$$

Maximum particle velocity will be at $\frac{nT}{2}$, nT or at $\frac{n\lambda}{2}$, $n\lambda$ *i.e.*, at maximum $\frac{dy}{dt}$

$$\cos (100\pi t - 0.4\pi x) = 1$$

$$v_{max} = 500\pi \text{ m/s.}$$

❑❑❑

Sample Question Paper-II

Q1. Modulus of rigidity of liquid is:

(*a*) infinity (*b*) zero

(*c*) unity (*d*) some finite small non zero constant value

Sol. (*b*): Modulus of rigidity $\eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

Stress does not require to change the shape of liquid.

Q2. If all other parameters except one mentioned in each of the options below be the same for two objects, in which case(s) they would have the same kinetic energy.

(*a*) Mass of object A is two times that of B

(*b*) Volume of the object A is half that of B

(*c*) Object A is falling freely while object B is moving upward with the same speed at any given point of time.

(*d*) Object A is moving horizontally with while object B is falling freely.

Sol. (*c*): If an object (B) is thrown upward and one object A is dropped freely from a height such that B can reach upto height from B is dropped, then the sum of their K.E. will be same.

Q3. If the sun and planet carries huge amount of opposite charges:

(*a*) All three Kepler's laws would still be valid.

(*b*) Only third law will be valid

(*c*) The second law will not change

(*d*) The first law will still be valid

Sol. (*c*): The force of attraction between sun and planet increase due to charges so radius of orbit become smaller so elliptical orbit change to circular orbit so first Kepler's law changed.

For IIIrd law centrifugal force = centripetal force

$$\frac{mv^2}{R} = \frac{GMm}{R^2} + \frac{Kq_1q_2}{R^2}$$

$$\frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{1}{R^2}[GMm + Kq_1q_2]$$

$$\frac{m4\pi^2 R^3}{T^2} = (GMm + Kq_1q_2)$$

as, m, G, M, K, q_1, q_2 are all constant then $\dfrac{R^3}{T^2}$ is constant. So the IIIrd law is valid.

Similarly, IInd law will not change.

Hence, verifies the option (*c*).

Q4. Which of the following pairs of physical quantities does not have same dimensional formula?

(*a*) Work and torque
(*b*) Angular momentum and Planck's constant
(*c*) Tension and surface tension
(*d*) Impulse and momentum

Sol. (*c*): (*a*) S.I. unit of work and torque are J and N-m, both are unit of energy.

So, has same dimensional formulae $[ML^2T^{-2}]$

(*b*) $\vec{L} = \vec{r} \times \vec{p}$ and $E = h\nu$ or $h = \dfrac{E}{\nu} = \dfrac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

$L = m$ kg ms^{-1} = $[ML^2T^{-1}]$ has same dimensional formulae $[ML^2T^{-1}]$

(*c*) Tension has unit of force so its dimensional formula $[MLT^{-2}]$. But surface tension $\sigma = \dfrac{F}{l}$ has dimensional formula $= \dfrac{[MLT^{-2}]}{[L]} = [M^1L^0T^{-2}]$

So, tension and surface tension both have different dimensional formulae.

Hence, verifies option (*c*).

(*d*) Impulse (F.*t*) has dimensional formula

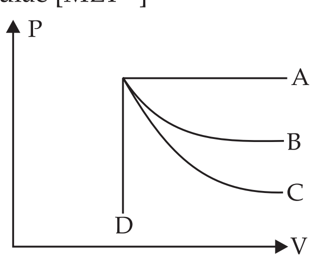
$= [MLT^{-2}].[T^1] = [MLT^{-1}]$

Momentum ($p = mv$) has dimensional formula

$=$ kg ms^{-1} $= [MLT^{-1}]$

So has same dimensional formulae $[MLT^{-1}]$

Q5. An ideal gas undergoes four different process from same initial state (figure). Four process are adiabatic, isothermal, isobaric and isochoric. Out of A, B, C and D, which is adiabatic?



(*a*) B (*b*) A
(*c*) C (*d*) D

Sol. (*c*): Adiabatic, A isobaric (P constt.), D is isochoric (constt. volume), B has smaller slope with volume axis, then slope of (C). So, B is isothermal (constant temperature). Remaining process (C) has larger slope with volume axis. So (C) it is **adiabatic process**. Verifies option (*c*).

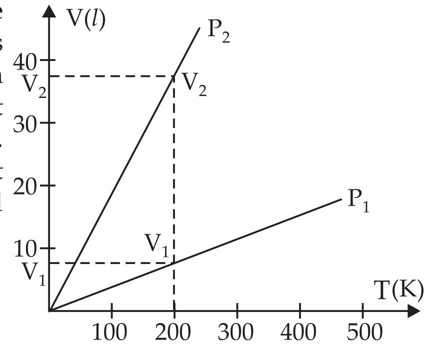
Q6. Why do two layers of a cloth of equal thickness provides warmer covering than a single layer of cloth with double thickness?

Sol. Air enclosed between the two layers of cloth and air is bad conductor of heat so prevents (or resists) the transmission of heat from body.

$$\boxed{Q = \frac{K(T_1 - T_2)A}{x}}$$

In this case K decreased (due to air), x increased due to air between two layers. So Q will be decreased.

Q7. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relations between P_1 and P_2.



(*a*) $P_1 > P_2$
(*b*) $P_1 = P_2$
(*c*) $P_1 < P_2$
(*d*) Data is insufficient.

Sol. We know that as temperature increases volume is increasing in both graph. For any particularly constant temperature let 200 K of a given quantity of ideal gas the volume are V_1 and V_2 as $V_1 < V_2$. By Boyle's law $P_1V_1 = P_2V_2$ (as T = 200 K). So $P_1 > P_2$ verifies the option (*a*).

Q8. Along a streamline:
(*a*) the velocity of a fluid particle remains constant.
(*b*) the velocity of all fluid particles crossing a given position is constant.
(*c*) the velocity of all fluid particles at a given instant is constant.
(*d*) the speed of a fluid particle remains constant.

Sol. Equation of continuity $A_1v_1 = A_2v_2$
So, velocity of fluid changes with change in area of cross-section

$$v \propto \frac{1}{A}$$

In streamline flow of fluid velocity of a particle will remain constant as at that point A does not changes. So, verifies option (*b*).

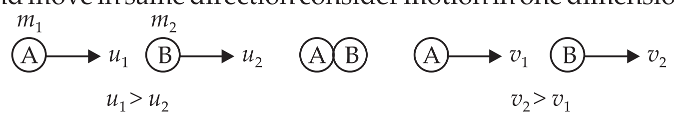
Q9. State Newton's third law of motion and use it to deduce the principle of conservation of linear momentum.

Sol. (*i*) **Newton's third law:** When a body applies action force on another body then this body will apply a reaction force exactly equal and opposite to that of action.

OR

"For every action, there is an equal and opposite reaction."

(*ii*) **Derivation of principle of conservation of linear momentum by using Newton's IIIrd law:** Consider two bodies A and B are moving in same direction. They collide elastically each other and move in same direction consider motion in one dimension.



When body 'A' collide with body B, then A apply action force F_{BA} on B and by Newton's IIIrd law B applies reaction force F_{AB} so that

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

(As both are equal and opp. by Newton's IIIrd law)

$$\frac{dp_A}{dt} = \frac{-dp_B}{dt} \quad \text{...}(i) \quad \text{(By Newton's II}^{nd}\text{ Law)}$$

As the time of collision of A and B are equal

dp_A is change in momentum of A before and after collision

dp_B is change in momentum of B before and after collision

$$dp_A = m_1v_1 - m_1u_1 \quad \text{...}(ii)$$

$$dp_B = m_2v_2 - m_2u_2 \quad \text{...}(iii)$$

From (*i*)

$$dp_A = -dp_B$$

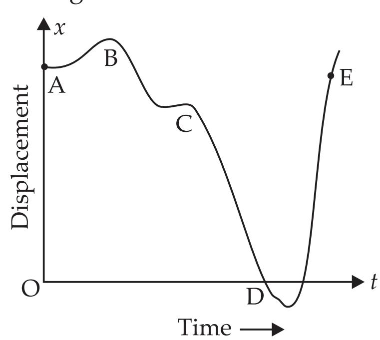
$$m_1v_1 - m_1u_1 = -[m_2v_2 - m_2u_2] \quad \text{[using } (i) \text{ and } (ii)]$$

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

Hence, total final momentum is equal to the total initial momentum if no external forces are acting on the system of particles.

Q10. A graph of x versus t is shown in figure. Choose correct alternatives from below.



(*a*) The particle is released from rest.

(*b*) At B acceleration $a > 0$

(*c*) At C the velocity and acceleration vanish.

(*d*) Average velocity for the motion between A and B is positive.

(*e*) The speed at D exceeds that at E.

Sol. (*a*, *c*, *d*): (*a*) As the slope of (x-t) graph at A is zero (parallel to time axis). So speed of particle at A is zero or particle released from A, verifies the option (*a*).

(*b*) At B again body is at rest so its $a = 0$ *i.e.*, $a \not> 0$ rejects the option (*b*).

(*c*) At C graph is again parallel to time axis. So v and acceleration both are zero, verifies option (*c*).

(*d*) As the displacement from A to B is positive so its velocity is positive, verifies option (*d*).

(*e*) Slope at D is downward *i.e.*, speed negative and at E slope is positive, so $v_E > v_D$ reject the option (*e*).

Q11. A vehicle travels half the distance L with speed v_1 and other half with speed v_2 then average speed is

(*a*) $\dfrac{v_1 + v_2}{2}$ (*b*) $\dfrac{2v_1 + v_2}{v_1 + v_2}$

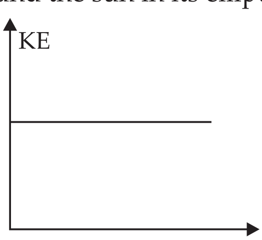
(*c*) $\dfrac{2v_1 v_2}{v_1 + v_2}$ (*d*) $\dfrac{L(v_1 + v_2)}{v_1 + v_2}$

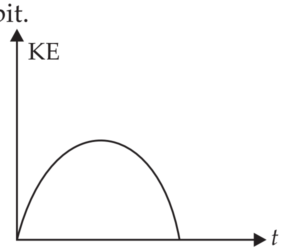
Sol. Average speed = $\dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{L + L}{\dfrac{L}{v_1} + \dfrac{L}{v_2}}$

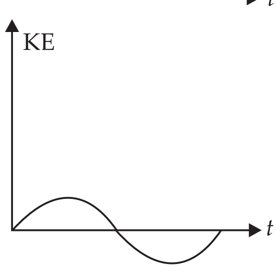
$$\text{Average speed} = \frac{2\cancel{L}}{\cancel{L}\left[\dfrac{1}{v_1} + \dfrac{1}{v_2}\right]} = \frac{2}{\dfrac{v_1 + v_2}{v_1 v_2}}$$

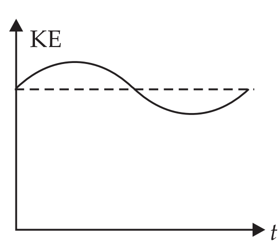
$v_{av} = \dfrac{2v_1 v_2}{v_1 + v_2}$, verifies option (*c*).

Q12. Which of the diagram shown in figure most closely shows the variation in kinetic energy of the earth at it moves once around the sun in its elliptical orbit.

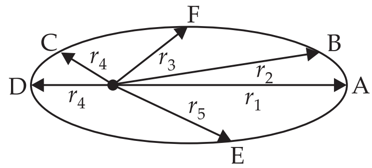
(*a*)


(*b*)


(*c*)


(*d*)


Sol. By Kepler's second law of planetary motion, the areal velocity (area swept by radius vector per unit time) of a planet is always constant *i.e.*, velocity of planet around the sun is inversely proportional to the position vector



$$\therefore \qquad v \propto \frac{1}{\vec{r}}$$

from figure as $r_1 > r_2 > r_3 > r_4$ and $r_4 < r_5 < r_1$

So, velocity of planet increases from A to D and then again decreases from D to A and average velocity will be at points E and F, *i.e.*, **KE** increases from E to D, then decreases from D to A then again increase from A to F and tallies with option (*d*).

Q13. The Vernier scale of a travelling microscope has 50 divisions, which coincide with 49 main scale divisions. If each main scale division is 0.5 mm. Calculate the minimum, inaccuracy in the measurement of distance.

Sol. Minimum inaccuracy of measurement by Vernier calliper when *n* division of Vernier scale coincides (*n* – 1) parts of main scale or

$$\text{Least Count} = \frac{\text{LC of Main Scale}}{\text{No. of parts of Vernier Scale}}$$

$$\text{LC} = \frac{0.5 \text{ mm}}{50} = \frac{1}{100} \text{ mm} = 10^{-2} \text{ mm}$$

$$\text{LC} = 0.01 \text{ mm.}$$

Q14. A vessel contains two mono atomic gases in the ratio 1 : 1 by mass. The temperature of the mixture is 27°C. If their atomic masses are in the ratio 7 : 4. What is the (*i*) Average kinetic energy per molecule (*ii*) r.m.s. speed of the atoms of the gases?

Sol. (*i*) K.E. per molecule of a gas $E = \frac{3}{2} K_B T$

As temperature of both gases are 27°C *i.e.*, 300 K so their K.E. per molecule will be equal.

So, the ratio of their K.E. is 1 : 1.

(*ii*) r.m.s. Speed: $v_{rms} = \sqrt{\frac{3RT}{M}}$ (R, T are constant)

$$\therefore \qquad v_{rms} \propto \sqrt{\frac{1}{M}}$$

$$\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{4}{7}} = \sqrt{\frac{1}{1.75}} = \frac{1}{1.32}$$

$$\therefore \qquad (v_{rms})_1 : (v_{rms})_2 = 1 : 1.32$$

Q15. A 500 kg satellite is in a circular orbit of radius R_e about the earth. How much energy is required to transfer it to circular orbit of the radius 4 R_e? What are the changes in the kinetic energy and potential energy? ($R_e = 6.37 \times 10^6$ m, $g = 9.8$ ms^{-2})

Sol. Total energy of satellite orbiting around earth in radius r

$$E = \frac{-GMm}{2r}$$

$$g = \frac{GM}{R_e^2} \quad \text{or} \quad GM =$$

$$\therefore \qquad E = -\frac{gR_e^2\, m}{2r}$$

$$(i) \qquad E = \frac{-gR_e^2\, m}{2r} \qquad \ldots(i)$$

$$E_i = \frac{-gR_e^2\, m}{2R_e} \qquad \because r = R_e \text{ (Given)}$$

$$E_f = \frac{-gR_e^2\, m}{2(4R_e)} \qquad \because r = 4R_e \text{ (Given)}$$

$$\Delta E' = E_f - E_i = \frac{-gR_e^2\, m}{8R_e} + \frac{gR_e^2\, m}{2R_e}$$

$$= gR_e m\left[-\frac{1}{8} + \frac{1}{2}\right]$$

$$= gR_e m\left[\frac{-1+4}{8}\right] = \frac{3}{8} gR_e m$$

$$\Delta E = \frac{3}{\cancel{8}_4} \times \overset{4.9}{\cancel{9.8}} \times 6.37 \times 10^6 \times 500$$

$$= \frac{5 \times 3 \times 4.9 \times 6.37 \times 10^{6+2}}{4} = \frac{468.2}{4} \times 10^8$$

$$\Delta E = 117 \times 10^8 = 11.7 \times 10^9 \text{ J}$$

So, 11.7×10^9 J energy is required to shift satellite from radius R_e to $4R_e$.

(*ii*) K.E. of satellite = Total E = P.E. + K.E.

$$\frac{-gmR^2}{2r} = \frac{-gmR^2}{r} + E_k$$

$$\because \qquad U = \frac{-GMm}{r} \quad \text{and} \quad g = \frac{GM}{R_e^{\,2}} \quad \text{or} \quad GM = gR_e^2$$

$$U = -\frac{gmR_e^{\,2}}{r} \qquad R = \text{radius of earth}$$

$\because$ $$E_k = \frac{gmR_e^{\ 2}}{r} - \frac{1}{2}\frac{gmR_e^{\ 2}}{r} = \frac{gmR_e^{\ 2}}{2r}$$

$$E_k = \frac{gmR_e^{\ 2}}{2r}$$

$$\Delta E_k = \frac{gmR_e^{\ 2}}{2r_2} - \frac{gm\,R_e^{\ 2}}{2r_1} = \frac{gmR_e^{\ 2}}{2}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\Delta KE = \frac{+\,gmR_e^2}{2}\left[\frac{1}{4R_e} - \frac{1}{R_e}\right]$$

$\because$ $r_1 = R_e$ and $r_2 = 4R_e$ (Given)

$$\Delta KE = \frac{gmR_e^2}{2R_e}\left[\frac{1}{4} - 1\right] = \frac{gmR_e}{2}\left[\frac{1-4}{4}\right]$$

$$\Delta KE = \frac{-3}{8}gmR_e$$

$$= \frac{-9.8 \times 500 \times 6.37 \times 10^6 \times 3}{8}$$

$$= \frac{-93639}{8} \times 10^6$$

$$= -11704.8 \times 10^6 = -11.7 \times 10^9 \text{ J} \quad \text{Ans. II}$$

$$\Delta PE = \frac{-gmR_e^{\ 2}}{r_2} - \frac{-gmR_e^{\ 2}}{r_1} = -gmR_e^{\ 2}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= -gmR_e^2\left[\frac{1}{4R_e} - \frac{1}{R_e}\right]$$

$$= \frac{-gmR_e^2}{R_e}\left[\frac{1}{4} - \frac{1}{1}\right] = +gmR_e \times \frac{3}{4}$$

$$\Delta PE = +\frac{3}{4}gmR_e = 2\left(\frac{3}{8}gmR_e\right) = 2 \times 11.7 \times 10^9$$

$$\Delta PE = 23.4 \times 10^9 \text{ J.} \quad \text{Ans. III}$$

Q16. A pipe of 17 cm in length, closed at one end, is found to resonate with a 1.5 kHz source (*a*) which harmonic of the pipe resonate with the above source?

(*b*) Will resonance with the same source be observed if the pipe is open at both ends? Justify your answer. (Speed of sound in air is 340 ms^{-1})

Sol. For closed one end pipe. First mode of vibration

$$\nu_1 = \frac{1}{4L}\sqrt{\frac{\gamma P}{\rho}} \quad \text{and} \quad v_{air} = \sqrt{\frac{\gamma P}{\rho}} = 340 \text{ ms}^{-1}$$

$$\nu_1 = \frac{1 \times \overset{25}{\cancel{100}}}{\cancel{4} \times \cancel{17}} \times \cancel{340}^{20} = 500 \text{ Hz} \qquad L = .17 \text{ m}$$

Frequencies which can produced by one end closed are odd multiples of first mode of vibration *i.e.*,

$1\nu_1, 3\nu_1, 5\nu_1, ...$

So $3 \times 500 = 1500$ Hz = 1.5 kHz. So resonance can occur in same pipe with same source 1.5 kHz.

(*b*) Ist mode of vibration in both end open pipe

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{\gamma P}{\rho}} = \frac{1 \times 100 \times \cancel{340}^{\cancel{20}\,10}}{\cancel{2} \times \cancel{17}} = 1000 \text{ Hz}$$

Ratio of resonant frequencies are 1 : 2 : 3 ...

So in this case resonance can not occur (as 1000 n $\neq$ 1500 Hz) with same source and pipe both end open.

Q17. Show that average kinetic energy of a molecule of an ideal gas is directly proportional to the absolute temperature of the gas.

Sol. Pressure exerted by gas molecule on a vessel is $P = \frac{1}{3}\rho\bar{v}^2$

$\bar{v}$ = Average speed of molecule

$$\rho = \frac{M}{V}$$

$$\therefore \qquad P = \frac{1}{3}\frac{M}{V}\bar{v}^2$$

$$PV = \frac{1}{3}M\bar{v}^2$$

$$nRT = \frac{1}{3}M\bar{v}^2 \times \frac{2}{2}$$

$$nRT = \frac{2}{3}\frac{1}{2}M\bar{v}^2$$

$$RT = \frac{2}{3}KE$$

Total energy of ideal gas is only due to KE.

$n = 1$

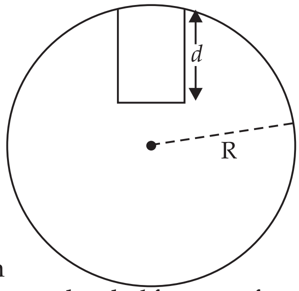
$$KE = \frac{3}{2}RT$$

So, KE or Energy of ideal gas is directly proportional to the absolute temperature.

Q18. Obtain an expression for the acceleration due to gravity at a depth h below the surface of earth.

Sol. 'g' acceleration due to gravity on surface of earth

$$g = \frac{GM}{R^2}$$



M = Mass of earth,
R = Radius of earth

$$M = \frac{4}{3}\pi R^3 \rho$$

$$\therefore \quad g = \frac{\frac{4}{3}\pi R^3 \rho G}{R^2} = \frac{4}{3}\pi R \rho G$$

$$g = \frac{4}{3}\pi R \rho G$$

ρ is average density of earth

Let g_d is the acceleration due to gravity at depth d from surface

$$g_d = \frac{GM_d}{(R-d)^2} = \frac{G.\frac{4}{3}\pi(R-d)^3 \rho}{(R-d)^2}$$

$$g_d = \frac{4}{3}\pi G(R-d)\rho$$

$$\frac{g_d}{g} = \frac{\frac{4}{3}\pi G\rho(R-d)}{\frac{4}{3}\pi G\rho R} = \left(1 - \frac{d}{R}\right)$$

$$g_d = g\left[1 - \frac{d}{R}\right]$$

Hence, on increase depth (d) the acceleration due to gravity (g_d) decreases.

Q19. The position of a particle given by $r = 6t\hat{i} + 4t^2\hat{j} + 10\hat{k}$ where r is in meter and t in seconds.

(a) Find the velocity and acceleration as a function of time.

(b) Find the magnitude and direction of velocity at $t = 2$ s.

Sol. (a)

$$\vec{r} = 6t\hat{i} + 4t^2\hat{j} + 10\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 6\hat{i} + 4 \times 2t\hat{j} + 0\hat{k}$$

$$\vec{v} = 6\hat{i} + 8t\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 0 + 8\hat{j}$$

$$\vec{a} = 8\hat{j}$$

(b)

$$\vec{v} = 6\hat{i} + 8t\hat{j}$$

$$\vec{v} = 6\hat{i} + 16\hat{j} \quad \text{at } t = 2 \text{ s}$$

$$|\vec{v}| = \sqrt{6^2 + 16^2} = \sqrt{36 + 256} = \sqrt{292}$$

$$|\vec{v}| = 17.09 \text{ m/s}$$

$$\tan\theta = \frac{|v_y|}{|v_x|} = \frac{16}{6} = \frac{8}{3}$$

$\therefore$ Angle with x-axis of velocity at $t = 2$ s.

$$\theta = \tan^{-1}\frac{8}{3} \quad \text{Ans. II}$$

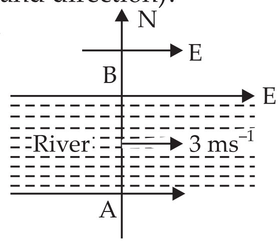
Q20. A river is flowing due east with speed 3 m/s. A swimmer can swim in still water at a speed of 4 m/s as in figure.

(*a*) If swimmer starts swimming due to north what will be his resultant velocity (magnitude and direction)?

(*b*) If he wants to start from point A on south bank and reach opposite point B on north bank.

(*i*) Which direction should he swim?

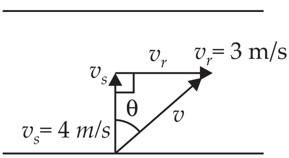
(*ii*) What will be the resultant speed?



(*c*) From two different cases as mentioned in (*a*) and (*b*), in which case will he reach opposite bank in shorter time?

Sol. (*a*)

$$v_S = 4 \text{ m/s}$$
$$v_R = 3 \text{ m/s}$$
$$v = \sqrt{v_s^2 + v_r^2} = \sqrt{4^2 + 3^2} = \sqrt{25}$$
$$|v| = 5 \text{ m/s}$$



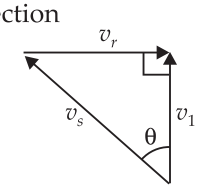
$$\tan\theta = \frac{3}{4}, \quad \angle\theta \text{ with north direction}$$

$$\theta = \tan^{-1}\frac{3}{4} = 37° \text{ to north down direction}$$

Magnitude of $v = 5$ m/s

(*b*) v_1 is resultant velocity toward north direction

$$|v_1| = \sqrt{v_s^2 - v_r^2} = \sqrt{4^2 - 3^2}$$
$$|v_1| = \sqrt{16-9} = \sqrt{7} \text{ m/s}$$
$$\tan\theta_1 = \frac{v_r}{v_1} = \frac{3}{\sqrt{7}}$$



$$\theta_1 = \tan^{-1}\frac{3}{\sqrt{7}} \text{ to north upstream.}$$

Time taken in (*a*) part $t_1 = \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{x \text{ (let)}}{5} = \dfrac{x}{5}$ second

Time taken in part (*b*) $t_2 = \dfrac{x}{\sqrt{7}}$

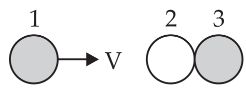
$$\frac{x}{\sqrt{7}} > \frac{x}{5} \quad \Rightarrow \quad t_2 > t_1$$

So, in Ist case, swimmer reaches the opposite bank in shortest time.

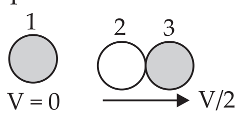
Q21. (*i*) A rain drop of mass 1 g fall from rest, from a height of 1 km and hits the ground with a speed of 50 m/s.

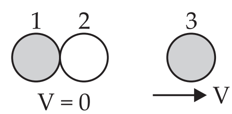
(*a*) What are the final KE of the drop and its initial PE? ($g = 10\ ms^{-2}$)
(*b*) How do you account for the difference between the two?

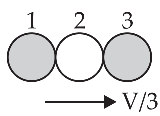
(*ii*) Two identical ball bearings (2, 3) in contact with each other and resting on a friction less table are hit head on by another ball bearing (1) of same mass moving initially with a speed V as shown in figure.

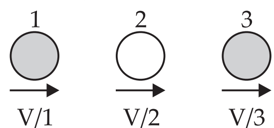


If the collision is elastic, which of the following (figures) is the possible result after collision.

(*a*)


(*b*)


(*c*)


(*d*)


Sol. (*i*) (*a*) Final KE of raindrop $= \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 1 \times 10^{-3} \times 50 \times 50$$

$$= 1250 \times 10^{-3} = 1.25\ \text{J}$$

(*b*) PE at top from where it was dropped $= mgh$

$$= 1 \times 10^{-3} \times 10 \times 1000$$

$$= 10\ \text{J}$$

The difference in energies in two cases that some parts of energy of raindrop used against viscous force of air.

(*ii*) As collision is elastic, momentum and KE conserved so from figure, initial momentum $m\text{V} + (m + m)0 = m\text{V}$ and initial KE $= \frac{1}{2}m\text{V}^2 + 0$. Final momentum and KE are

(*a*) $p_f = m(0) + (m + m)\frac{\text{V}}{2} = 0 + 2m\frac{\text{V}}{2} = m\text{V}$

$$\text{KE}_f = \frac{1}{2}m(0) \quad \frac{1}{2}(2m)\frac{\text{V}}{4} = \frac{m\text{V}^2}{4} \neq \frac{1}{2}m\text{V}^2 \text{ (initial KE)}$$

So, rejects the option (*a*).

(*b*) $p_f = (m + m)\ (0)^2 + m\text{V} = m\text{V} = p_i$

$$\text{KE}_f = \frac{1}{2}2m(0)^2 + \frac{1}{2}m\text{V}^2 = \frac{1}{2}m\text{V}^2 = \text{KE}_i$$

As both momentum p and KE are conserved, so verifies the option (*b*).

(*c*) $p_f = (m+m+m)\dfrac{V}{3} = 3m\dfrac{V}{3} = mV$

$$KE_f = \frac{1}{2}(3m)\left(\frac{V}{3}\right)^2 = \frac{3}{2}m\frac{V^2}{9} = \frac{mV^2}{6} \neq \frac{1}{2}mV^2 \text{ (initial KE)}$$

So, rejects the option (*c*).

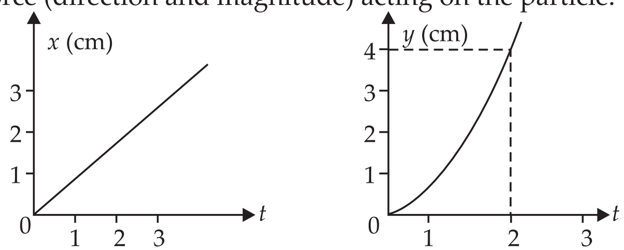
(*d*) $p_f = mV + m\dfrac{V}{2} + m\dfrac{V}{3} \neq mV = p_i$

So, rejects the option (*d*).

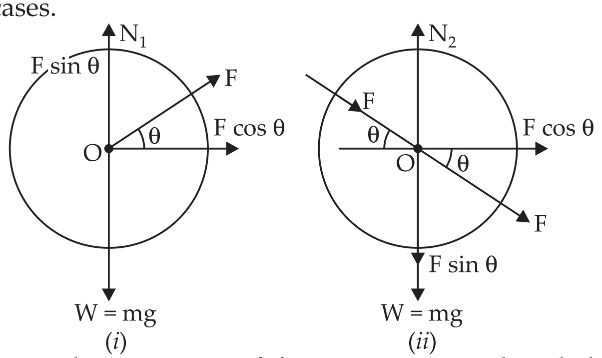
Q22. Explain why:

(*a*) It is easier to pull a hand cart than to push it.

(*b*) Figure shows (x, t), (y, t) diagrams of a particle moving in 2 dimensions. If the particle has a mass of 500 g. Find the force (direction and magnitude) acting on the particle.



Sol. (*a*) Let Force (F) is applied at an angle θ with horizontal in two ways to drag a cart by (*i*) pulling (*ii*) by pushing. The horizontal component F cos θ drag the cart in forward direction in both cases.



Vertical component of force is in upward and downward while pulling and pushing respectively. Then, the weight W acts in downward direction

∴ $$W = F \sin\theta + N_1 \quad \text{(Case pulling)}$$

$$N_1 = W - F\sin\theta \quad \text{(I)}$$

and while pushing

$$W + F\sin\theta = N_2 \text{ II} \quad \text{(II)}$$

in IInd case normal reaction is larger. So, force of friction F = μN in pushing becomes larger and makes the cart difficult to drag than by pulling.

Hence, pulling is easier than pushing a cart to drag forward.

(*b*) From the shape of group (*i*), (*ii*) given in figure of question $x \propto t$ or $x = kt$ and $y \propto t^2$ and $y = k't^2$ respectively

$v_x = \dfrac{dx}{dt} = k$	$v_y = \dfrac{dy}{dt} = \dfrac{d}{dt}k't^2 = 2k't$
$a_x = \dfrac{dv_x}{dt} = 0$	$a_y = \dfrac{dv_y}{dt} = 2k'\ \text{m/s}^2$
$F_x = ma_x$	$a_y = 2k'$
$F_x = 0.5 \times 0$	$F_y = ma_y = 0.5 \times 2k'$
$F_x = 0$ Newton	For SI unit $k' = 1$
	$\therefore F_y = 2$

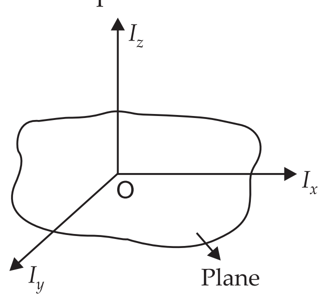
$$|F| = \sqrt{F_x^2 + F_y^2} = \sqrt{0^2 + 2^2} = 2\ \text{N}$$

As $F_x = 0$ $\therefore$ F = 2 N force acts along Y-direction only.

Q23. (*a*) State axis theorem of moment of inertia.

(*b*) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameter to be $\frac{2}{5}MR^2$, where M is the mass of the sphere and R is radius of sphere.

Sol. (*a*) **Perpendicular axis theorem:** The moment of inertia (I_z) of a body about a point (O) and perpendicular to its any plane (X and Y) is equal to the sum of moment of inertia (I_x and I_y) of body about any two mutually perpendicular (X and Y) axis is in the same and intersecting each other at the point where the perpendicular axis intersect on the plane.



OR

If I_x and I_y are the moment of inertia of a body are perpendicular to each other, then the moment of inertia about the axis perpendicular to both I_x and I_y and passing through the intersecting point of I_x and I_y is equal to the sum of moment inertia of I_x and I_y *i.e.*, $I_z = I_x + I_y$.

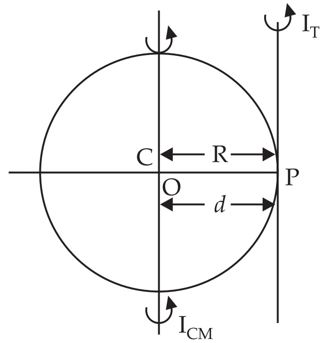
(*b*) **Moment of inertia of sphere along tangential axis:** By parallel axis theorem

$$I_T = I_{CM} + Md^2$$

I_T = Moment of inertia of sphere about tangent

I_{CM} = MI of sphere about centre of mass and parallel to I_T

d = Perpendicular distance between C.M. and the axis



$\therefore\ d = R \qquad I_{CM} = \frac{2}{5}MR^2$ (As diameter symmetric to CM)

$$I_T = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$I_T = \frac{7}{5}MR^2$ is required moment of inertia of sphere about any tangent (as all tangents are identical)

Q24. A 3 m long ladder weighing 20 kg leans on a frictionless wall. It's feet rest on the floor 1 m from the wall. Find the reaction forces of the wall and floor.

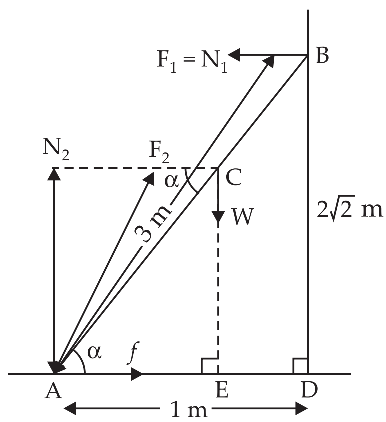
Sol. AB is a ladder AB = 3 cm (Given)

$F_1 = N_1$ = Normal reaction caused by frictionless wall ($\mu = 0$)

N_2 = Normal reaction caused due to weight

f = Force of friction which resist the ladder to slip outward

F_2 = Resultant reaction force $\angle\alpha$ with horizontal caused by floor *i.e.*, $\vec{F}_2 = \vec{f} + N_2$



W = Downward weight of ladder AB at its centre *i.e.*, AC = BC = 1.5 m

AD = 1 m (Given) and AE = $\frac{1}{2}$ m = 0.5 m

By Pythagoras theorem in right angled ΔADB

$$BD = \sqrt{3^2 - 1^2} = \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}\text{ m}$$

From figure forces in vertical direction in balanced condition

$N_2 = W$ and $W = mg = 20 \times 9.8$

$W = 196$ N

$\therefore \quad N_2 = 196 \text{ N}$

Taking moments of forces about a point A then

$$N_1 \times BD - W \times AE = 0$$

$$F_1 \times 2\sqrt{2} - 196 \times 0.5 = 0$$

$$F_1 = \frac{\cancelto{49}{196} \times /}{\cancel{2}\sqrt{2} \times \cancel{10}\,/} = \frac{49}{\sqrt{2}}\,\frac{\sqrt{2}}{\sqrt{2}} = \frac{49}{/} \times \cancelto{0.707}{1.414}$$

$$F_1 = 34.6 \text{ N}$$

$$F_2 = \sqrt{N_2^2 + f^2}$$

Taking forces in horizontal direction

$$f = N_1 = F_1 = 34.6 \text{ N}$$

$$f = \left(\frac{49}{\sqrt{2}}\right) \text{N}$$

$$\therefore \quad F_2 = \sqrt{(196)^2 + \left(\frac{49}{\sqrt{2}}\right)^2}$$

$$= \sqrt{(49 \times 4)^2 + \frac{(49)^2}{2}} = 49\sqrt{16 + \frac{1}{2}}$$

$$= 49\sqrt{16.5}$$

$$F_2 = 49 \times 4.06 = 199 \text{ N}$$

and $\quad \tan\alpha = \dfrac{N_2}{f} = \dfrac{\cancelto{4}{196} \times \sqrt{2}}{\cancel{49}}$

$$\tan\alpha = 4\sqrt{2}$$

$$\alpha = \tan^{-1} 4\sqrt{2} = 80°$$

Net reaction force (F_2) due to floor is 199 N at an angle 80° with horizontal and reaction force F_1 due to wall is $N_2 = 196$ N

$F_1 = 34.6$ N (Horizontally)

$F_2 = 199$ N at $\alpha = 80°$ (with horizontal)

Q25. Fully loaded Boeing aircraft has a mass of 3.3×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 960 km/hr.

(*a*) Estimate the pressure difference between the upper and lower surfaces of the wings.

(*b*) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface.

Sol. (*a*) Level flight *i.e.*, Boeing in balance condition upthrust

= Weight of Boeing aircraft

Pressure difference × Area = mg

$$\Delta p \times 500 = 3.3 \times 10^5 \times 9.8$$

$$\Delta p = \frac{33 \times 98}{50000} \times 10^5 = \frac{3234 \times 10}{5}$$
$$\Delta p = 6.46 \times 10^3 \text{ N/m}^2$$

(*b*) v_1, p_1 = Speed and pressure of air on lower surface of wing
v_2, p_2 = Speed and pressure of air on upper surface of wing
By Bernoulli's principles

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$
$$p_1 - p_2 = +\frac{1}{2}\rho(v_2^2 - v_1^2)$$
$$\Delta p = \frac{1}{2}\rho(v_2 + v_1)(v_2 - v_1)$$
$$\frac{v_1 + v_2}{2} = v_{av} = 960 \text{ km/hr}$$
$$= \frac{\cancelto{160}{960} \times 5}{\cancelto{3}{18}} = \frac{800}{3} = 266.6 = 267 \text{ m/s}$$

$\therefore$ $$\Delta p = \rho . v_{av} \Delta v$$

or $$\Delta v = \frac{\Delta p}{\rho . v_{av}}$$

Fractional increase in velocity Δv of the air w.r.t. v_1

$\therefore$ $$\frac{\Delta v}{v_{av}} = \frac{\Delta p}{\rho v_{av}^2} = \frac{\cancelto{323}{646} \times 10^3}{\cancelto{60}{120} \times 267 \times 267} = \frac{32300}{6 \times 267 \times 267}$$
$$\frac{\Delta v}{v_{av}} = \frac{32300}{427734} = 0.076$$
$$\frac{\Delta v}{v_{av}} \times 100 = 7.6\%$$

Q26. Explain briefly the working principle of a refrigerator and obtain an expression for it's coefficient of performance.

Sol. **Working Principle of Refrigerator:** Refrigerator is a Carnot's engine working in reverse order.

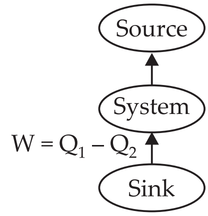
Heat energy is pumped out from cold (T_2) body into a hot (T_1) body by doing an external work done by motor (on compressor).

Coefficient of performance: It may be defined as the ratio of the amount of heat removed (Q_2) per cycle to the mechanical work done (W) required to be done on it.

$$\beta = \frac{Q_2}{W}$$

By first law of thermodynamics $W = Q_1 - Q_2$

$$\therefore\ \beta = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{Q_2\left[\frac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\frac{T_1}{T_2} - 1\right]}$$



$$\beta = \frac{T_2}{T_1 - T_2}$$

T_1 is temperature of sink.

T_2 is temperature of source.

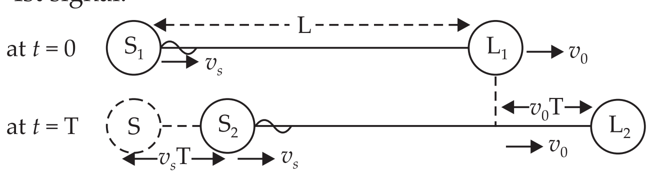
Q27. Derive an expression for the sound heard by a listener when source of sound and listener both move in the same direction.

Sol. Let, v = Speed of sound in air

v_s = Speed of sources of sound in favourable (+ve) direction

v_0 = speed of listeners in the same direction as source *i.e.*, in forward direction (+ve)

$\boxed{S_1L_1 = L}$ is the first position of S and L when S sends Ist signal.



S_2 and L_2 are the second positions of S and L after one time period when IInd signal is send by source S

$$\therefore \qquad S_2L_2 = L - v_sT + v_0T \qquad ...(i)$$

Time taken by Ist signal send by S to reach upto observer is

$$t_1 = \frac{\text{Distance}}{\text{Relative speed } (v_r) \text{ of sound wave w.r.t. observer L}}$$

$$t_1 = \frac{L}{v - v_0} \qquad ...(ii)$$

As both v and v_0 are in same direction $v_r = v - v_0$

Now time to reach the second signal emitted after time T is

$$t_2 = T + \frac{L - v_s T + v_0 T}{v - v_0}$$

$$= T + \frac{L}{v - v_0} + \frac{(-v_s + v_0)T}{v - v_0}$$

$$t_2 = T + \frac{L}{v - v_0} + \frac{(v_0 - v_s)T}{(v - v_0)}$$

Time difference (Time period) for nearest two signal, Listen by observer $T' = t_2 - t_1$

$$T' = T + \frac{\cancel{L}}{(\cancel{v} - v_0)} + \frac{(v_0 - v_s)T}{(v - v_0)} - \frac{\cancel{L}}{(\cancel{v} - v_0)}$$

$$T' = \left[1 + \frac{(v_0 - v_s)}{(v - v_0)}\right]T = \left[\frac{v - v_0 + v_0 - v_s}{v - v_0}\right]T$$

$$T' = \left(\frac{v - v_s}{v - v_0}\right)T$$

So, observed frequency ν by Listener's is $\nu' = \frac{1}{T'}$,

$$\nu' = \left(\frac{v - v_0}{v - v_s}\right)\nu \quad \text{where } \nu = \frac{1}{T}$$

Case I: If $v_s > v_0$ then $(v - v_0) > (v - v_s)$

$\nu' > \nu$ as the distance between SL decrease *i.e.*, $S_2L_2 < S_1L_1$

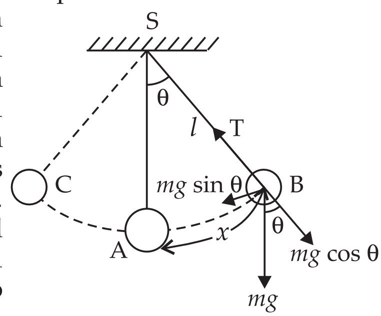
Case II: If $v_s < v_0$ then $(v - v_0) < (v - v_s)$

$\nu > \nu'$ as the distance $S_2L_2 > S_1L_1$

Q28. (*a*) Show that for small amplitudes the motion of a simple pendulum is simple harmonic and hence obtain an expression for its time period.

(*b*) Consider pair of identical pendulums, which oscillates independently such that when one pendulum is at its extreme position making an angle of 2° to the right with vertical, the other pendulum is at it's extreme position making an angle 1° to the left of vertical. What is the phase difference between the pendulums?

Sol. (*a*) Consider a simple pendulum of bob of point mass suspended with a rigid support at S with a string (flexible, inelastic and weightless) of length *l*. In equilibrium position bob is below the suspension point S. Now, it is displace very small angle θ as shown and released then bob oscillates to and fro about it mean position.



Let any instant the position of bob is at B then

(*i*) $x = l\theta$ = is displacement of bob from mean position.

(*ii*) mg = weight of bob act vertically downward.

(*iii*) Tension T in string acts toward rigid support S.

The two components of *mg* are (*i*) ***mg* sin θ towards the mean position and perpendicular to string which is equal to restoring force**, and (*ii*) $mg \cos\theta$ acting along the string away from S and balances the tension T of string *i.e.*, $T = mg\cos\theta$.

So, restoring force $F = -\,mg \sin\theta$

(–) sign shows the restoring force acts toward its mean position A
$\sin\theta \cong \theta$ for small θ

$$\therefore \qquad \sin\theta = \frac{x}{l} \quad \text{or} \quad F = -mg\frac{x}{l}$$

$$F = -\frac{mg}{l}x$$

m, g, l are constant for a simple pendulum

$$ma = -\frac{mg}{l}x$$

$$a = -\frac{g}{l}x$$

i.e., accn. (a) of bob directly proportional to the displacement (x). Hence, motion of pendulum for small angle is Simple Harmonic motion

$$a = -\frac{g}{l}x$$

$$\frac{d^2x}{dt^2} = -\omega x$$

is again equation of S.H.M. oscillator

$$\omega^2 = \frac{g}{l} \quad \text{or} \quad \omega = \sqrt{\frac{g}{l}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

gives the time period of simple pendulum.

(*b*) When pendulum Ist is at extreme right then

$$\theta_1 = \theta_0 \sin(\omega t + \delta_1) \qquad ...(i)$$

Similarly for pendulum IInd is at extreme left then

$$\theta_2 = \theta_0 \sin(\omega t + \delta_2) \qquad ...(ii)$$

For I become, $\theta_1 = 2°$ at extreme right so $\theta_0 = 2°$

$$+2 = 2\sin(\omega t + \delta_1)$$
$$\sin(\omega t + \delta_1) = 1$$
$$\sin(\omega t + \delta_1) = \sin 90°$$
$$\omega t + \delta_1 = 90°$$
$$\delta_1 = 90° - \omega t \qquad ...(i)$$

For (*ii*) eqn. $\qquad \theta_0 = |-1°| = 1, \theta_1 = -1°$

$$|\sin(\omega t + \delta_2)| = -1$$
$$= -\sin 90°$$
$$\sin(\omega t + \delta_2) = \sin(180° + 90°)$$
$$\omega t + \delta_2 = 270°$$
$$\delta_2 = 270° - \omega t$$
$$\delta_2 - \delta_1 = 270° - \omega t - (90° - \omega t)$$
$$\delta_2 - \delta_1 = 180°$$

Q29. (*a*) What is capillary rise? Derive an expression for the height to which a liquid rise in a capillary tube of radius r.

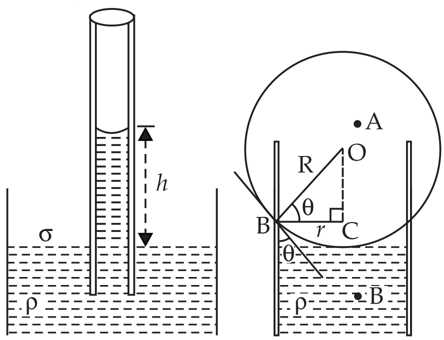
(*b*) Why small drops of liquid are always spherical in shape.

Sol. (*a*) **Capillary Rise:** When a capillary tube open at both ends dipped into liquid (partially) then the level of liquid outside and inside tube is different.

This phenomenon of rise or fall in liquid level in the capillary tube in comparison to the surrounding level is called **capillarity** or **capillary rise**.

(*i*) Ascent formula for capillary tube or

(*ii*) **Expression for height of liquid in capillary tube:** When capillary tube dipped in liquid.



ρ = Density of liquid

σ = Surface tension of liquid

r = Radius of capillary tube

θ = Angle of contact of meniscus of liquid surface excess pressure on concave surface (A) of liquid, then in liquid at B = $p = \dfrac{2\sigma}{R}$

$\cos\theta = \dfrac{r}{R} \quad \Rightarrow \quad R = \dfrac{r}{\cos\theta}$

$\therefore\ p = \dfrac{2\sigma\cos\theta}{r}$

due to the excess pressure p liquid rises up and balance when liquid rise upto height h so

$$p = h\rho g$$

$$\frac{2\sigma\cos\theta}{r} = h\rho g$$

or
$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

If we take into account the volume of liquid in meniscus then above formula reduced to

$h = \dfrac{2\sigma\cos\theta}{r\rho g} - \dfrac{r}{3}$

However, for capillary tube r is very small and $\dfrac{r}{3}$ can be neglected.

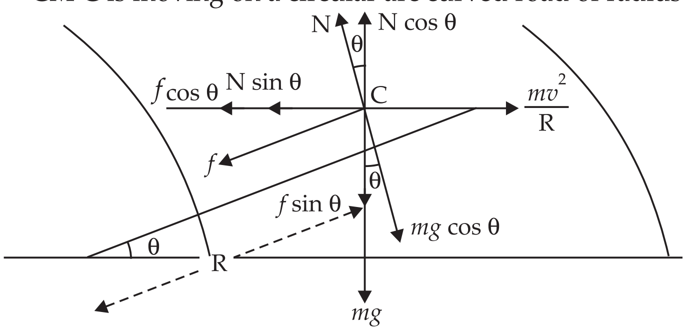
(*b*) **Small Drops of Liquid are Spherical:** Due to surface tension liquid minimise its surface. And for a particular volume, the surface area of sphere is minimum.

So, small drops become spherical to minimise its surface due to surface tension.

Q30. (*a*) Derive an expression for maximum safe speed for a car on a banked track inclined at an angle θ to the horizontal. μ is the coefficient of friction between the tracks and tyres.

(*b*) A 100 kg gun fires a ball of 1 kg from a cliff of height 500 m. If falls on ground at a distance of 400 m from the bottom of the cliff. Find the recoil velocity of the gun $g = 10\ ms^{-2}$.

Sol. (*a*) **Circular Motion of a Car on a Banked Road:** Let a car of CM C is moving on a circular arc curved road of radius R.



mg = Weight of car downward

f = Inward or downward to plane friction of force between tyre of car and road as car can skid outward.

N = Normal section of road on car perpendicular to plane of road resolving the component of f and N vertical and horizontal as shown in figure.

m = mass of car, v = velocity of car

R = Radius of circular road at the point C (of car)

$\therefore$ Centrifugal force (F) acting on car horizontal outward to the circular road $F = \dfrac{mv^2}{R}$

$$N = mg \cos\theta$$

In balance motion of car, net vertical and horizontal components of forces are zero.

$$N\cos\theta - f\sin\theta - mg = 0$$

$$N\cos\theta - \mu N\sin\theta = mg \quad ...(i) \qquad (f = \mu N)$$

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

$$N\sin\theta + \mu N\cos\theta = \frac{mv^2}{R} \quad ...(ii) \qquad (\because f = \mu N)$$

Divide (*ii*) by (*i*),

$$\frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta} = \frac{\frac{mv^2}{R}}{mg}$$

$$\frac{N\cos\theta\,[\tan\theta + \mu]}{N\cos\theta\,[1 - \mu\tan\theta]} = \frac{v^2}{Rg}$$

$$v^2 = Rg\left[\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right]$$

$$v = \sqrt{\frac{Rg\,(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$$

v_{max} for safe turning is $v_{max} = \sqrt{\dfrac{Rg\,(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$

Case I: For no wear and tear of tyre (or rod) $\mu = 0$

$$v_{max} = \sqrt{Rg\tan\theta}$$

Case II: Minimum value of θ for no wear and tear of tyre (or road)

$$v_{max}^2 = Rg\tan\theta$$

$$\tan\theta = \frac{v_{max}^2}{Rg}$$

$$\theta = \tan^{-1}\frac{v_{max}^2}{Rg}$$

(*b*) Gun — Ball

M = 100 kg — m = 1 kg

Cliff.

$h = 500$ m

$R_x = 400$ m

Vertical motion of ball

$u = 0,\quad g = 10\text{ m s}^{-2}$

$h = 500,\quad t = ?$

$$h = ut + \frac{1}{2}gt^2$$

$$500 = 0 + \frac{1}{2}\times 10t^2$$

$$t = \sqrt{\frac{500}{5}} = \sqrt{100} = 10\text{ sec}$$

Horizontal motion of ball

$R_x = 400$ m, $\quad t = 10$ s

$\therefore\ U_x = \dfrac{R_x}{t} = \dfrac{400}{10} = 40$ m/s

By law of conservation of momentum as external forces are zero

$$\therefore\qquad MV + mu = 0$$

$$100\,V + 1\times 40 = 0$$

$$V = \frac{-40}{100} = -0.4\text{ m/s}$$

$\therefore$ Recoil speed (backward) of the gun = 0.4 m s^{-1}

❑❑❑

Formulae

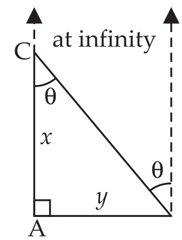
Chapter 2 Units and Measurements

1. Degree to radian (angle)

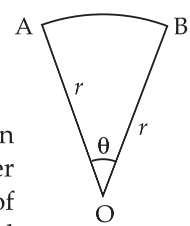
$$1^\circ = 1.745 \times 10^{-2} \text{ radian}$$
$$1 \text{ minute} = 1' = 2.91 \times 10^{-4} \text{ radian}$$
$$1 \text{ second} = 1'' = 4.85 \times 10^{-6} \text{ radian}$$

2. Distance of nearby object 'C', by parallax (θ) method

$$\boxed{x = y \cot \theta}$$



3. Angle (radian) $= \dfrac{\text{arc}}{\text{radius}} = \dfrac{\overset{\frown}{AB}}{r}$



4. Size of molecules (10^{-8} m) cannot be even measured by optical microscope, due to lower (1 Å) resolving power by light rays. In place of light ray; by using electron beam we can find out the molecular size. RP = .6 Å.

5. Measuring the diameter (x) of oleic acid

$$1 \text{ cm}^3 \text{ oleic acid + Alcohol} = 20 \text{ cm}^3 \text{ solution}$$
$$1 \text{ cm}^3 \text{ solution + Alcohol} = 20 \text{ cm}^3 \text{ solution}$$

Now, concentration of oleic acid

$$= \left(\frac{1}{20} \times \frac{1}{20}\right) \text{ oleic acid per cm}^3$$

V = Vol. of one drop of solution

Vol. or amount of oleic acid in n drops of film of oleic acid

$$V' = nV\left(\frac{1}{20 \times 20}\right) \text{cm}^3$$

∴ Thickness of diameter (x) of oleic acid molecules spread on lycopodium powder on water $= \dfrac{V'}{\text{Area of film}}$

$$\boxed{x = \frac{nV}{20 \times 20 \text{ A}}}$$

6. 1 Unified atomic mass unit (amu) = 1.66×10^{-27} kg
1 AU (astronomical unit) (mean dist. between sun and earth)
$= 1.496 \times 10^{15}$ m $= 1.5 \times 10^{15}$ m
1 light year = 9.46×10^{15} m $= 6.3 \times 10^{4}$ AU

$$1 \text{ Parsec} \left(= \frac{1 \text{ AU}}{1''}\right) = 3.08 \times 10^{16} \text{ m} = 3.26 \text{ ly}$$

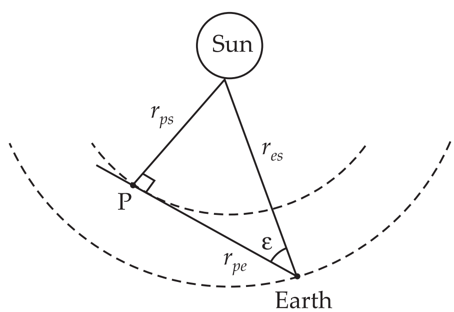
Unit of area 1 barn = 10^{-28} m^2 (ar. of cross-section of nucleus)
1 acre = 4047 m^2
1 slug = 14.57 kg
1 Chandrashekher limit = 1 CSl = 1.4 × mass of sun
White dwarf to supernova stage of star.
1 sidereal day = 1.0027 solar day
1 lunar month = 27.3 days
1 shake = 10^{-8} s
1 bar = 1 atmospheric pressure
$= 10^5$ Pascal (Pa) $\cong 10^5$ N/m^2

7. If the intensities of two stars at on a photographic plate are I_1 and I_2 and their respective distances are r_1 and r_2 then

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}$$

8. By reflection of wave or by echo method $= x = \dfrac{\text{speed} \times \text{time}}{2}$

9. Copernicus Method (for inferior planets)



ε angle of elongation (maximum ∠ between sun and planet at earth)

$$\sin \varepsilon = \frac{r_{ps}}{r_{es}}$$

$r_{pe} = 1$ AU
∴ $r_{ps} = 1 \text{ AU} \sin \varepsilon$

10. Distance for superior planet by Kepler's third law

$$T^2 \propto r^3 \quad \text{or} \quad \frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

11. Inertial mass: $\dfrac{m_1}{m_2} = \dfrac{T_1^2}{T_2^2}$

T_1, T_2 are time period of vibration in horizontal plane.
Out of m_1 and m_2 one is known.

12. Gravitational mass W = mg

13. Molar volume of gas at NTP = 22.4 lit

14. Linear magnification = $\dfrac{\text{Final size}}{\text{Initial size}} = \dfrac{\text{I (image sizes)}}{\text{O (size of object)}}$

Areal magnification = $\dfrac{\text{Area of image}}{\text{Area of object}}$

Linear magnification = $\sqrt{\text{Areal magnification}}$

15. Fundamental physical quantities are Mass (M) (kg), Length (L) (m), Time (T) (second), Temperature (K) (Kelvin), Electric current A (ampere), Luminous intensity (Cd) (Candela) quantity of matter (mol) (mole).

Supplementary unit: Plane angle θ (radian) = $\dfrac{\text{Arc}}{\text{radius}}$

and solid angle = $\dfrac{\text{Surface area}}{(\text{radius})^2} = \Omega$ (steradian)

16. If a physical quantity Q is measure n_1, n_2 in two system of units (e.g., M.K.S. or F.P.S. or S.I.) u_1 and u_2 respectively

(*i*) then $Q = n_1u_1 = n_2u_2$

(*ii*) If dimensional formula of physical quantity is

$[M_1^a\, L_1^b\, T_1^c]$ and $M_2^a\, L_2^b\, T_2^c$ then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

(*iii*) Dimensions of all terms in any relation are same.

17. Significant figures after

(*i*) **Addition or subtraction:** Final result will be in same number of decimal places as the any number with minimum number of decimal places.

(*ii*) **Multiplication or division:** Final result will be in the same numbers of significant figures as that of the number with minimum number of significant figure.

18. Errors in measurements: Basic readings $a_1, a_2, a_3, \ldots a_n$ are called **True Values**.

(*i*) Arithmetic or True value simply means

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

(*ii*) Absolute error $|\Delta a_i| = |\bar{a} - x_i|$

(*iii*) Final or average absolute error

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$

(*iv*) Relative error or fractional error $\delta a = \dfrac{\text{Final absolute error}}{\text{True value}}$

$$\delta a = \frac{\Delta\bar{a}}{\bar{a}}$$

(*v*) Relative or percent error $= \dfrac{\Delta\bar{a}}{\bar{a}} \times 100$

19. (*i*) If Z = A + B then maximum possible error ΔZ = ΔA + ΔB or Z = (A + B) ± (ΔA + ΔB)

(*ii*) If Z = A – B $\quad \Delta Z_{max} = \Delta A + \Delta B$
Z = A – B ± (ΔA + ΔB)

(*iii*) If Z = A.B then $\dfrac{\Delta Z_{max}}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

(*iv*) If $Z = \dfrac{A}{B}$ then $\dfrac{\Delta Z_{max}}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

(*v*) $Z = \dfrac{A^p B^q C^{r/n}}{D^s E^{t/e}}$

then $\dfrac{\Delta Z}{Z} = p\dfrac{\Delta A}{A} + q\dfrac{\Delta B}{B} + \dfrac{r}{n}\dfrac{\Delta C}{C} + s\dfrac{\Delta D}{D} + \dfrac{t}{e}\dfrac{\Delta E}{E}$

(*vi*) Percentage $Z = \dfrac{\Delta Z}{Z} \times 100$

Chapter 3 Motion in a Straight Line

1. Distance: Length of actual path traversed by body = Δs
Displacement $\Delta\vec{s} = \Delta\vec{s}_2 - \Delta\vec{s}_1$, magnitude initial position is subtracted from final position, direction from initial to final position.

2. Speed $v = \dfrac{s}{t} = \dfrac{s_2 - s_1}{t_2 - t_1}$ $\qquad s$ = distance (m)
v = speed (ms^{-1})

$$\vec{v} = \frac{\vec{s}}{t} = \frac{\Delta\vec{s}_2 - \Delta\vec{s}_1}{t_2 - t_1}$$

$\vec{v}$ = velocity (ms^{-1})

$\vec{s}$ = displacement (m)

$$v_{av} = \frac{\text{Total distance}}{\text{Total time}} = \frac{s_1 + s_2 + s_3 + ... + s_n}{t_1 + t_2 + ... + t_n}$$

v_{av} = average speed (ms^{-1})

$$\vec{v}_{av} = \frac{\vec{s}_2 - \vec{s}_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

$\vec{v}_{av}$ = average velocity (ms^{-1})

$$\vec{v}_{av} = \frac{u + v}{2}$$ if acceleration of body constant.

a_{av} = average acceleration (ms^{-2})

3. Acceleration $a = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{dv}{dt}$

u = initial speed

v = speed after time t sec

a = uniform acceleration (ms^{-2})

Instantaneous acceleration = $\lim\limits_{t \to 0} \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$

s = distance covered by body in time t

4. Equation of motion in conventional form

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as \quad \text{and} \quad S_n^{th} = u + \frac{a}{2}(2n - 1)$$

S_n^{th} distance traversed by body in nth second.

5. Equation of motion in Cartesian form

(*i*) $v(t) = v(0) + at$

(*ii*) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(*iii*) $x(t) - x(0) = v(0)t + \frac{1}{2}at^2$

(*iv*) $x(t_2) - x(t_1) = v(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$

(*v*) $v^2(t_2) - v^2(t_1) = 2a[x(t_2) - x(t_1)]$

6. Equation of motion for a freely falling body

(*i*) $v = u + gt$ (*ii*) $h = ut + \frac{1}{2}gt^2$

(*iii*) $v^2 - u^2 = 2gh$

7. If a body is thrown up vertically then g is taken negative in above formulae.

8. (*i*) Slope ($\tan\theta$) of s-t graph with time axis gives speed or velocity.

(*ii*) As the angle of s-t graph with time axis increases v of body increases $v = \dfrac{s_2 - s_1}{t_2 - t_1}$

(*iii*) If slope ($\tan\theta$) of s-t graph with time axis is zero then velocity (or speed) of body zero i.e., body is at rest.

(*iv*) Slope of v-t graph represents acceleration

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

(*v*) As the angle of v-t graph with time axis increases 'a' also increases.

(*vi*) If v-t graph parallel ($\tan\theta = 0°$) then $a = 0$ and vice-versa.

(*vii*) If v-t graph is straight line then a is constant.

(*viii*) If s-t graph is straight line then v is constant and vice-versa.

(*ix*) Area of v-t graph with time axis represents distance (s).

9. If v_A and v_B are the velocities of two bodies with respect to ground, the relative velocity of B with respect to A is $v_{BA} = v_B - v_A$ (vice versa also)

10. If velocity changes with time then for maximum velocity (which is again constant value) $\dfrac{dv(t)}{dt} = 0$

Chapter 4 Motion in a Plane

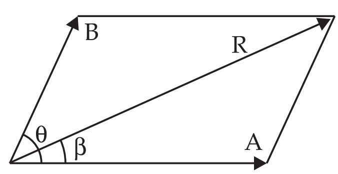
Formula:

1.

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\cos\theta = \frac{|R|^2 - |A|^2 - |B|^2}{2|A||B|}$$

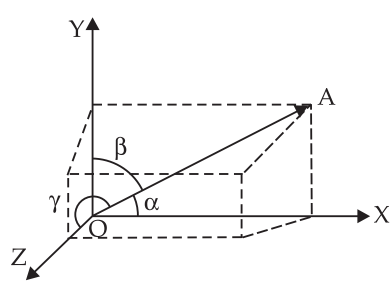
$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$



2. A_x, A_y and A_z are rectangular components of A and $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in the direction of X, Y and Z axis

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|A| = \sqrt{[A_x^2 + A_y^2 + A_z^2]}$$



$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

If vector $\vec{A}$ makes angle α, β and γ with X, Y and Z axis then

$A_x = \vec{A}\cos\alpha$, $A_y = \vec{A}\cos\beta$

and $A_z = \vec{A}\cos\gamma$ and $A = A\cos\alpha + A\cos\beta + A\cos\gamma$

$\cos\alpha = \dfrac{A_x}{|\vec{A}|}$, $\cos\beta = \dfrac{A_y}{|\vec{A}|}$ and $\cos\gamma = \dfrac{A_z}{|\vec{A}|}$

3. (*i*) $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ (*ii*) $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

(*iii*) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ (*iv*) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(*v*) $\vec{A}.\vec{B} = B.A$

(*vi*) $\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$

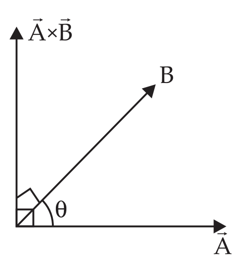
4. (*i*) $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\hat{n}$ (*ii*) $\sin\theta = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}||\vec{B}|}$

(*iii*) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ (*iv*) $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}||\vec{B}|\sin\theta}$

(*v*)
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$|\vec{A}\times\vec{B}| = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

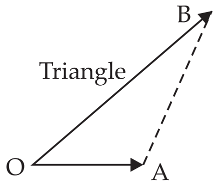
(*vi*) The direction vector $\vec{A}\times\vec{B}$ by right hand thumb rule is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ in this figure $\vec{A}\times\vec{B}$ is upward to the plane $\vec{A}$ and $\vec{B}$.

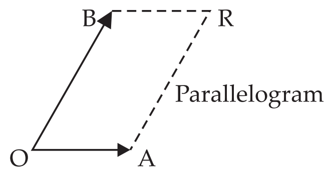


(*vii*) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$ and $\hat{i}\times\hat{j} = \hat{k}$ and $\hat{j}\times\hat{i} = -\hat{k}$

(*viii*) Area of parallelogram formed by two vectors = $|\vec{A}\times\vec{B}|$

(*ix*) Area of Δ formed by $\vec{A}$ and $\vec{B}$ = $\dfrac{1}{2}|\vec{A}\times\vec{B}|$



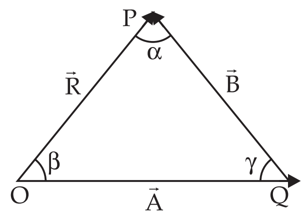


(x) A vector $\vec{A}$ is unit vector if $\left| a\hat{i} + b\hat{j} + c\hat{k} \right| = 1$

(xi) A vector $\vec{C}$ which is parallel to A (or in the direction of A) and of magnitude equal to $|B|$ is $\vec{C} = |\vec{B}|\,\hat{A}$

5. Law of sine

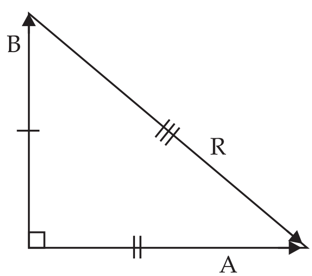
$$\frac{|\vec{A}|}{\sin\alpha} = \frac{|\vec{B}|}{\sin\beta} = \frac{|\vec{R}|}{\sin\gamma}$$



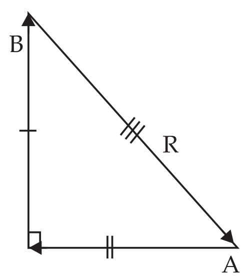
6. $\vec{B} + \vec{R} = \vec{A}$ from Figure (i)

$\therefore\ \vec{A} - \vec{B} = \vec{R}$

$\vec{A} + \vec{B} = \vec{R}$ from Figure (ii)



(i)



(ii)

$\therefore\ \left|\vec{A} - \vec{B}\right| = \left|\vec{A} + \vec{B}\right|$ if $\angle$ between $\vec{A}$ and $\vec{B}$ is 90°

7. If $\hat{a}$ and $\hat{b}$ are unit vectors then $\left|\hat{a} - \hat{b}\right| = 2\sin\dfrac{\theta}{2}$

8. (i) W.D = $\vec{F}.\vec{s}$ W.D = scalar J

(ii) $\vec{\tau} = \vec{r} \times \vec{F}$ $\vec{F}$ = Force (V) N

$\vec{s}$ = Displacement m (V)

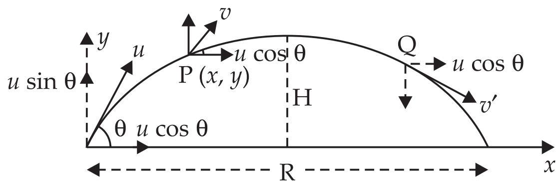
$\vec{\tau}$ = Torque (V) N-m

(iii) P = $\vec{F}.\vec{v}$ P = Power Watt (S)

(iv) $\vec{p} = m\vec{v}$ $\vec{v}$ = Velocity (V) ms^{-1}

$\vec{r}$ = $\perp^r$ distance of F from axis to Force (V)

9. Projectile's Trajectory



If projectile fired with an angle θ with horizontal

$u_x = u\cos\theta \quad u_y = u\sin\theta \quad a_y = -g \quad a_x = 0 \quad PE_{max} = mg\,H$

(*i*) Position of projectile after time t at P(x, y)

$$x = (u\cos\theta)t \qquad y = (u\sin\theta)t - \frac{1}{2}gt^2$$

(*ii*) Position after time t or equation of projectile

$$y = x\tan\theta - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

(*iii*) $H_{max} = \dfrac{u^2\sin^2\theta}{2g}$ (*iv*) $T = \dfrac{2u\sin\theta}{g}$

(*v*) $R = \dfrac{u^2\sin 2\theta}{g}$ (*vi*) $R_{max} = \dfrac{u^2}{g}$ at $\theta = 45°$

(*vii*) Velocity of projectile after time t:

$v_x = u\cos\theta \qquad v_y = u\sin\theta - gt$

$|v| = \sqrt{v_x^2 + v_y^2} \qquad \tan\beta = \dfrac{v_y}{v_x}$ [β ∠ with X-axis]

If projectile is projected horizontally from height h

then $\theta = 0°$ $\qquad H_{max} = h \qquad PE_{max} = mg\,H_{max}$

(*viii*) $x = ut \qquad y = \dfrac{1}{2}gt^2$

(*ix*) Equation of trajectory $y = \dfrac{g}{2u^2}x^2$

(*x*) Velocity at any time

$v_x = u \qquad v_y = -gt$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2t^2}$, $\tan\beta = \dfrac{gt}{u}$

(*xi*) $T = \sqrt{\dfrac{2h}{g}} \qquad R = u \times T = u\sqrt{\dfrac{2h}{g}}$

(*xii*) For maximum range (const. for a projectile)

$\frac{dR}{d\theta} = 0$ (θ is angle of velocity at any time t)

10. (*i*) Instantaneous velocity $\vec{v} = \lim_{dt\to 0} \frac{d\vec{r}}{dt}$

(*ii*) Instantaneous acceleration $\vec{a} = \lim_{dt\to 0} \frac{d\vec{v}}{dt}$

(*iii*) Relative velocity of A with respect to B = $v_{AB} = v_A - v_B$

$|v_{AB}|^2 = |v_A|^2 + |v_B|^2 - 2|v_A||v_B| \cos\theta$

(*iv*) If Angle of v_{AB} with v_A is β then

$\tan\beta = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$, θ is angle between v_A and v_B.

Chapter 5 Law of Motion

1. (*i*) $\vec{p} = m\vec{v}$ $\vec{p}$ = linear momentum (kg ms^{-1})

(*ii*) $\vec{F} = \frac{d\vec{p}}{dt} = \frac{\vec{d}(mv - mu)}{dt}$ m = mass (kg)

$\vec{v}$ = final velocity (ms^{-1})

$\vec{u}$ = initial velocity (ms^{-1})

(*iii*) $\vec{F} = m\vec{a}$ $\vec{F}$ = Force (N)

(*iv*) Effect of Force = Impulse $\vec{I} = \vec{F} \times t = mv - mu$

(*v*) $I = \int_{t_1}^{t_2} F.dt$ (for variable F), $\vec{I}$ = Impulse = N-s, kg ms^{-1}

(*vi*) Measurement of unknown Force by spring

$F = k\Delta x$ and $F_1 = k\Delta x_1$

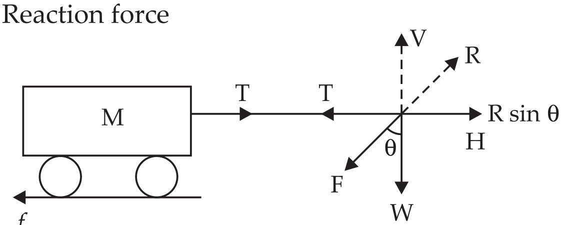
$F = \frac{\Delta x_1}{\Delta x} F_1$ F_1 is any known force

2. Horse Cart

$|W| = |V| = |R| \cos\theta$

F = action force on road by horse at an angle θ with vertical

R = Reaction force



H > f for a time

$T - f = Ma$

$H - T = ma$ H > f for a +ive

$H = R \sin\theta$ $|R| = |F|$

3. Weight in lift R is reaction force which is apparent wt

$R - mg = ma$ (upward motion) or $R = m(g + a)$

$mg - R = ma$ (downward motion) or $R = m(g - a)$

4. Velocity of rocket w.r.t. earth after time t its start velocity becomes (v_R)

(*i*) $v_R = u_g . \log_e \left(\dfrac{m_0}{m}\right) - gt$

u_g = velocity of burnt gases w.r.t. earth

m_0, m are the masses of rocket initially and after time t.

(*ii*) Burnt out speed (when fuel finished $g = 0$) of Rocket

$v_b = v_0 + u \log_e \left(\dfrac{m_0}{m_r}\right)$

v_0 = initial velocity of rocket (either zero or v_0)

(*iii*) Thrust on rocket $F = -u_g \dfrac{dm}{dt}$ (when $g = 0$)

(*iv*) If $g \neq 0$, Net thrust on rocket $= F = +u_g \dfrac{dm}{dt} - mg$

(*v*) $a = \left[\dfrac{u_g}{\left[m_0 - t.\dfrac{dm}{dt}\right]}\right] . \dfrac{dm}{dt} - g$

(*vi*) If $g \neq 0$ velocity of rocket after time (t)

$v_b = v_0 + v_g \log_e \left(\dfrac{m_0}{m}\right) - gt$

5. Law of conservation of momentum (p) $F_{ext} = 0$ then

(*i*) $\dfrac{dp}{dt} = 0$

(*ii*) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

(*iii*) Recoil speed of gun $V = \dfrac{-mv}{M}$

m and M are masses of bullet and gun respectively.

v and V are the velocities of bullet and gun respectively.

6. Coefficient of limiting μ_s or static friction

$$\mu_s = \frac{\text{limiting friction}}{\text{normal reaction}} = \frac{f_s^{max}}{R}$$

(*i*) $f_s = \mu_s R$ $\quad f_s^{max} \geq f_s$ $\quad$ f = force of friction

R = normal reaction

(*ii*) $f_k = \mu_k R$ $\quad f_k \leq f_s^{max}$

(*iii*) If θ is angle of repose of friction then

$$\mu_s = \tan\theta$$

(*iv*) A body is moving on horizontal surface $\mu = \frac{a}{g}$

$$\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g}$$

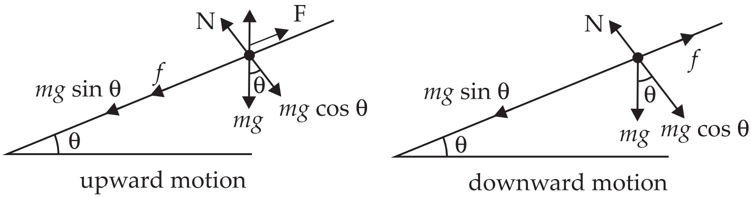
(*v*) Coefficient of rolling friction $\mu_r < \mu_k < \mu_s$

Inclined Plane

(*i*) Normal Reaction $N = mg \cos\theta$

(*ii*) $mg \sin\theta - f = ma$ (downward motion)

(*iii*) $f = \mu N = \mu\, mg \cos\theta$



(*iv*) $F - f - mg\sin\theta = ma$

$F - \mu N - mg \sin\theta = ma$

$F = \mu\, mg\cos\theta + mg\sin\theta + ma$

$F = m\,[\mu g \cos\theta + g\sin\theta + a]$

(*v*) $W.F = \vec{F}.\vec{s} = m\left[\mu g \cos\theta + g \sin\theta + a\right].\vec{s}$

(*vi*) $\mu = \tan\theta$

8. Circular Motion in Horizontal Plate

(*i*) $\vec{a}$ = (towards centre of path) centripetal acceleration

$$\vec{a} = \frac{\vec{v}^2}{r}$$

(*ii*) $v = \omega r$

(*iii*) $a_c = r\omega^2$

(*iv*) Centripetal Force (Towards centre) of path $= \frac{m\vec{v}^2}{r} = m\vec{r}\omega^2$

(*v*) $\omega = 2\pi\nu$

(*vi*) $\therefore\ \vec{F} = 4\pi^2 m\vec{r}\nu^2 = \frac{4\pi^2 m\vec{r}}{T^2}$

9. Circular Motion of Vehicle

(*i*) Leaning of vehicle or cycle with vertical on a turn on plane road.

$$\tan\theta = \frac{v^2}{rg}$$

θ = angle of plane of perpendicular of vehicle with vertical

(*ii*) Maximum velocity of vehicle for safely take circular turn of radius $r = v_{max} = \sqrt{\mu rg}$

(*iii*) On Banking road maximum speed with $\mu = 0$ on road

$$v = \sqrt{rg\tan\theta}$$

(*iv*) v_{max} on frictional Banked Road = $v_{max} = \sqrt{rg\left[\dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}\right]}$

θ is the angle of road with horizontal

10. Circular Motion of Particle in Vertical Plane

(*i*) Velocity (v) of particle at any point h m above lowest point

$$v = \sqrt{u^2 - 2gh} \quad u = \text{velocity at lowest point}$$

(*ii*) Tension at any point in string $T = \dfrac{m}{r}[u^2 - 3gh + gr]$

(*iii*) Tension at lowest point $h = 0$, $T_L = \dfrac{m}{r}[u^2 + gr]$

(*iv*) Tension at highest point where $h = 2r$, $T_H = \dfrac{m}{r}[u^2 - 5gr]$

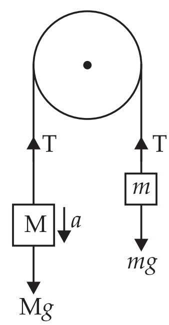
(*v*) $T_L - T_H = 6\,mg$

(*vi*) Minimum velocity at lowest point for looping the loop in vertical loop $v_L = \sqrt{5gr}$

(*vii*) Velocity at the highest point for looping

$$v_H = \sqrt{gr}$$

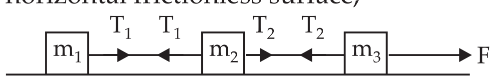
11. Motion of connected masses over a frictionless pulley and mass (negligible) of string.



(*i*) $T - mg = ma$ (*ii*) $Mg - T = Ma$

(*iii*) $a = \left[\dfrac{M - m}{M + m}\right]g$ (*iv*) $T = \dfrac{2Mmg}{(M + m)}$

12. Motion of connected masses by string on horizontal frictionless surface,



Frictionless surface

(*i*) $T_1 = m_1 a$ (*ii*) $T_2 = (m_1 + m_2)a$

(*iii*) $\vec{F} = (m_1 + m_2 + m_3)\vec{a}$

13. (*i*) When whole system is going up

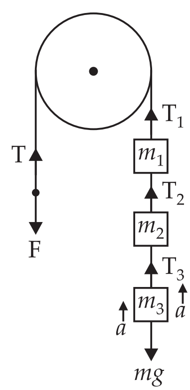
$$T_1 = F$$

$$T_1 - (m_1 + m_2 + m_3)\,g = (m_1 + m_2 + m_3)\,a$$

$$T_1 = (m_1 + m_2 + m_3)\,(g + a)$$

$$T_2 = (m_2 + m_3)\,(a + g)$$

$$T_3 = m_3\,(a + g)$$



(*ii*) Whole system is at rest $a = 0$

$$T_1 = (m_1 + m_2 + m_3)\,g$$

$$T_2 = (m_2 + m_3)\,g$$

$$T_3 = m_3\,g$$

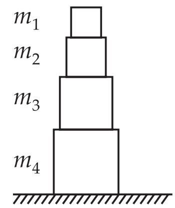
14. Masses on one another

Force on m_2 due to $m_1 = m_1 g$

Force on m_1 due to $m_2 = m_1 g$

Force on m_4 due to $m_3 = (m_1 + m_2 + m_3)\, g$

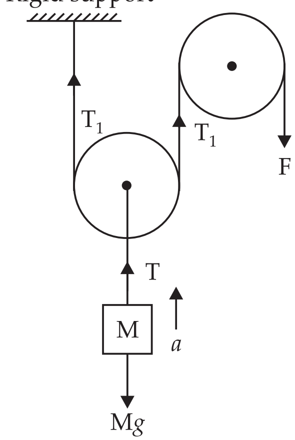
Force on m_3 due to $m_4 = (m_1 + m_2 + m_3)\, g$



15. Mass M is pulled up by force F with the help of force F

$F = 2T_1$ and $T = 2T_1$

$T - Mg = Ma$



16. (*i*) Pushing a roller

$$R_1 = W + F \sin \theta$$

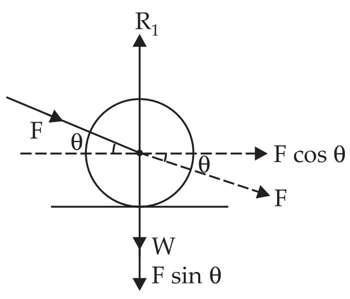
(*ii*) Pulling a roller

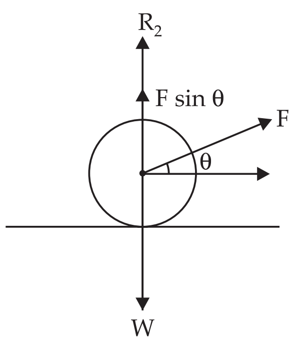
$$R_2 + F \sin \theta = W$$

$$R_2 = W - F \sin \theta$$

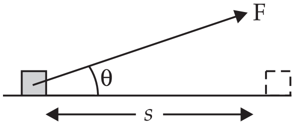
$$\therefore \quad R_1 > R_2$$

So pulling is more comfortable than pushing a roller or wheel.





Chapter 6 Work, Energy and Power



1. W.D = $\vec{F}.\vec{s}\cos\theta$

2. W.D = PE = mgh

3. W.D in moving up on inclined plane, inclined at θ with horizontal

$$W = mg\sin\theta \times s$$

h = vertical height lifted to mass m

4. W.D = ΔU = $\int_{s_1}^{s_2} \vec{F}.\vec{ds}$ For variable force i.e., $\vec{F}(s)$ Force is a function of $\vec{s}$

5. Work energy theorem, conservative forces,

$$\text{W.D} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

6. $p = \sqrt{2mk}$

7. Conservative force

(*i*) W.D depends only on the initial and final position e.g., Gravitational, Electrostatic, elastic forces.

(*ii*) When work is done by conservative forces the K.E + P.E = constt.

i.e. change in total Energy $\frac{dU}{ds} = 0$

(*iii*) $dU = -F.ds$ (attractive)

$\therefore$ $-\frac{dU}{ds} = F$

(*iv*) Potential or P.E can be taken negative for force of attraction. e.g., gravitational, electrostatic, extension of spring ($F = -kx$) etc.

(*v*) W.D in closed path W.D = $\oint_s F(x)\,dx = 0$

W.D = $\frac{1}{2}kx^2$

(*vi*) Hooke's law $F = -kx$ or $k = -\frac{F}{x}$

8. $P = \frac{W}{t}$ $P = \vec{F}.\vec{v}$

9. $E = mc^2$

m unit mass converts completely into energy E, c is speed of light.

10. Linear momentum is conserved in elastic and inelastic collision

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

11. K.E is conserved only in elastic collision

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

12. In one dimensional elastic collision velocities after collision are

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2m_2}{(m_1 + m_2)} u_2$$

$$v_2 = \frac{(m_2 - m_1)}{(m_1 + m_2)} u_2 + \frac{2m_1}{(m_1 + m_2)} u_1$$

13. Coefficient of restitution (e) for a collision is

(*i*) $e = -\frac{(v_1 - v_2)}{(u_1 - u_2)} = \frac{|v_1 - v_2|}{|u_1 - u_2|}$

(*ii*) e for a bouncing ball from a floor

$e = \frac{v}{u}$

(*iii*) For elastic collision no loss of K.E, $e = 1$

(*iv*) For inelastic collision when loss of K.E. $e < 1$

Chapter 7 System of Particles and Rotational Motion

1. (*i*) Mass m_i, position vectors r_i, then position vector R_{CM} of **centre of mass** is

$$R_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_1\vec{r}_i}{\sum_{i=1}^{n} m_i}$$

(*ii*) Co-ordinates of centre of mass

$$x = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}, \quad y = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$

(*iii*) Position vector of R_{CM} for a continuous mass distribution

$$R_{CM} = \frac{1}{M}\int \vec{r}.dm$$

dm is mass of small element at position vector $\vec{r}$

(*iv*) Co-ordinates of R_{CM}

$$x_{cm} = \frac{1}{m}\int x\,dm, \quad y_{cm} = \frac{1}{m}\int y\,dm \text{ and } z_{cm} = \frac{1}{m}\int z\,dm$$

(*v*) Velocity of C.M. = $V_{CM} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}$

2. For rotational motion under constant angular acceleration (α)

(*i*) $\omega = \omega_0 + \alpha t$ (*ii*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(*iii*) $\omega^2 = \omega_0^2 + 2\alpha\theta$

(*iv*) $\omega = \frac{2\pi n}{t} \Rightarrow \omega = \frac{2\pi}{T}, \frac{n}{t} = \frac{1}{T}$

(*v*) $\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta$

(*vi*) $\tau_{ext} = \dfrac{dL}{dt}$; $\tau = I\alpha = \dfrac{I\,d\omega}{dt}$ ω = angular speed (rad/s)

α = rad s^{-2}

(*vii*) $a = R\alpha$

(*viii*) Power (P) of torque $P = \tau\omega$ τ = Torque (N-m)

(*ix*) W.D by Torque $W = \tau.\theta$ θ = radian

(*x*) Angular momentum $L = \vec{r} \times \vec{p} = rp \sin\theta$

$L = mvr$ [if $\theta = 90°$] L = kg m^2 s^{-1}

(*xi*) Angular impulse $\vec{\tau}.\Delta t = I(\omega_f - \omega_i)$

3. Kepler's Law of Planet or Motion

(*i*) $\dfrac{\Delta A}{\Delta t}$ = constant ΔA = change in area in Δt time

(*ii*) $\dfrac{\Delta A}{\Delta t} = \dfrac{1}{2m}(\vec{r} \times \vec{p}) = \dfrac{L}{2m}$ $\dfrac{\Delta A}{\Delta t}$ = areal velocity

m = mass revolving

$p = mv$ of revolving body (kg ms^{-1})

4. Moment inertia

(*i*) $I = \sum m_i r_i^2$ I = kg m^2

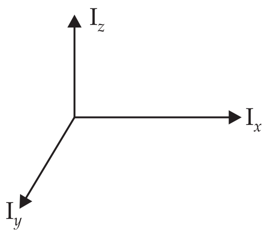
(*ii*) Radius of gyration K K = metre

$I = MK^2$

(*iii*) When all particles of same mass

$K = \sqrt{\dfrac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$

K = rms of distance



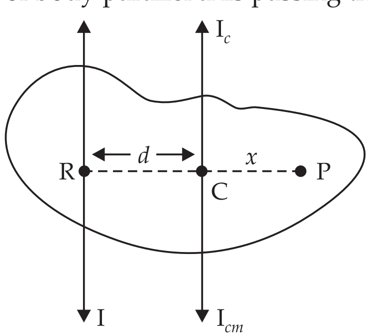
5. (*i*) $I_z = I_x + I_y$ is perpendicular axis theorem

(*ii*) parallel axis theorem

$$I = I_{CM} + Md^2$$

M = mass of whole body

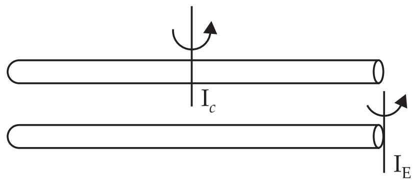
I = M.I of body parallel axis passing through R, I ∥ I_C



6. MI (I) and **radius of Gyration** (K) of a body of mass M

(*i*) Thin Rod of length L, negligible thickness.

I_C, MI of rod, $\perp^r$ to length and passing CM, $I_C = \frac{1}{12}ML^2$



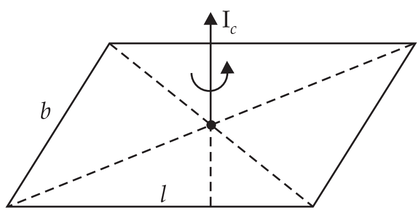
$$K_C^2 = \frac{1}{12}L^2$$

I_E, MI of rod axis $\perp^r$ to length and passing through one end

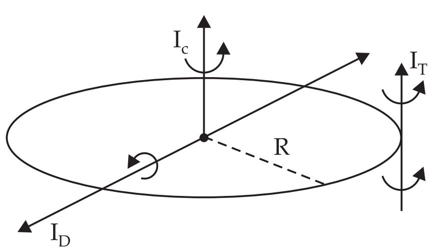
$$I_E = \frac{1}{3}ML^2 \quad K_E = \sqrt{\frac{1}{3}L^2}$$

(*ii*) Rectangular lamina of length l, breadth b, axis passing through C.M. i.e.

$$I_C = M\left(\frac{l^2 + b^2}{12}\right) \quad K_C^2 = \frac{l^2 + b^2}{12}$$



(*iii*) Circular Ring of Mass M axis passing $\perp^r$ to plane of ring and through centre (I_C) and through one end (I_T)



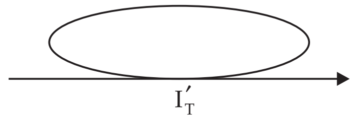
(*a*) $I_C = MR^2$ and $K_C^2 = R$ (*b*) $I_T = 2MR^2$ and $K_T^2 = 2R^2$

(*c*) $I_D = \frac{1}{2}MR^2, K_D^2 = \frac{1}{2}R^2$

(*d*) M.I. I'_T tangentially and in the plane of ring

$$I'_T = \frac{3}{2}MR^2$$

and $K'^2_T = \frac{3}{2}R^2$

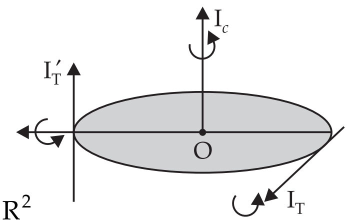


(*iv*) Disc of radius R negligible thickness

(*a*) I_C M.I perpendicular to the plane of disc and passing through centre

$$I_C = \frac{1}{2}MR^2 \text{ and}$$

$$K_C^2 = \frac{1}{2}R^2$$



(*b*) $I_D = \frac{1}{4}MR^2$ and $K_D^2 = \frac{1}{4}R^2$

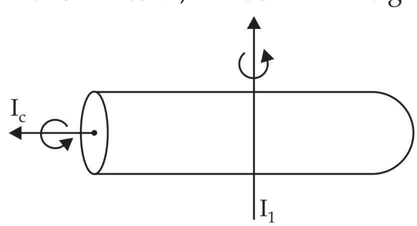
(*c*) I_T, M.I along the plane passing tangentially

$$I_T = \frac{5}{4}MR^2 \text{ and } K_T^2 = \frac{5}{4}R^2$$

(*d*) I'_T M.I perpendicular to plane of disc passing tangentially

$$I'_T = \frac{3}{2}MR^2 \text{ and } K'^2_T = \frac{3}{2}R^2$$

(*v*) Cylinder of mass M, radius R and height L



(*a*) If cylinder is solid M.I through axis of cylinder

$$I_C = \frac{1}{2}MR^2$$

(*b*) I_1 the M.I through axis perpendicular to length and at it's mid-point of length

$$I_1 = M\left[\frac{R^2}{4} + \frac{L^2}{12}\right] \text{ (Solid cylinder)}$$

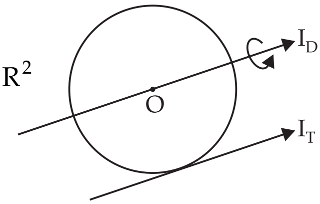
(*c*) If cylinder is hollow,

I_C = through axis $I_C = MR^2$, $K_C = R$

(*vi*) **Solid Sphere**

(*a*) $I_D = I_C = \frac{2}{5}MR^2$ and $K_C^2 = \frac{2}{5}R^2$

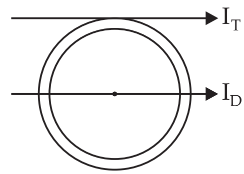
(*b*) $I_T = \frac{7}{5}MR^2$ and $K_T = \frac{7}{5}R^2$



(*vii*) **Hollow Sphere**

(*a*) $I_D = I_C = \frac{2}{3}MR^2$ and $K_C^2 = \frac{2}{3}R^2$

(*b*) $I_T = \frac{5}{3}MR^2$ and $K_T = \frac{5}{3}R^2$



7. Rotational

(*i*) Total K.E = $\frac{1}{2}I\omega^2$

(*ii*) Angular momentum $L = I\omega$ — If external τ acts

(*iii*) If $\tau = 0$ (torque not acts) — L = constt.

$I_1\omega_1 = I_2\omega_2$

Chapter 8 Gravitation

1. (*i*) Newton's law of gravitation: $F = \frac{Gm_1m_2}{r^2}$

(*ii*) Intensity of gravitational field $E = \frac{F}{m} = \frac{GM}{r^2}$

2. Mass of planet revolving around the sun in radius r and time period T

$$M = \frac{4\pi^2 r^3}{GT^2}$$

3. (*i*) Acceleration due to gravity (g) on a planet of mass M with its radius R

$$g = \frac{GM}{R^2}$$

(*ii*) Variation of 'g' with altitude (h)

$$g_h = g\left[1 - \frac{2h}{R}\right] \quad \text{when } h <<< R$$

$$g_h = \frac{gR^2}{(R+h)^2} \quad \text{when } h \text{ is comparable with R}$$

(*iii*) Variation of 'g' with depth (d)

$$g_d = g\left(1 - \frac{d}{R}\right)$$

(*iv*) If $g_h = g_d$ then $d = 2h$

(*v*) Variation of g due to rotation of earth

(*a*) At latitude λ (varies 0° to 90°) from equator to pole

$$g_\lambda = g - R\omega^2 \cos^2 \lambda$$

(*b*) At poles $\lambda = 90°$, $g_p = g$

(*c*) At equator $\lambda = 0°$, so $g_e = g - R\omega^2$

(*d*) $g_p - g_e = R\omega^2$

4. Mean density (ρ) of a planet with mass M and radius R

$$\rho = \frac{3g}{4\pi GR}$$

5. Gravitational Potential at distance r from the centre of earth is

(*i*) $V = \dfrac{W.D}{m} = \dfrac{-GM}{r}$

(*ii*) $P.E. = U = \dfrac{-GMm}{r}$

(*iii*) Electric field intensity $\vec{E} = \dfrac{-dV}{dr}$

(*iv*) Total of K.E + P.E = $\int \frac{1}{2}mv^2 + \left(\frac{-GM}{r}\right)$

6. Satellite

(*i*) Escape velocity of a satellite

$$v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{2gR} = \sqrt{\frac{8}{3}\pi G\rho R^2}$$

(*ii*) Orbital velocity v_0 at height (h)

$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

(*iii*) When a satellite revolve close to earth (Remote sensing satellite) surface *i.e.*, when $h \approx 0$, $v_0 = \sqrt{gR}$

$$v_e = \sqrt{2}v_0 = \sqrt{2gR}$$

(*iv*) Time period of a satellite

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