Physical Quantities and Units

1. Overview

Physics begins with observations of phenomena. Through rigorous and controlled experimentation and logical thought process, the physical phenomena are described quantitatively using mathematical tools. Any quantitative description of a property requires comparison with a reference For example, length needs a meter-stick. In this process we recognize a very obvious fact that properties of different kinds cannot be compared. You cannot compare the time of travel from point A to B with the distance between two points, although two quantities may be related. The time of travel is a physical quantity, *time* while the distance is a physical quantity, *length*. They are completely different types of physical quantities measured by different references and units.

Suppose you are measuring the size of your room to estimate the amount of wooden panels for your floor. You will probably use a tape measure and read out the lengths of the room in feet and inches. However, in Paris, people would do the exactly the same thing but they will write their measurements in meters and centimeters. So one can measure the same physical quantity but can express the size of the quantity in different ways (units). A physical quantity can have many different units depending on the location and culture. Scientists found that this was very inefficient and confusing and decided to set up a universal system of units, **International System of Unit (SI)**. *In this class we will use SI.*

2. Physical Quantities

Name physical quantities that you know: mass, time, speed, weight, energy, power, ... Scientists can even make up a completely new physical quantity that has not been known if necessary. However, there is a set of limited number of physical quantities of fundamental importance from which all other possible quantities can be derived. Those fundamental quantities are called **Base Physical Quantities**, and obviously the other derivatives are called **Derived Physical Quantities**. SI is built upon 7 base quantities and their associated units (see Table I).

TABLE I: SI Base Quantities and Units

Property	Symbol	Unit	Dimension
Length	L	meter (m)	L
Mass	m	kilogram (kg)	M
Time	t	second (s)	T
Temperature	T	kelvin (K)	θ
Electric Current	I	ampere (A)	I
Amount of Substance	N	mole (N)	1
Luminous Intensity	F	candela (cd)	J

TABLE II: SI Examples of Derived Quantities and Their Units

Property	Symbol	Unit	Dimension
Force	$\boldsymbol{F}$	newton (N)	$kg{\cdot}m{\cdot}s^{-2} = kg{\cdot}m/s^2$
Speed	$\boldsymbol{v}$	meter per second (m/s)	$m{\cdot}s^{-1} = m/s$
Pressure	$\boldsymbol{P}$	pascal (Pa)	(force per unit area) $kg{\cdot}m^{-1}{\cdot}s^{-2}$
Energy	$\boldsymbol{E}$	joule (J)	$kg{\cdot}m^2{\cdot}s^{-2}$
Power	$\boldsymbol{W}$	watt (W)	(energy per unit time) $kg{\cdot}m^2{\cdot}s^{-3}$

A physical quantity can be expressed with a *unique combination* of 7 base quantities. One can also make a physical quantity with a combination of derived quantities. But it will be eventually reduced to a combination of base quantities. For example, *Kinetic Energy* (E) is a type of energy represented in joule (J) and is a derived quantity through $\frac{1}{2}mv^2$ *i.e.* (1/2)(**mass**)x(speed)x(speed). Since speed is a derived quantity itself: (speed) = (**length**)/(**time**), one can express energy using base quantities: (1/2)(**mass**)x(**length**)^2/(**time**)^2. Note that the base quantities are in bold. Some of the important derived physical quantities are listed in Table II.

3. Conversion of Units

Below is the table for commonly used unit conversion. It is also useful to know metric prefixes (Table IV). Let us do a couple of examples of unit conversion.

TABLE III: Unit Conversion of Base Quantities

Quantity	From	To	Operation
Length	inch (in)	m	(inch) × 0.0254
	foot (ft)	m	(foot) × 0.3048
	mile (mi)	m	(mile) × 1609.34
Mass	pound (lb)	kg	(pound) × 0.4536
	metric ton (t)	kg	(ton) × 1000
	ounce	kg	(ounce) × 0.02835
Volume	liter (l)	m^3	(*liter*) × 0.001
	gallon (ga)	m^3	(gallon) × 0.00379
Temperature	fahrenheit (F)	K	$\{(\text{fahrenheit}) - 32\} \times \frac{5}{9} + 273.15$
	celcius (C)	K	(celcius) + 273.15

Examples

1. Length 0.02 in can be converted into SI unit in meters using Table I: (0.02in)×0.0254 = 0.000508 m. Too many zeros below decimal points here. Since 0.000508 = 0.508×10^{-3} = 508×10^{-6}, it is also 0.503 mm (millimeter) or 508 μm (micrometer).
2. Honda Fit weighs about 2,500 lb. It is equivalent to 2500lb × 0.4536 = 1134.0kg.

TABLE IV: Metric Prefix

Prefix Name	Prefix Symbol	Base 10	Decimal	English Word
peta	P	1,000,000,000,000,000	10^{15}	quadrillion
tera	T	1,000,000,000,000	10^{12}	trillion
giga	G	1,000,000,000	10^{9}	billion
mega	T	1,000,000	10^{6}	million
kilo	k	1,000	10^{3}	thousand
deca	da	10	10^{1}	ten
		1	one	
centi	c	0.01	10^{-2}	hundredth
milli	m	0.001	10^{-3}	thousandth
micro	μ	0.000,001	10^{-6}	millionth
nano	n	0.000,000,001	10^{-9}	billionth
pico	p	0.000,000,000,001	10^{-12}	trillionth
femto	f	0.000,000,000,000,001	10^{-15}	quadrillionth

Let us look into a derived quantity. Julia is driving her Honda Fit on I-75 at 70 mph (miles per hour). Any moving object carries a physical quantity called *kinetic energy*; you will soon learn about this. The kinetic energy is given by $\frac{1}{2}mv^2$. So if you use the conventional units, it will give $0.5 \times 2500 \times 70 \times 70$ lb(mph)^2. This quantity of energy needs to be expressed in SI units in this class. So convert the mass into kg and the speed mph into m/s (meter per second).

Step 1 Convert mass. It is already done in the above example m = 1134 kg.

Step 2 Convert speed. v = 70 mile/hr = 70 (1609.34 m)/(3600 s) = 70 $\cdot$1609.34/3600 = 31.29 m/s.

Step 3 Carry out the calculation using the quantities in SI. (kinetic energy) = $\frac{1}{2}mv^2$ = $0.5 \cdot 1134 \cdot (31.29)^2$ = 555, 129.3 kg m^{2}s^{-2}. This is equivalent to 555,129.3 J (joule) (see Table I).

Q1 The length scale in astronomy is much larger than what we are used to. So scientist uses a different length unit called *light-year* (ly) which is the distance that light travels for one year (speed of light is 3×10^8 m/s). One of the nearest star from the solar system is about 4.2 ly away. Express 4.2 ly in km.

Answer: about $4 \times 10^{+13}$ km.

Sirius is the brightest star in the night sky. It is 8.6 ly away from Earth. When you look at Sirius on a clear night, light from the star was emitted 8.6 years ago and traveled at the speed of light for 8.6 years to reach your eyes.

4. Weight and mass, they are not the same quantities in physics!

Measure your weight on a scale. Suppose that the scale reads 120 lb. Let's investigate this little further. Table I says pound is a conventional unit for a base quantity, **mass**. So your scale measures your mass 120 lb = 54.43 kg. However, if you bring your scale to the moon and measure your weight, it will give you 19.9 lb = 9.03 kg about 1/6 of the quantity on

Earth. Has your mass changed? Mass is the total amount of material or atoms–everything is composed of atoms–in an object. Ignoring the possible weight loss during the space travel, the amount of substance in your body should not change. What happened?

When you step on a scale, you apply a force on the top of the scale. The force that you are applying is due to gravitational pull (proportional to mass) of you by the earth. This force will press down the top of your scale and the scale measures internally the force you applied on the surface. The gravitational pull is weaker on the moon than on Earth. That is why the scale reads 1/6 of the reading on Earth. The scale is calibrated (designed to be used on Earth) to show you the mass equivalent to the force detected on Earth. *Weight* is the *force* due to gravitational pull. So it should be expressed in unit of newton (N) not in kg!

Note: When a quantity A is proportional to a quantity B, it means mathematically $A = c\,B$ where c is a constant (fixed) number. Knowing the value of c, the knowledge of one quantity immediately produce the value of the other quantity. Conversely, if you know both A and B, then you can calculate $c = \frac{A}{B}$.

It took a long time through Galileo and Newton to figure out the force of gravitational pull is proportional to mass; $F = gm$. The proportional constant g is called *gravitational acceleration* on Earth (g = 9.8 m/s^2). The gravitational acceleration on the moon is 1.6 m/s^2 which is about 1/6 of the Earth value. You can directly measure g in many ways, which we will do in our laboratory session.

5. The Length Scale of the Universe

Length is one of the fundamental base quantities. We are very familiar with length: height, distance of travel, size of a cell phone, ... You will be surprised to know what range of length physicists are dealing with. The size of the largest virus is about 10^{-6} m, the size of atom is about 10^{-10} m, and the radius of Earth is about 6×10^{6} m. You may think the numbers are outrageously small or large. But physicists have a quite good idea how to understand phenomena occurring in these outrageous length scales: quantum physics describes phenomena in very small length scale such as subatomic world and astrophysics for, as you imagine, astronomical phenomena such as expansion of universe.

Q2 What are the physical entities in the femtometer (fm) range? What is the typical size of galaxies? *You can get the answers from the web, http://scaleofuniverse.com/.*

Q3 How many years would it take light to traverse a typical galaxy (light-year)? How long would it take to pass through an atom in seconds?

6. How cold it can get?

On a day of winter storm, the temperature in Antarctica can reach -100 F. This is much colder than your freezer. Then what is the lowest temperature one can get? Deep in space far far away from the sun, is that the coldest spot in the universe? Is -1,000,000 F possible? If you convert Fahrenheit into kelvin using the formula in Table III, -100 F is equivalent to 199.82 K. One reason that physicists use kelvin scale rather than Celcius or Fahrenheit is directly related to the question posed above. The lowest temperature allowed in physics–therefore, one can reach– is zero kelvin, 0 K. One can not reach negative kelvin temperature, very convenient! And we call this *absolute zero*. Then what is the temperature of deep space? Surprisingly it is not absolute zero. Physicists know the average temperature of deep space with very high accuracy, about 2.73 K *now*. It used to be hotter than now and will get colder in the future.

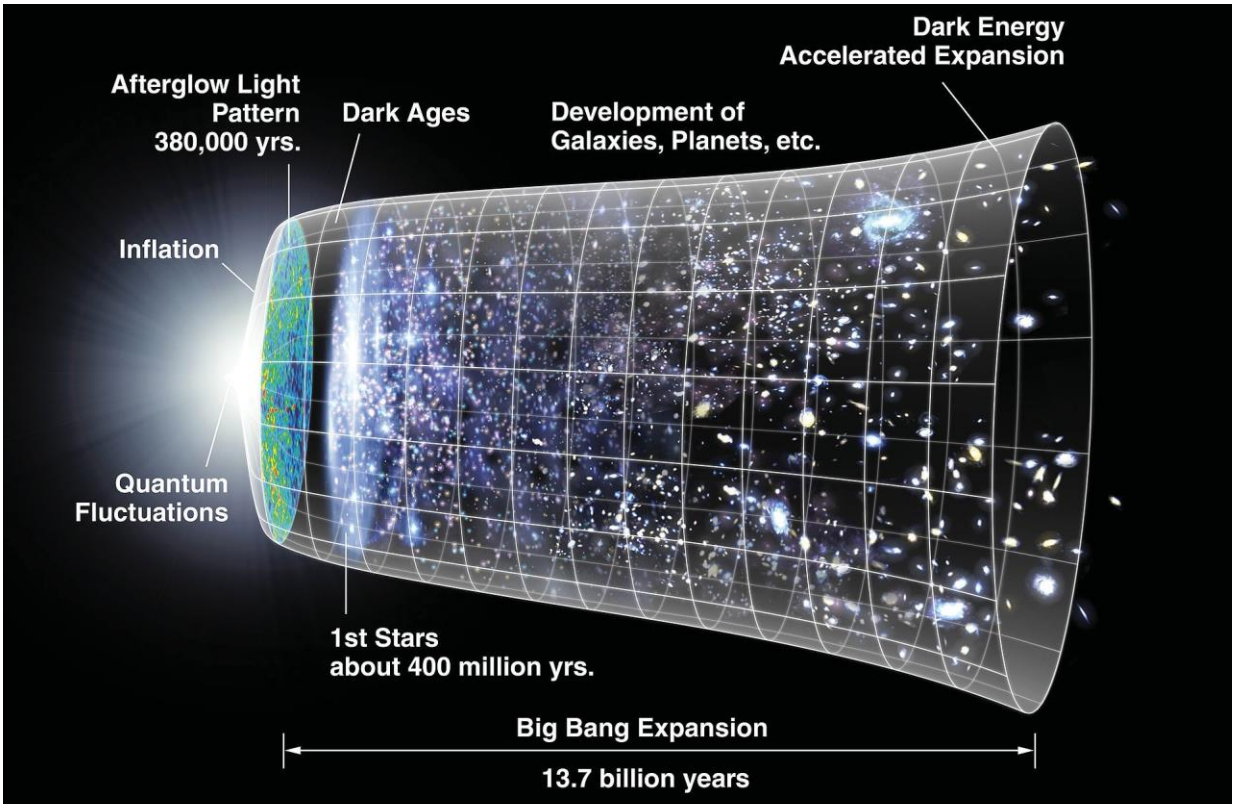


FIG. 1: Diagram of evolution of the universe from the Big Bang. From wikipedia, modified from the original NASA/WMAP.

The current understanding of the universe is that it started from a point in which all matter and energy are contained. At this stage, it is unimaginably hot. The universe started to expand and accordingly cooled down very rapidly. Around 10^{-43} s after the Big Bang, the universe cooled down to 10^{32} K (I know it is still unimaginably high temperature!). Within about 1 s, the universe cooled to 10^{9} K, and reached to the current temperature, 2.73 K after about 14 billion years after the Big Bang.

"... while the sources of heat were obvious the sun, the crackle of a fire, the life force of animals and human beings cold was a mystery without an obvious source, a chill associated with death, inexplicable, too fearsome to investigate."

Absolute Zero and Conquest of Cold

by T. Shachtman

Current technology can reach around 10^{-9} K (nanokelvin). It is 0.000,000,001 K above absolute zero. You can think of absolute zero like the speed of light in the sense that it is a barrier nature does not allow to go beyond and one can only reach as close as possible. As the temperature lowers close to absolute zero, nature reveals fascinating character, quantum nature which is so different from what we experience. Remember that when the length scale gets shorter in subatomic level, quantum physics is needed. If you lower temperature low enough, the quantum nature appears even in a macroscopic length scale.

APPENDIX 1.1 PHYSICAL QUANTITIES AND THEIR SI UNITS

	symbol	*SI measurement units*	*symbol*	*unit dimensions*
distance	***d***	**meter**	**m**	**m**
mass	***m***	**kilogram**	**kg**	**kg**
time	***t***	**second**	**s**	**s**
electric charge*	***Q***	**coulomb**	**C**	**C**
temperature	***T***	**Kelvin**	**K**	**K**
amount of substance	***n***	**mole**	**mol**	**mol**
luminous intensity	***I***	**candela**	**cd**	**cd**
acceleration	*a*	meter per second squared	m/s^2	m/s^2
area	*A*	square meter	m^2	m^2
capacitance	*C*	farad	F	$C^2{\cdot}s^2/kg{\cdot}m^2$
concentration	*[C]*	molar	M	mol/dm^3
density	*D*	kilogram per cubic meter	kg/m^3	kg/m^3
electric current	*I*	ampere	A	C/s
electric field intensity	*E*	newton per coulomb	N/C	$kg{\cdot}m/C{\cdot}s^2$
electric resistance	*R*	ohm	^	$kg{\cdot}m^2/C^2{\cdot}s$
emf	ξ	volt	V	$kg{\cdot}m^2/C{\cdot}s^2$
energy	*E*	joule	J	$kg{\cdot}m^2/s^2$
force	*F*	newton	N	$kg{\cdot}m/s^2$
frequency	*f*	hertz	Hz	s^{-1}
heat	*Q*	joule	J	$kg{\cdot}m^2/s^2$
illumination	*E*	lux (lumen per square meter)	lx	cd/m^2
inductance	*L*	henry	H	$kg{\cdot}m^2/C^2$
magnetic flux	ϕ	weber	Wb	$kg{\cdot}m^2/C{\cdot}s$
potential difference	*V*	volt	V	$kg{\cdot}m^2/C{\cdot}s^2$
power	*P*	watt	W	$kg{\cdot}m^2/s^3$
pressure	*p*	pascal (newton per square meter)	Pa	$kg/m{\cdot}s^2$
velocity	*v*	meter per second	m/s	m/s
volume	*V*	cubic meter	m^3	m^3
work	*W*	joule	J	$kg{\cdot}m^2/s^2$

* The official SI quantity is electrical current, and the base unit is the ampere. Electrical current is the

UNIT & DIMENSION

Preface

IIT - JEE Syllabus : Unit & Dimension
Unit & Dimensions, Dimensional analysis, Least count, Significant figure, Methods of measurement and Error analysis for physical quantities.

Fundamental concepts of the Physics start from this chapter. Basically the terms & concepts which are illustrated in this topic will be used in so many ways because all Physical quantities have units. It is must to measure all Physical quantities so that we can use them. In this chapter we will have an over view of different units of different Physical quantities. We will learn the dimension and dependence of the unit of any Physical quantity on fundamental quantities or unit. Entire topic is illustrated very systematically with respective examples so that the students can understand the fundamentals very easily & quickly. Students are advised to read every point of supplementary very carefully which is given at the end of the topic. Generally, students are not able to find out the Dimension of unseen or new quantity as their basic concepts are not clear & then they read the dimensions like a parrot. It should be avoided & they should develope themselves, so that they can find out the dimensions of any given quantity.

Total number of Questions in **Units & Dimension** are :

In chapter Examples...27

Physics :

Physics is the study of the laws of nature from the observed events.

1. PHYSICAL QUANTITIES

The quantities by means of which we describe the laws of physics are called physical quantities. There are two type of physical quantities.

1.1 Fundamental quantities

1.2 Derived quantities

1.1 Fundamental quantities:

Physical quantities which are independent of each other and cannot be further resolved into any other physical quantity are known as fundamental quantities. There are seven fundamental quantities.

Fundamental quantity	Units	Symbol
(a) Length	Metre	m
(b) Mass	Kilogram	kg
(c) Time	Second	s
(d) Electric current	Ampere	A
(e) Thermodynamic temperature	Kelvin	K
(f) Luminous intensity	Candela	Cd
(g) Amount of substance	Mole	Mol.

1.2 Derived Quantities :

Physical quantities which depend upon fundamental quantities or which can be derived from fundamental quantities are known as derived quantities.

2. UNITS

Definition : Things in which quantity is measured are known as units.

Measurement of physical quantity

= (Magnitude) × (Unit)

Ex.1 A physical quantity is measured and the result is expressed as nu where u is the unit used and n is the numerical value. If the result is expressed in various units then :

(A) $n \propto$ size of u (B) $n \propto u^2$

(C) $n \propto \sqrt{u}$ (D) $n \propto \frac{1}{u}$

Answer : (D)

There are three types of units

2.1 Fundamental or base units

2.2 Derived units

2.3 Supplementary units

2.1 Fundamental or base units:

Units of fundamental quantities are called fundamental units.

2.1.1 Characteristics of fundamental units:

(i) they are well defined and are of a suitable size

(ii) they are easily reproducible at all places

(iii) they do not vary with temperature, time pressure etc. i.e. invariable.

(iv) there are seven fundamental units.

2.1.2 Definitions of fundamental units:

2.1.2.1 Metre :

The distance travelled by light in Vacuum in $\frac{1}{299,792,458}$ second is called 1m.

2.1.2.2 Kilogram :

The mass of a cylinder made of platinum iridium alloy kept at international bureau of weights and measures is defined as 1kg.

2.1.2.3 Second :

Cesium -133 atom emits electromagnetic radiation of several wavelengths. A particular radiation is selected which corresponds to the transistion between the two hyperfine levels of the ground state of Cs - 133. Each radiation has a time period of repetition of certain characteristics. The time duration in 9, 192, 631, 770 time periods of the selected transistion is defined as 1s.

2.1.2.4 Ampere :

Suppose two long straight wires with negligible cross-section are placed parallel to each other in vacuum at a seperation of 1m and electric currents are established in the two in same direction. The wires attract each other. If equal currents are maintained in the two wires so that the force between them is 2×10^{-7} newton per meter of the wire, then the current in any of the wires is called 1A. Here, newton is the SI unit of force.

2.1.2.5 Kelvin :

The fraction $\frac{1}{273.16}$ of the thermodynamic temperature of triple point of water is called 1K.

2.1.2.6 Mole :

The amount of a substance that contains as many elementary entities (Molecules or atoms if the substance is monoatomic) as there are number of atoms in .012 kg of carbon - 12 is called a mole. This number (number of atoms in 0.012 kg of carbon-12) is called Avogadro constant and its best value available is 6.022045×10^{23} .

2.1.2.7 Candela:

The S.I. unit of luminous intensity is 1cd which is the luminous intensity of a blackbody of surface area $\frac{1}{600,000}$ m^2 placed at the temperature of freezing platinum and at a pressure of 101,325 N/m^2, in the direction perpendicular to its surface.

Examples based on **Definition of fundamental Units**

Ex.2 A man seeing a lighting starts counting seconds until he hears thunder. He then claims to have found an approximate but simple rule that if the count of second is divided by an integer, the result directly gives in km, the distance of the lighting source. What is the integer if the velocity of sound is 330 m/s

Sol. If n is the integer then according to the

problem $\frac{t \text{ in s}}{n}$ = dist in km.

$$\frac{t \text{ in s}}{n} = (v)\, t$$

$$n = \frac{1}{v} = \frac{1}{330 \times 10^{-3}} = 3$$

Ex.3 In defining the standard of length we have to specify the temperature at which the measurement should be made. Are we justified in calling length a fundamental quantity if another physical. quantity, temperature, has to be specified in choosing a standard.

Sol. Yes, length is a fundamental quantity. One metre is the distance that contains 1650 763.73 wavelength of orange-red light of Kr - 86. Hence, the standard metre is independent of temperature. But the length of object varies with temperature and is given by the relation .

$$L_t = L_0 (1 + \alpha t)$$

∴ We usually specify the temperature at which measurement is made.

Ex.4 Which of the following sets cannot enter into the list of fundamental quantities in any system of units

(A) length ; mass ; velocity
(B) length ; time ; velocity
(C) mass ; time; velocity
(D) length ; time, mass

Sol.[B] Since velocity = $\frac{\text{length}}{\text{time}}$ i.e. in this set a quantity is dependent on the other two quantities Where as fundamental quantities are independent.

2.2 Derived units :

Units of derived quantities are called derived units.

	Physical quantity	units
Illustration	Volume = $(\text{length})^3$	m^3
	Speed = length/time	m/s

2.3 Supplementary units :

The units defined for the supplementary quantities namely plane angle and solid angle are called the supplementary units. The unit for plane angle is rad and the unit for the solid angle is steradian.

Note :

The supplemental quantities have only units but no dimensions (will be discussed later)

3. PRINCIPAL SYSTEM OF UNITS

3.1 C.G.S. system [centimetre (cm) ; gram (g) and second (s)]

3.2 F.P.S system [foot ; pound ; second]

3.3 M.K.S. system [meter ; kilogram ; second]

3.4 S.I. (system of international)

In 1971 the international Bureau of weight and measures held its meeting and decided a system of units. Which is known as the international system of units.

Examples based on **Units**

Ex.5 The acceleration due to gravity is 9.80 m/s^2. What is its value in ft/s^2 ?

Sol. Because 1 m = 3.28 ft, therefore

$9.80\ m/s^2 = 9.80 \times 3.28\ ft/s^2$

$= 32.14\ ft/s^2$

Ex.6 A cheap wrist watch loses time at the rate of 8.5 second a day. How much time will the watch be off at the end of a month ?

Sol. Time delay = 8.5 s/day

= 8.5 × 30 s/ (30 day)

= 255 s/month = 4.25 min/month.

4. DIMENSIONS

Dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.

Illustration :

Force (Quantity) = mass × acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}} = \text{mass} \times \frac{\text{length}}{(\text{time})^2}$$

$$= \text{mass} \times \text{length} \times (\text{time})^{-2}$$

So dimensions of force : 1 in mass

1 in length

–2 in time

and Dimensional formula : $[MLT^{-2}]$

5. DIMENSIONAL FORMULA

It is an expression which shows how and which of the fundamental units are required to represent the unit of physical quantity.

Different quantities with units. symbol and dimensional formula,

Quantity	Symbol	Formula	S.I. Unit	D.F.
Displacement	s	—	Metre or m	M^0LT^0
Area	A	$l \times b$	$(\text{Metre})^2$ or m^2	$M^0L^2T^0$
Volume	V	$l \times b \times h$	$(\text{Metre})^3$ or m^3	$M^0L^3T^0$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	–	F × t	N.sec	MLT^{-1}
Work	W	F. d	N.m	ML^2T^{-2}
Energy	KE or U	$K.E. = \frac{1}{2}mv^2$ P.E. = mgh	Joule or J	ML^2T^{-2}
Power	P	$P = \frac{W}{t}$	watt or W	ML^2T^{-3}
Density	d	d = mass/volume	kg/m^3	$ML^{-3}T^0$

Pressure	P	P = F/A	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m.	$ML^{2}T^{-2}$
Angular displacement	θ	$\theta = \frac{arc}{radius}$	radian or rad	$M^{0}L^{0}T^{0}$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^{0}L^{0}T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	$M^{0}L^{0}T^{-2}$
Moment of Inertia	I	$I = mr^2$	kg-m^2	$ML^{2}T^{0}$
Angular momentum	J or L	J = mvr	$\frac{kgm^2}{s}$	$ML^{2}T^{-1}$
Frequency	ν or f	$f = \frac{1}{T}$	hertz or Hz	$M^{0}L^{0}T^{-1}$
Stress	—	F/A	N/m^2	$ML^{-1}T^{-2}$
Strain	—	$\frac{\Delta l}{l}; \frac{\Delta A}{A}; \frac{\Delta V}{V}$	—-	$M^{0}L^{0}T^{0}$
Youngs modulus (Bulk modulus)	Y	$Y = \frac{F/A}{\Delta l / l}$	N/m^2	$ML^{-1}T^{-2}$
Surface tension	T	$\frac{F}{l}$ or $\frac{W}{A}$	$\frac{N}{m}; \frac{J}{m^2}$	$ML^{0}T^{-2}$
Force constant (spring)	k	F = kx	N/m	$ML^{0}T^{-2}$
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S)	$ML^{-1}T^{-1}$
Gravitational constant	G	$F = \frac{Gm_1m_2}{r^2}$ $\Rightarrow G = \frac{Fr^2}{m_1m_2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^{3}T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$M^{0}L^{2}T^{-2}$
Temperature	θ	—	Kelvin or K	$M^{0}L^{0}T^{0}\theta^{+1}$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	$ML^{2}T^{-2}$
Specific heat	S	$Q = m \times S \times \Delta t$	$\frac{Joule}{kg.Kelvin}$	$M^{0}L^{2}T^{-2}\theta^{-1}$
Latent heat	L	Q = mL	$\frac{Joule}{kg}$	$M^{0}L^{2}T^{-2}$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$	$\frac{Joule}{m\ secK}$	$MLT^{-3}\theta^{-1}$

Universal gas constant	R	PV = nRT	$\frac{\text{Joule}}{\text{mol.K}}$	$ML^2T^{-2}\theta^{-1}$
Mechanical equivalent of heat	J	W = JH	—	$M^0L^0T^0$
Charge	Q or q	$I = \frac{Q}{t}$	Coulomb or C	M^0L^0TA
Current	I	—	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$	$\frac{(\text{coul.})^2}{\text{N.m}^2}$ or $\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}A^2T^4$
Electric Potential	V	$V = \frac{\Delta W}{q}$	Joule/coul	$ML^2T^{-3}A^{-1}$
Intensity of electric field	E	E =	N/coul.	$MLT^{-3}A^{-1}$
Capacitance	C	Q = CV	Farad	$M^{-1}L^{-2}T^4A^2$
Dielectric constant or relative permittivity	ε_r	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	—	$M^0L^0T^0$
Resistance	R	V = IR	Ohm	$ML^2T^{-3}A^{-2}$
Conductance	S	$S = \frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{-3}A^2$
Specific resistance or resistivity	ρ	$\rho = \frac{RA}{l}$	Ohm × meter	$ML^3T^{-3}A^{-2}$
Conductivity or specific conductance	σ	$\sigma = \frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^3A^2$
Magnetic induction	B	$F = qvB\sin\theta$ or F = BIL	Tesla or weber/m^2	$MT^{-2}A^{-1}$
Magnetic flux	ϕ	$e = \frac{d\phi}{dt}$	Weber	$ML^2T^{-2}A^{-1}$
Magnetic intensity	H	$B = \mu H$	A/m	$M^0L^{-1}T^0A$
Magnetic permeability of free space or medium	μ_0	$B = \frac{Idl \sin\theta}{r^2}$	$\frac{N}{amp^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e = L \cdot \frac{dI}{dt}$	Henery	$ML^2T^{-2}A^{-2}$
Electric dipole moment	p	$p = q \times 2l$	C.m.	M^0LTA
Magnetic dipole moment	M	M = NIA	amp.m^2	$M^0L^2AT^0$

Examples based on Dimensions

Ex.7 (a) Can there be a physical quantity which has no unit and dimensions

(b) Can a physical quantity have unit without having dimensions

Sol. (a) Yes, strain

(b) Yes, angle with units radians

Ex.8 Fill in the blanks

(i) Three physical quantities which have same dimensions are ....

(ii) Mention a scalar and a vector physical quantities having same dimensions ....

Sol. (i) Work, energy, torque

(ii) Work, torque

Ex.9 Choose the correct statement (s)

(A) all quantities may be represented dimensionally in terms of the base quantities

(B) all base quantity cannot be represented dimensionally in terms of the rest of the base quantities

(C) the dimension of a base quantity in other base quantities is always zero.

(D) the dimension of a derived quantity is never zero in any base quantity.

Sol. **[A,B,C]**

(B) all the fundamental base quantities are independent of any other quantity

(C) same as above

Ex.10 If velocity (V), time (T) and force (F) were chosen as basic quantities, find the dimensions of mass.

Sol. Dimensionally :

Force = mass × acceleration

Force = mass ×

$$\text{Mass} = \frac{\text{Force} \times \text{time}}{\text{velocity}}$$

mass = FTV^{-1}

Ex.11 In a particular system, the unit of length, mass and time are chosen to be 10cm, 10gm and 0.1s respectively. The unit of force in this system will be equivalent to

(A) $\frac{1}{10}$ N (B) 1N

(C) 10N (D) 100 N

Sol. Dimensionally

$F = MLT^{-2}$

In C.G.S system

1 dyne = 1g 1 cm $(1s)^{-2}$

In new system

1x = (10g) (10 cm) $(0.1s)^{-2}$

$$\frac{1\text{dyne}}{1\text{x}} = \frac{1\text{g}}{10\text{g}} \times \frac{1\text{cm}}{10\text{cm}}\left(\frac{10\text{s}}{1\text{s}}\right)^{-2}$$

$$1 \text{ dyne} = \frac{1}{10{,}000} \times 1\text{x}$$

10^4 dyne = 1x

10 x = 10^5 dyne = 1 N

$$x = \frac{1}{10}\text{N}$$

Ex.12 If the units of length and force are increased four times, then the unit of energy will

(A) increase 8 times

(B) increase 16 times

(C) decreases 16 times

(D) increase 4 times

Sol. Dimensionally

$E = ML^2T^{-2}$

$E = (MLT^{-2})\ (L)$

$E' = (4)\ (MLT^{-2})\ (4L)$

$E' = 16\ (ML^2T^{-2})$

$\frac{\text{veloc}}{\text{tim}}$

Note :

5.1 Two physical quantities having same dimensions can be added or subtracted but there is no such restriction in division and multiplication. (Principle of homogeneity)

Illustration : Using the theory of dimensions, determine the dimensions of constants ‘a’ and ‘b’ in Vander Wall’s equation. $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Sol. $\frac{a}{V^2}$ must have the same dimensions as that of P (because it is added to P)

Dimension of b must be same as that of V.

$$\frac{[a]}{L^6} = ML^{-1}T^{-2}$$

$[a] = ML^5T^{-2}$

$[b] = L^3$

5.2 Expressions such as sinx ; cosx (trigonometric functions) e^x, a^x, log x, lnx , have no dimensions. In these quantities 'x' has also no dimensions.

Examples based on Remarks

Ex.13 The time dependence of physical quantity P is found to be of the form

$P = P_0 e^{-\alpha t^2}$

Where 't' is the time and α is some constant.

Then the constant α will

(A) be dimensionless

(B) have dimensions of T^{-2}

(C) have dimensions of P

(D) have dimensions of P multiplied by T^{-2}

Sol. Since in e^x, x is dimensionless

$\therefore$ In ; αt^2 should be dimensionless

$\alpha t^2 = M^0L^0T^0$

$\alpha = M^0L^0T^{-2}$

6. APPLICATION OF DIMENSIONAL ANALYSIS

6.1 To find the unit of a given physical quantity in a given system of units

Illustration F = $[MLT^{-2}]$

6.2 In finding the dimensions of physical constants or coefficients.

Examples based on Application of dimensional analysis

Ex.14 To find the dimensions of physical constants, G, h, η etc.

Sol. Dimension of (Gravitational constant)

$G : F = \quad \Rightarrow [MLT^{-2}] = \frac{[G][M^2]}{L^2}$

$G = M^{-1}L^3T^{-2}$

Dimensions of h : Plank's constant

$E = h\nu$

$ML^2T^{-2} = h . \frac{1}{T}$

$h = ML^2T^{-1}$

Dimension of η : Coefficient of viscosity.

$F = 6\pi\eta vr$

$[MLT^{-2}] = \eta[LT^{-1}][L]$

$\eta = [ML^{-1}T^{-1}]$

6.2.1 To convert a physical quantity from one system to the other

Example based on Conversion of units from one system to other system of units

Ex.15 Conversion of Newton to Dyne

(MKS) (C.G.S.)

Dimensional formula of F = MLT^{-2}

$$1N = \frac{1kg \times 1m}{(|\sec|^2)} = \frac{1000\,g \times 100\,cm}{(|\sec|)^2}$$

$$= 10^5 \frac{g.cm}{sec^2}$$

$1N = 10^5$ Dyne

Ex.16 Conversion of G from SI system to C.G.S.

Dimensional formula = $M^{-1}L^3T^{-2}$

$$G = 6.67 \times 10^{-11} \times \frac{m^3}{kg.s^2}$$

$$G = 6.67 \times 10^{-11} \times \frac{(100cm)^3}{1000g.(1sec)^2}$$

$$G = 6.67 \times 10^{-11} \times \frac{10^6}{10^3} \frac{cm^3}{g.sec^2}$$

$$G = 6.67 \times 10^{-8} \frac{cm^3}{g.sec^2}$$

Ex.17 If the units of force, energy and velocity in a new system be 10 N, 5J and 0.5 ms^{-1} respectively, find the units of mass, length and time in that system.

Sol. Let M_1, L_1 and T_1 be the units of mass, length and time in SI and M_2, L_2 and T_2 the corresponding units in new system.

The dimensional formula for force is $(M^1 L^1 T^{-2})$

Hence the conversion formula for force becomes

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-2}$$

Here n_1= 10 N, n_2 = 1, Substituting we get

$$1 = 10 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-2} \quad ...(1)$$

The dimensional formula for work is
$(M^1 L^2 T^{-2})$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^2 \left[\frac{T_1}{T_2}\right]^{-2}$$

$n_1 = 5$ J, $n_2 = 1$
Substituting values we get

$$1 = 5 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^2 \left[\frac{T_1}{T_2}\right]^{-2} \quad ...(2)$$

Similarly the dimensional formula for velocity is $(M^o L^1 T^{-1})$.
Hence, conversion formula for velocity is

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^0 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-1}$$

Here $n_1 = 0.5\ ms^{-1}$, $n_2 = 1$,
Substituting values we get

$$1 = 0.5 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-1} \quad ...(3)$$

Dividing (2) by (1), $1 = \left[\frac{L_1}{L_2}\right]$,

$$L_2 = \frac{L_1}{2} = \frac{1}{2} m = 0.5\ m$$

Substituting value of $\left[\frac{L_1}{L_2}\right]$ in (3), we get

$$1 = 0.5 \times 2 \left[\frac{T_1}{T_2}\right]^{-1}, \quad \frac{T_1}{T_2} = 1, T_2 = 1s$$

Substituting value of $\left[\frac{L_1}{L_2}\right]$ and $\left[\frac{T_1}{T_2}\right]$ in (1)

$$1 = 10 \left[\frac{M_1}{M_2}\right] \times 2 \times 1,$$

$$1 = 20 \left[\frac{M_1}{M_2}\right],$$

$M_2 = 20\ M_1$ as $M_1 = 1$kg, $M_2 = 20$ kg.
Hence units of mass, length and time are 20 kg, 0.5 m and 1 sec respectively

6.2.2 Conversion of a quantity from a given system to new hypothetical system

Examples

Ex.18 The density of a substance is 8 g/cm^3. Now we have a new system in which unit of length is 5cm and unit of mass 20g. Find the density in this new system

Sol. In the new system ; Let the symbol of unit of length be La and mass be Ma.

Since 5cm = 1 La $\Rightarrow$ 1cm = $\frac{1\ La}{5}$

20g = 1Ma $\Rightarrow$ 1g = $\frac{1\ Ma}{20}$

$$D = 8\ g/cm^3 = \frac{\left(8 \times \frac{1}{20}\right) Ma}{\left(\frac{1 La}{5}\right)^3}$$

$D = 50\ Ma/(La)^3$
= 50 units in the new system

6.3 To check the dimensional correctness of a given relation

Examples based on **Dimensional correctness**

Ex.19 Find the correct relation

$F = \frac{mv^2}{r^2}$ or $F = \frac{mv^2}{r}$

Sol. Checking the dimensionally correctness of relation

$F = \frac{mv^2}{r^2}$
L.H.S. = MLT^{-2}

R.H.S. = $\frac{M(LT^{-1})^2}{L^2} = ML^0T^{-2}$; LHS $\neq$ RHS

$F = \frac{Mv^2}{r}$
LHS = MLT^{-2}

RHS = $\frac{M(LT^{-1})^2}{L} = MLT^{-2}$; LHS = RHS

Hence dimensionally second relation is correct

Limitation :

It is not necessary that every dimensionally correct relation, physically may be correct

6.4 As a research tool to derive new relation

Examples based on **Deriving new relation**

Ex.20 To derive the Einstein mass - energy relation

Sol. $E = f(m, c)$

$E = k M^x C^y$

$ML^2T^{-2} = M^x (LT^{-1})^y$ $ML^2T^{-2} = M^xL^yT^{-y}$

Comparing the coefficients

$x = 1$; $y = +2$

Through experiments ; k = 1

$\therefore E = mc^2$

Ex.21 When a small sphere moves at low speed through a fluid, the viscous force F opposing the motion, is found experimentaly to depend on the radius 'r', the velocity v of the sphere and the viscosity η of the fluid. Find the force F (Stoke's law)

Sol. $F = f(\eta ; r ; v)$

$F = k . \eta . r. v$

$MLT^{-2} = (ML^{-1}T^{-1})^x (L)^y (LT^{-1})^z$

$MLT^{-2} = M^x L^{-x+y+z} T^{-x-z}$

comparing coefficients

$x = 1$, $-x + y + z = 1$; $-x - z = -2$

$x = y = z = 1$

$F = k\eta vr$

$F = 6\pi\eta vr$

As through experiments : $k = 6\pi$

Ex.22 A gas bubble from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$ where p is the static pressure, d is the density of water and E is the total energy of explosion. Find the values of a,b, and c.

Sol. a = – ; $b = \frac{1}{2}$ and $c = \frac{1}{3}$

7. LIMITATIONS OF THE APPLICATION OF DIMENSIONAL ANALYSIS

7.1 If the dimensions are given, then the physical quantity may not be unique as many physical quantities can have same dimensions.

Examples based on **Limitation of dimensional analysis**

Ex.23 If there is a physical quantity whose dimensional formula is $[ML^2T^{-2}]$. Determine the physical quantity.

Sol. It may be torque, work or energy.

7.2 Since numerical constant have no dimensions.

Such as , 1, 6π etc, hence these can't be deduced by the methods of dimensions.

7.3 The method of dimensions cannot be used to derive relations other than product of power functions.

Illustration : $S = ut + \frac{1}{2}at^2$, $y = a\sin\omega t$

Note :

However the dimensional correctness of these can be checked

7.4 The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than three physical quantities. As then there will be less number (=3) of equations than the unknowns. However the dimensional correctness of the equation can be checked

Illustration : $T = 2\pi\sqrt{\frac{I}{mgl}}$ cannot be derived by theory of dimensions.

7.5 Even if a physical quantity depends on three physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions.

Illustration : Formula of the frequency of a tunning fork $f = \left(\frac{d}{L^2}\right)v$

Note :

However the dimensional correctness can be checked.

8. SIGNIFICANT DIGITS

8.1 Normally decimal is used after first digit using powers of ten,

Illustration : 3750 m will be written as 3.750×10^3 m

8.2 The order of a physical quantity is expressed in power of 10 and is taken to be 1 if $\le (10)^{1/2} = 3.16$ and 10 if > 3.16

Illustration : speed of light = 3×10^8, order = 10^8

Mass of electron = 9.1×10^{-31}, order = 10^{-30}

8.3 Significant digits : In a multiplication or division of two or more quantities, the number of significant digits in the answer is equal to the number of significant digits in the quantity which has the minimum number of significant digit

Illustration : 12.0/7.0 will have two significant digits only.

8.4 The insignificant digits are dropped from the result if they appear after the decimal point. They are replaced by zeroes if they appear to the left of two decimal point. The least significant digit is rounded according to the rules given below.

Rounding off : If the digit next to one rounded as more then 5, the digit to be rounded is increased by 1; if the digit next to the one rounded is less than 5, the digit to be rounded is left unchanged, if the digit next to one rounded is 5, then the digit to be rounded is increased by 1 if it odd and is left unchanged if it is even.

8.5 For addition and subtraction write the numbers one below the other with all the decimal points in one line now locate the first column from left that has doubtful digits. All digits right to this column are dropped from all the numbers and rounding is done to this column. The addition and subtraction is now performed to get the answer.

8.6 Number of 'Significant figure' in the magnitude of a physical quantity can neither be increased nor decreased.

Illustration :: If we have 3.10 kg than it can not be written as 3.1 kg or 3.100 kg.

Examples based on significant digits

Ex.24 Round off the following numbers to three significant digits

(a) 15462

(b) 14.745

(c)14.750

(d) 14.650×10^{12}.

Sol.(a) The third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore , be increased by 1. The digits to be dropped should be replaced by zeros because they appear to the left of the decimal. Thus, 15462 becomes 15500 on rounding to three significant digits.

(b) The third significant digit in 14.745 is 7. The number next to it is less than 5. So 14.745 becomes 14.7 on rounding to three significant digits.

(c) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

(d) 14.650×10^{12} will become 14.6×10^{12} because the digit to be rounded is even and the digit next to it is 5.

Ex.25 Evaluate $\frac{25.2 \times 1374}{33.3}$. All the digits in this expression are significant.

Sol. We have $\frac{25.2 \times 1374}{33.3}$ = 1039.7838 .....

Out of the three numbers given in the expression 25.2 and 33.3 have 3 significant digits and 1374 has four. The answer should have three significant digits. Rounded 1039.7838 .... to three significant digits, it becomes 1040.

Thus , we write.

$$\frac{25.2 \times 1374}{33.3} = 1040.$$

Ex.26 Evaluate 24.36 + 0.0623 + 256.2

Sol.

24.36
0.0623
256.2

Now the first column where a doubtful digit occurs is the one just next to the decimal point (256.2). All digits right to this column must be dropped after proper rounding. The table is rewritten and added below

24.4
0.1
256.2
―――
280.7 The sum is 280.7

SUPPLEMENTRY

9. FRACTIONAL AND PERCENTAGE ERRORS

9.1 Absolute error

= |experimental value – standard value|

9.2 If Δx is the error in measurement of x, then

Fractional error = $\frac{\Delta x}{x}$

Percentage error = × 100%

Percentage error in experimental measurement

= × 100%

9.3 Propagation of error (Addition and Subtraction) :

Let error in x is ± Δx, and error in y is ± Δy, then the error in x + y or x – y is ± (Δx + Δy). The errors add.

9.4 Multiplication and Division :

Let errors in x, y, z are respectively ± Δx, ± Δy and ± Δz. Then error in a quantity f (defined as)

f = is obtained from the relation

$\frac{\Delta f}{f} = |a| \quad + |b| \left|\frac{\Delta y}{y}\right| + |c| \left|\frac{\Delta z}{z}\right|$. The fraction errors (with proper multiples of exponents) add. The error in f is ± Δf.

9.5 Important Points :

9.5.1 When two quantities are added or subtracted the absolute error in the result is the sum of the absolute error in the quantities.

9.5.2 When two quantities are multiplied or divided, the fractional error in the result is the sum of the fractional error in the quantities to be multiplied or to be divided.

9.5.3 If the same quantity x is multiplied together n times (i.e. x^n), then the fractional error in x^n is n times the fractional error in x,

Examples based on

i.e. $\pm n \frac{\Delta x}{x}$

Errors

Ex.27 In an experiment to determine acceleration due to gravity by simple pendulum, a student commit positive error in the measurement of length and 3% negative error in the measurement of time period. The percentage error in the value of g will be-

(A) 7% (B) 10%
(C) 4% (D) 3%

Sol. We know T = k

$\therefore T^2 = k' \left(\frac{l}{g}\right) \Rightarrow g = k' \frac{l}{T^2}$

$\frac{\Delta g}{g} \times 100 = \quad \times 100 + \quad \times 100$

= 1% + 2 x 3% = 7%

Hence correct answer is **(A)**

10. MEASURMENTS OF LENGTH, MASS & TIME

10.1 Distance of a hill :

To find the distance of a hill, a gun is fired towards the hill and the time interval t between the instant of firing the gun and the instant of hearing the echo of the gun is determined. Clearly, during this time interval sound first travels towards the hill from the place of firing and then back from the hill to the place of firing. If v be the velocity of sound, and s the distance of hill from the place of firing , then

$$2s = v \times t$$

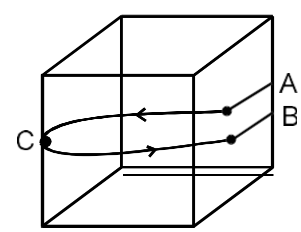
$$\text{or } s = \frac{v \times t}{2}$$

10.2 Distance of moon :

A laser beam is a source of very intense, monochromatic and unidirectional beam. By sending a laser beam towards the moon instead of sound waves, the echo method becomes useful in finding the distance of moon from the earth. If t is the total time taken by laser beam in going towards moon and back, then distance of moon from the earth's surface is given by : $S = \frac{c \times t}{2}$

Where c = 3 × 10^8 m/s ; is the velocity of laser beam.

10.3 Thickness of matter sheet :



For finding the thickness of some matter sheet, a signal from point A on the front surface of sheet is sent to the back surface. The signal gets reflected from point C on the back surface and is again received back at point B on the front surface. If the time interval between the instants of sending the signal from point A and receiving the signal back at B, is t, then thickness of sheet

$$S = \frac{C \times t}{2}$$ (where C is the velocity of signal)

10.4 Distance of submerged objects or submarines in sea (Sound navigation and ranging - Sonar)

Sonar in a instrument which uses ultrasonic waves (waves having frequency > 20,000 Hz) to detect and locate the submerged objects, submarines etc. in sea. Ultrasonic waves produced from a transmitter are sent towards the distant objects under water. When the object comes in the direction of ultrasonic waves, then the waves are reflected back from it. Measuring the time interval t between the instants the ultrasonic waves are sent and received back, the distance S of the object can be calculated by the relation.

$S = \frac{C \times t}{2}$ (where C is the velocity of ultrasonic waves)

10.5 Distance of aeroplane (Radio detection and Ranging - Radar). Radar is an instrument which uses. radiowaves for detecting and locating an aeroplane. Radiowaves produced by a transmitter at the radar station, are sent towards the aeroplane in space. These waves are reflected from the aeroplane. The reflected waves are received by a receiver at the radar station. By noting the time interval between the instants of transmission of waves and their detection, distance of aeroplane can be measured. If t is the required time interval and C the velocity of light (=equal to velocity of radio waves) then distance of aeroplane

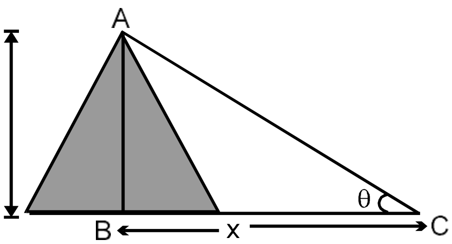
$$S = \frac{C \times t}{2}$$

10.6 Triangulation method :

This method uses the geometry of the triangle and is useful for measuring the heights in following cases

10.6.1

Height of a tower or height of an accessible object.

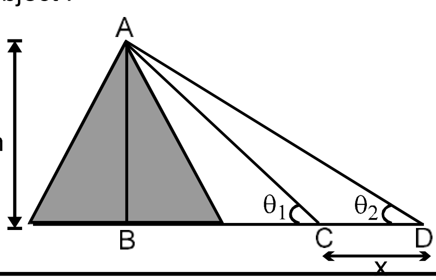


$\tan\theta = \frac{AB}{BC} = \frac{h}{x}$

$h = x \tan\theta$

10.6.2

Height of a mountain or height of an accessible object :



This method is useful in cases when it becomes impossible to measure the distance between the object and the observation point.

$\tan\theta_1 = \frac{AB}{BC} = \frac{h}{BC}$

$BC = h \cot\theta_1$

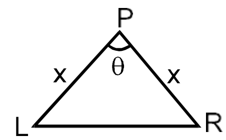
Similarly $\tan\theta_2 = \frac{AB}{BD}$

$BD = h \cot\theta_2$

$BD - BC = x = h(\cot\theta_2 - \cot\theta_1)$

10.7 Parallax method :

Parallax (Definition) : When we observe the object P by closing our right and left eye alternately, we observe a shift in the position of object w.r.t the background. This is known as parallax.



$\theta = \frac{LR}{x} = \frac{b}{x}$ (assuming distance LR as a circular arc of radius x)

$x = \frac{b}{\theta}$

10.7.1

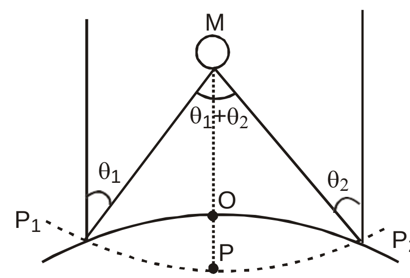
Determination of distance of moon from earth.

$\theta = \theta_1 + \theta_2$

Because $\theta = \frac{P_1PP_2}{PM}$

Hence $PM = \frac{P_1PP_2}{\theta}$

As astronomical bodies are at very large distances from earth



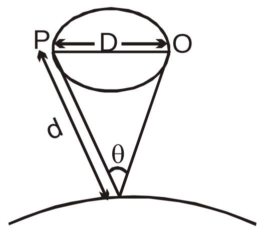
P_1PP_2 P_1P_2 PM MO

OM =

$\tilde{P}$ P 1 2 θ

10.7.2

Determination of size of an astronomical object (Moon)



$\theta = \quad = \frac{D}{d}$

$D = \theta d$

10.8 Measurement of very small distances :

Various devices are used to measure very small distances like vernier calliper, screw gauge, spherometer, optical microscopes, electron-microscopes, X-ray diffraction etc.

POINTS TO REMEMBER

1. Relation between some practical units of the standard of length

(i) 1 light year = 9.46×10^{15} m

$\frac{PO}{d}$

(ii) 1 par sec = 3.06×10^{16} m

= 3.26 light year

(iii) 1 AU = 1.496×10^{11} m

(iv) 1 X-ray unit = 10^{-3} m

2. Relation between some practical units of the standard mars

(i) 1 C.S.L. (chandra shekhar limit)

= 1.4 time the mass of sun

= 2.8×10^{30} kg

(ii) 1 amu = 1.67×10^{-27} kg

(iii) 1 slug = 14.57 kg

3. Relation between some practical unit of standards of time

(i) 1 solar day's = 86400 sec

(ii) 1 Lunar Month = 27.3 days

(iii) 1 solar year = 365.25 average solar day = 366.25 sedrial day

(iv) 1 shake = 10^{-8} sec

4. In mechanics, dimensions of a quantity are given in terms of powers of mass (M), length (L) and time (T). In heat and thermodynamics, power of temperature (θ) comes in addition to powers of M, L and T. In electricity and magnetism, dimensions are given in terms of M, L, T and I, the current.

5. Only like quantities having the same dimensions can be added to or subtracted from each other.

6. The dimensional formula of a physical quantity does not depend upon the system of units used to represent that quantity.

7. The value (magnitude) of a physical quantity remains the same in all systems of measurement. However, the numerical value changes.

In general, $n_1u_1 = n_2u_2 = n_3u_3 = ...$

8. All of the following have the same dimensional formula $[M^0L^0T^{-1}]$. Frequency, angular frequency, angular velocity and velocity gradient.

9. All of the following are dimensionless. Angle, Solid angle, T-ratios, Strain, Poisson's ratio, Relative density, Relative permittivity, Refractive index and Relative permeability.

10. Following three quantities have the same dimensional formula $[M^0L^2T^{-2}]$. Square of velocity, gravitational potential, latent heat.

11. Following quantities have the same dimensional formula $[ML^2T^{-2}]$. Work, energy, torque, heat.

12. Force, weight, thrust and energy gradient have the same dimensional formula $[MLT^{-2}]$.

13. Entropy, gas constant, Boltzmann constant and thermal capacity have the same dimensions in mass, length and time.

14. Light year, radius of gyration and wavelength have the same dimensional formula $[M^0L^1T^0]$

15. Rydberg constant and propagation constant have the same dimensional formula $[M^0L^{-1}T^0]$.

16. The decimal point does not separate the certain and uncertain digit, only last digit may be uncertain.

17. Significant figures indicate the precision of measurement which depend on least count of the measuring instruments.

18. So far as significant figures are concerned, in mathematical operations like addition and subtraction, the result would be correct upto minimum number of decimal places in any of the quantities involved. However, in multiplication and division, number of significant figures in the result will be limited corresponding to the minimum number of significant figures in any of the quantities involved.

To represent the result to a correct number of significant figures, we round off as per the rules already stated.

19. Whenever two measured quantities are multiplied or divided, the maximum possible relative or percentage error in the computed result is equal to the sum of relative or percentage errors in the observed quantities. Therefore maximum possible error in

$Z = \frac{A^m B^n}{C^p}$ is :

$$\frac{\Delta Z}{Z} \times 100 = m\frac{\Delta A}{A} \times 100 + n\frac{\Delta B}{B} \times 100 + p \times \frac{\Delta C}{C} \times 100$$