Class – XII
MATHEMATICS- 041
SAMPLE QUESTION PAPER 2019-20

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

(i) All the questions are compulsory.
(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

	SECTION A	
Q1 - Q10 are multiple choice type questions. Select the correct option		
1	If A is any square matrix of order 3 × 3 such that $\lvert A\rvert = 3$, then the value of $\lvert adjA\rvert$ is ? (a) 3 (b) $\frac{1}{3}$ (c) 9 (d) 27	1
2	Suppose P and Q are two different matrices of order 3 × n and n × p , then the order of the matrix P × Q is? (a) 3 × p (b) p × 3 (c) n × n (d) 3 × 3	1
3	If $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$,then the values of p and q are ? (a) p= 6 ,q=27(b)p=3,q=$\frac{27}{2}$ (c) p=6,q= $\frac{27}{2}$ (d) p=3 ,q=27	1
4	If A and B are two events such that P(A)=0.2 , P(B)=0.4 and $P(A \cup B)=0.5$, then value of P(A/B) is ? (a)0.1 (b)0.25 (c)0.5 (d) 0.08	1
5	The point which does not lie in the half plane $2x + 3y - 12 \leq 0$ is (a) (1,2) (b) (2,1) (c) (2,3) (d)$(-3, 2)$	1
6	If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is _____ (a)$\frac{2\pi}{3}$(b)$\frac{\pi}{3}$(c)$\frac{\pi}{2}$(d) π	1

7	An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is (a) $\frac{2}{5}$(b) $\frac{1}{15}$ (c) $\frac{8}{15}$(d) $\frac{4}{15}$	1
8	$\int \frac{dx}{\sqrt{9-25x^2}}$ (a) $\sin^{-1}\left(\frac{5x}{3}\right)$ + c(b)$\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)$ + c (c) $\frac{1}{6}\log\left(\frac{3+5x}{3-5x}\right)$ + c(d)$\frac{1}{30}\log\left(\frac{3+5x}{3-5x}\right)$ + c	1
9	What is the distance(in units) between the two planes 3x + 5y + 7z = 3 and 9x + 15y + 21z = 9 ? (a) 0(b) 3(c) $\frac{6}{\sqrt{83}}$(d) 6	1
10	The equation of the line in vector form passing through the point(−1,3,5) and parallel to line $\frac{x-3}{2} = \frac{y-4}{3}$, z = 2. is (a) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$. (b) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j})$ (c) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$ (d) $\vec{r} = (2\hat{i} + 3\hat{j}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$	1
(Q11 - Q15) Fill in the blanks		
11	If f be the greatest integer function defined asf(x) = [x] and g be the modulus function defined as g(x) = |x| , then the value of g of $\left(-\frac{5}{4}\right)$ is_____	1
12	If the function$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{when } x \neq 1 \\ k & \text{when } x = 1 \end{cases}$ is given to be continuous at $x = 1$, then the value of k is ____	1
13	If$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$, then value of y is _____.	1
14	If tangent to the curve $y^2 + 3x - 7 = 0$ at the point (h, k) is parallel to line $x - y = 4$, then value of k is ___? **OR** For the curve $y = 5x - 2x^3$,if x increases at the rate of 2units/sec, then at $x = 3$the slope of the curve is changing at_____	1
15	The magnitude of projection of $(2\hat{i} - \hat{j} + \hat{k})$ on $(\hat{i} - 2\hat{j} + 2\hat{k})$is_____ **OR** Vector of magnitude 5 units and in the direction opposite to $2\hat{i} + 3\hat{j} - 6\hat{k}$ is___	1
(Q16 - Q20) Answer the following questions		
16	Check whether (l + m + n) is a factor of the determinant$\begin{vmatrix} l+m & m+n & n+l \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$or not. Give reason.	1
17	Evaluate $\int_{-2}^{2}(x^3 + 1)dx$.	1
18	Find $\int \frac{3+3\cos x}{x+\sin x}dx$.	1

	OR Find $\int(cos^2 2x - sin^2 2x)dx$	
19	Find $\int xe^{(1+x^2)}dx$.	1
20	Write the general solution of differential equation $\frac{dy}{dx} = e^{x+y}$	1
	SECTION – B	
21	Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$;where$-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form. **OR** Let R be the relation in the set Z of integers given by R = {(a, b) : 2 divides a – b}.Show that the relation R transitive? Write the equivalence class [0].	2
22	If = $ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.	2
23	A particle moves along the curve $x^2 = 2y$. At what point, ordinate increases at the same rate as abscissa increases?	2
24	For three non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, prove that $[\ \vec{a} - \vec{b}\quad \vec{b} - \vec{c}\quad \vec{c} - \vec{a}\ ] = 0$. **OR** If $\vec{a} + \vec{b} + \vec{c} = 0\ and\ \lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 5, \lvert\vec{c}\rvert = 7$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.	2
25	Find the acute angle between the lines$\frac{x-4}{3} = \frac{y+3}{4} = \frac{z+1}{5}$ and$\frac{x-1}{4} = \frac{y+1}{-3} = \frac{z+10}{5}$	2
26	A speaks truth in 80% cases and B speaks truth in 90%cases. In what percentage of cases are they likely to agree with each other in stating the same fact?	2
	SECTION – C	
27	Let $f: A \rightarrow B$ be a function defined as $f(x) = \frac{2x+3}{x-3}$, whereA = R – {3}and B = R – {2}. Is the function f one –one and onto? Is f invertible? If yes, then find its inverse.	4
28	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$. **OR** If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find$\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.	4
29	Solve the differential equation $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$.	4

30	Evaluate $\int_1^3 \lvert x^2 - 2x \rvert dx$.	4
31	Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X. Also, find mean of the distribution. **OR** There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If It shows head. What is probability that it was the two headed coin ?	4
32	Two tailors A and B earn ₹150 and ₹200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Form a L.P.P to minimize the labour cost to produce (stitch) at least 60 shirts and 32 pants and solve it graphically.	4
	SECTION D	
33	Using the properties of determinants, prove that $\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$. **OR** If A = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$	6
34	Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 1,\ x + y \geq 1,\ x \geq 0,\ y \geq 0\}$	6
35	A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi-circular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of semi-circular ends is $\pi : \pi + 2$. **OR** Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.	6
36	Find the equation of a plane passing through the points $A(2,1,2)$ and $B(4,-2,1)$ and perpendicular to plane $\vec{r}.(\hat{i} - 2\hat{k}) = 5$. Also, find the coordinates of the point, where the line passing through the points (3,4,1) and (5,1,6) crosses the plane thus obtained.	6

Class – XII
MATHEMATICS (041)
SQP Marking Scheme (2019-20)

TIME: 3 Hrs. **Maximum Marks: 80**

	SECTION A	
1	(c) 9	1
2	(a) 3 × p	1
3	(b)p=3,q=$\frac{27}{2}$	1
4	(b)0.25	1
5	(c) (2,3)	1
6	(b)$\frac{\pi}{3}$	1
7	(c) $\frac{8}{15}$	1
8	(b) $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)$ + c	1
9	(a) 0	1
10	(b) $\vec{r} = \left(-\hat{i} + 3\hat{j} + 5\hat{k}\right) + \lambda(2\hat{i} + 3\hat{j})$	1
11	$g\left(\left[-\frac{5}{4}\right]\right) = g(-2) = 2$	1
12	2	1
13	$y = 2$	1
14	$\frac{-3}{2}$ OR decreasing at rate of 72 units/sec.	1
15	2 units OR $\frac{5}{7}\left(-2\hat{i} - 3\hat{j} + 6\hat{k}\right)$	1
16	Apply$R_1 \to R_1 + R_2$ $\begin{vmatrix} l+m+n & m+n+l & n+l+m \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$ $= 2(l + m + n)\begin{vmatrix} 1 & 1 & 1 \\ n & l & m \\ 1 & 1 & 1 \end{vmatrix}$; yes (l + m + n) is a factor	1
17	$\int_{-2}^{2}(x^3 + 1)dx = \int_{-2}^{2}(x^3)dx + \int_{-2}^{2} 1dx = I_1 + I_2$ $= 0 + [x]_{-2}^{2}$ (As I_1 is odd function) =2+2 = 4	1

18	Let $x + \sin x = t$ So $(1 + \cos x)dx = dt$ $I = 3\int \frac{dt}{t} = 3\log\|t\| + c = 3\log\|(x + \sin x)\| + c$ or directly by writing formula $$\int \frac{f'(x)}{f(x)}dx = \log\|f(x)\| + c$$ **OR** $\int \cos 4x\, dx = \frac{\sin 4x}{4} + c$	1
19	let $(1 + x^2) = t$ so $2xdx = dt$ $\Longrightarrow I = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + c = \frac{1}{2}e^{(1+x^2)} + c$	1
20	$\frac{dy}{dx} = e^x e^y$ $\Longrightarrow \frac{dy}{e^y} = e^x dx$ integrating both sides $\Rightarrow -e^{-y} + c = e^x$ $\Rightarrow \mathbf{e^x + e^{-y} = c}$	1
	SECTION B	
21	$= \sin^{-1}\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)$ if $-\frac{\pi}{4} < x < \frac{\pi}{4}$ $=\sin^{-1}\left(\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$ if $-\frac{\pi}{4} + \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4}$ $=\sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right)$ if $0 < \left(x + \frac{\pi}{4}\right) < \frac{\pi}{2}$ i.e. principal values $=\left(x + \frac{\pi}{4}\right)$ **OR** Let 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$ So 2 divides $[(a - b) + (b - c)]$ 2 divides $(a - c)$: Yes relation R is transitive [0] = {0, ± 2, ± 4, ± 6,...}	1 1 1 1 1
22	$y = ae^{2x} + be^{-x}$……(1) $\frac{dy}{dx} = 2ae^{2x} - be^{-x}$ ……(2) $\frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x}$ ……(3) putting values on LHS $=\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$ $=(4ae^{2x} + be^{-x}) - (2ae^{2x} - be^{-x}) - 2(ae^{2x} + be^{-x})$ $=4ae^{2x} + be^{-x} - 2ae^{2x} + be^{-x} - 2ae^{2x} - 2be^{-x}$ $=0$	1 1

23	$x^2 = 2y$ ……(1) $\Rightarrow 2x\frac{dx}{dt} = 2\frac{dy}{dt}$ (given $\frac{dy}{dt} = \frac{dx}{dt}$) $\Rightarrow 2x\frac{dx}{dt} = 2\frac{dx}{dt}$ $\Rightarrow x = 1$ from (1) $y = \frac{1}{2}$ so point is $\left(1, \frac{1}{2}\right)$	1 1
24	$= (\vec{a} - \vec{b}).\{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$ $=(\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a}\}$ $=(\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\}$ ……$(\vec{c} \times \vec{c} = 0)$ $= (\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\}$ $= \vec{a}.(\vec{b} \times \vec{c}) + \vec{a}.(\vec{a} \times \vec{b}) + \vec{a}.(\vec{c} \times \vec{a}) - \vec{b}.(\vec{b} \times \vec{c}) - \vec{b}.(\vec{a} \times \vec{b}) - \vec{b}.(\vec{c} \times \vec{a})$ $= \vec{a}.(\vec{b} \times \vec{c}) + 0 + 0 - 0 - 0 - \vec{b}.(\vec{c} \times \vec{a})$ $= \vec{a}.(\vec{b} \times \vec{c}) - \vec{b}.(\vec{c} \times \vec{a})$ $=0$ (STP remains same if vectors $\vec{a}, \vec{b}, \vec{c}$ are changed in cyclic order) **OR**	1 1
	$(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$ $\Rightarrow \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c} = 0.$ $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow 3^2 + 5^2 + 7^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -(9 + 25 + 49)$ $\Rightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -\frac{83}{2}$	1 $\frac{1}{2}$ $\frac{1}{2}$
25	Vector in the direction of first line $\vec{b} = (3\hat{i} + 4\hat{j} + 5\hat{k})$ Vector in the direction of second line $\vec{d} = (4\hat{i} - 3\hat{j} + 5\hat{k})$ Angle θ between two lines is given by $\cos\theta = \frac{\vec{b}.\vec{d}}{\lvert\vec{b}\rvert\lvert\vec{d}\rvert}$ $\cos\theta = \frac{(3\hat{i} + 4\hat{j} + 5\hat{k}).(4\hat{i} - 3\hat{j} + 5\hat{k})}{\lvert(3\hat{i} + 4\hat{j} + 5\hat{k})\rvert\lvert(4\hat{i} - 3\hat{j} + 5\hat{k})\rvert}$ $\Rightarrow \cos\theta = \frac{12 - 12 + 25}{\sqrt{9 + 16 + 25}\sqrt{9 + 16 + 25}}$ $\Rightarrow \cos\theta = \frac{25}{\sqrt{50}\sqrt{50}}$ $\Rightarrow \cos\theta = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{3}$	1 $\frac{1}{2}$ $\frac{1}{2}$
26	P(A)=$\frac{80}{100} = \frac{4}{5}$, P(B)=$\frac{90}{100} = \frac{9}{10}$ P(Agree)=P(Both speaking truth or both telling lie) $=P(AB \text{ or } \bar{A}\bar{B})$	1

	$= P(A)P(B) or P(\bar{A})P(\bar{B})$ $=\left(\frac{4}{5}\right)\left(\frac{9}{10}\right)+\left(\frac{1}{5}\right)\left(\frac{1}{10}\right)$ $=\frac{36+1}{50}=\frac{37}{50}$ $=\frac{74}{100}$ = 74%	1
	SECTION C	
27	Let y = f(x) = $\frac{2x+3}{x-3}$ ……(1) Let $x_1, x_2 \in$ A = R – {3} Let $f(x_1) = f(x_2)$ $\Rightarrow \frac{2x_1+3}{x_1-3} = \frac{2x_2+3}{x_2-3}$ $\Rightarrow (2x_1+3)(x_2-3) = (2x_2+3)(x_1-3)$ $\Rightarrow (2x_1x_2-6x_1+3x_2-9) = (2x_1x_2-6x_2+3x_1-9)$ $\Rightarrow -6x_1+3x_2 = -6x_2+3x_1$ $\Rightarrow 9x_1 = 9x_2$ $\Rightarrow x_1 = x_2$ Now $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ so $f(x)$ is one-one For onto $y = \frac{2x+3}{x-3}$ $\Rightarrow xy-3y = 2x+3$ $\Rightarrow xy-2x = 3y+3$ $\Rightarrow x(y-2) = 3(y+1)$ $\Rightarrow x = \frac{3(y+1)}{(y-2)}$ ……(2) equation (2) is defined for all real values of y except 2 i.e $y \in$ R – {2} which is same as given set B = R – {2} (co-domain=range) Also y = f(x) $f(x) = f\left(\frac{3(y+1)}{(y-2)}\right)$ $= \frac{2\left[\frac{3(y+1)}{(y-2)}\right]+3}{\frac{3(y+1)}{(y-2)}-3}\left(\text{since } f(x) = \frac{2x+3}{x-3}\right)$ $\frac{2(3y+3)+3(y-2)}{3y+3-3y+6} = \frac{9y}{9} = y$ Thus for every y $\in$ B,there exists x $\in$ A such that f(x) = y Thus function is onto. Since f(x) is one-one and onto so f(x)is invertible. Inverse is given by $x = f^{-1}(y) = \frac{3(y+1)}{(y-2)}$	$\frac{1}{2}$ 1 $1\frac{1}{2}$ 1
28	$\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$ Let $x = \sin A$, $y = \sin B$ $\sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B} = a(\sin A - \sin B)$ $\cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow \cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right)$	$\frac{1}{2}$ 1

	$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$ $\Rightarrow \frac{A-B}{2} = \cot^{-1} a$ $\Rightarrow A - B = 2\cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ differentiating w.r.t. x $\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ **OR**	1 1 $\frac{1}{2}$
	$x = a(\cos 2\theta + 2\theta \sin 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(-2\sin 2\theta + 2\sin 2\theta + 4\theta\cos 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(4\theta\cos 2\theta)$……..(1) $y = a(\sin 2\theta - 2\theta\cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(2\cos 2\theta + 4\theta\sin 2\theta - 2\cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(4\theta\sin 2\theta)$……(2) using (1)and (2) $\Rightarrow \frac{dy}{dx} = \frac{a(4\theta\sin 2\theta)}{a(4\theta\cos 2\theta)}$ $\Rightarrow \frac{dy}{dx} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$ Differentiating again with respect to x, we get $\Rightarrow \frac{d^2y}{dx^2} = 2\,sec^2 2\theta.\frac{d\theta}{dx}$ $\Rightarrow \frac{d^2y}{dx^2} = 2\,sec^2 2\theta.\frac{1}{a(4\theta\cos 2\theta)}$ $\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{8}} = 2\,sec^2\frac{\pi}{4}.\frac{1}{a\left(4\frac{\pi}{8}\cos\frac{\pi}{4}\right)}$ $= \frac{8\sqrt{2}}{\pi a}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1
29	$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$ $\Rightarrow x\frac{dy}{dx} = y + \sqrt{x^2+y^2}$ $\Rightarrow \frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$……(1) let y = vx differentiating with w.r.t. x $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ put in (1)	1

	$\Rightarrow v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2x^2}}{x}$ $\Rightarrow v + x\frac{dv}{dx} = \frac{x(v + \sqrt{1 + v^2})}{x}$ $\Rightarrow x\frac{dv}{dx} = v + \sqrt{1 + v^2} - v$ $\Rightarrow x\frac{dv}{dx} = \sqrt{1 + v^2}$ $\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ integrating both sides $\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$ $\Rightarrow \log\left(v + \sqrt{1 + v^2}\right) = \log x + \log c$ $\Rightarrow \log\left(v + \sqrt{1 + v^2}\right) = \log cx$ $\Rightarrow \left(v + \sqrt{1 + v^2}\right) = cx$ $\Rightarrow \left(\frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}\right) = cx$ $\Rightarrow y + \sqrt{x^2 + y^2} = cx^2$	1 $1\frac{1}{2}$ $\frac{1}{2}$
30	Consider I=$\int_1^3 \lvert x^2 - 2x\rvert dx$ $\lvert x^2 - 2x\rvert = \begin{cases} -(x^2 - 2x) & when\ 1 \le x < 2 \\ (x^2 - 2x) & when\ \ 2 \le x \le 3 \end{cases}$ I=$\int_1^2 \lvert x^2 - 2x\rvert dx \quad + \int_2^3 \lvert x^2 - 2x\rvert dx$ I=$\int_1^2 -(x^2 - 2x)\ dx + \int_2^3 (x^2 - 2x) dx$ I=$-\left[\frac{x^3}{3} - x^2\right]_1^2 + \left[\frac{x^3}{3} - x^2\right]_2^3$ I=$-\left(-\frac{4}{3} + \frac{2}{3}\right) + \left(\frac{4}{3}\right)$ I=$\frac{6}{3} = 2$	1 1 1 1
31	Let X denotes the smaller of the two numbers obtained So X can take values 1,2,3,4,5,6 P(X=1 is smaller number) P(X=1)=$\frac{6}{{}^7C_2} = \frac{6}{21} = \frac{2}{7}$ (Total cases when two numbers can be selected from first 7 numbers are ${}^7C_2$) P(X=2)=$\frac{5}{{}^7C_2} = \frac{5}{21}$ P(X=3)=$\frac{4}{{}^7C_2} = \frac{4}{21}$ P(X=4)=$\frac{3}{{}^7C_2} = \frac{3}{21} = \frac{1}{7}$ P(X=5)=$\frac{2}{{}^7C_2} = \frac{2}{21}$ P(X=6)=$\frac{1}{{}^7C_2} = \frac{1}{21}$ <table><tr><td>x_i</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>	$\frac{1}{2}$ 2

p_i	$\frac{6}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{1}{21}$
$p_i x_i$	$\frac{6}{21}$	$\frac{10}{21}$	$\frac{12}{21}$	$\frac{12}{21}$	$\frac{10}{21}$	$\frac{6}{21}$

$\frac{1}{2}$

Mean $=\sum p_i x_i = \frac{6}{21} + \frac{10}{21} + \frac{12}{21} + \frac{12}{21} + \frac{10}{21} + \frac{6}{21} = \frac{56}{21} = \frac{8}{3}$ 1

OR

Let E_1 = event of selecting a two headed coin

E_1 = event of selecting a biased coin, which shows 75% times Head

E_3 =event of selecting a unbiased coin.

A = event that tossed coin shows head. 1

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

. $P\left(A/E_1\right)$ = P(coin showing head given that it is two headed coin) $\frac{1}{2}$

$$= 1$$

$P\left(A/E_2\right)$ = P(coin showing head given that it is a biased coin)

$$= \frac{75}{100} = \frac{3}{4}$$

$P\left(A/E_3\right)$ = P(coin showing head given that it is unbiased coin) 1

$$= \frac{1}{2}$$

By Bayes theorem

P(gettingtwo headedcoin when it is known that it shows Head)

$$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_1)P\left(A/E_3\right)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)} = \frac{\frac{1}{3}}{\frac{1}{3}\times\frac{9}{4}} = \frac{4}{9}$$ $1\frac{1}{2}$

Required probability$=\frac{4}{9}$

32 Let tailor A works for x days and tailor B works for y days $\frac{1}{2}$

Objective function :

To minimize labour cost $Z = 150x + 200y$ (in ₹)

Subject to constraints

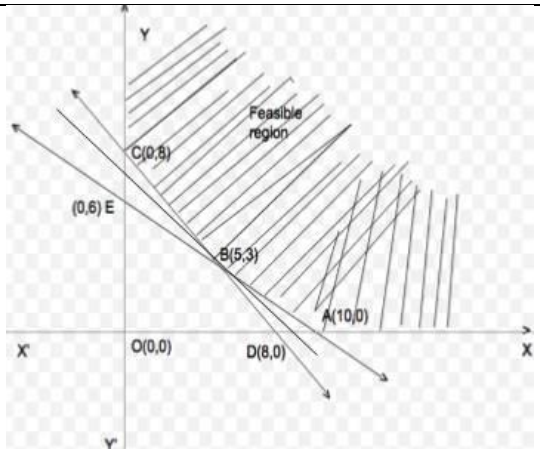
$6x + 10y \geq 60$ i.e. $3x + 5y \geq 30$

$4x + 4y \geq 32$ i.e. $x + y \geq 8$ $1\frac{1}{2}$

$x \geq 0, y \geq 0$

consider equations to draw the graph and then we will shade feasible region

$$3x + 5y = 30$$
$$x + y = 8$$



corner points of feasible region are A(10,0),B(5,3) and C(0,8) 1

Value of Z at these corner points

Point	$Z = 150x + 200y$ (in ₹)
A(10,0)	=1500+0=1500
B(5,3)	=750+600=1350 (minimum)
C(0,8)	=0+1600=1600

So minimum value of Z is ₹1350 when tailor A works for 5 days and tailor B works for 3 days.

To check draw $150x + 200y < 1350$ i.e $3x + 4y < 27$

As there is no region common with feasible region so minimum value is ₹1350 1

SECTION D

33

$$\text{LHS}=\begin{vmatrix}(y+z)^2 & x^2 & x^2\\ y^2 & (z+x)^2 & y^2\\ z^2 & z^2 & (x+y)^2\end{vmatrix}$$

$Apply\ C_2 \to C_2 - C_1,\quad C_3 \to C_3 - C_1$

$$=\begin{vmatrix}(y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2\\ y^2 & (z+x)^2-y^2 & 0\\ z^2 & 0 & (x+y)^2-z^2\end{vmatrix}$$ 1

$$=\begin{vmatrix}(y+z)^2 & (x+y+z)(x-y-z) & (x+y+z)(x-y-z)\\ y^2 & (z+x+y)(z+x-y) & 0\\ z^2 & 0 & (x+y+z)(x+y-z)\end{vmatrix}$$ 1

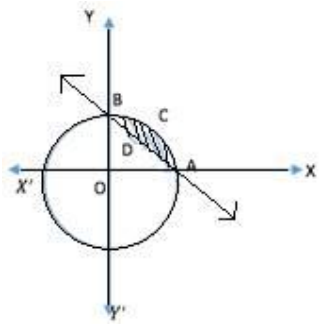
Taking $(x+y+z)$common from C_2as well as C_3

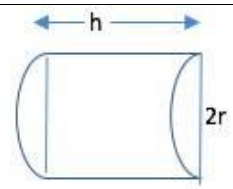
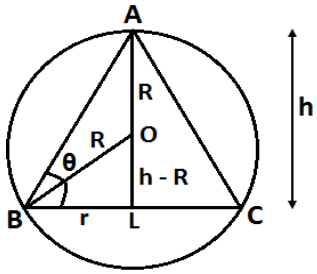
$$=(x+y+z)^2\begin{vmatrix}(y+z)^2 & (x-y-z) & (x-y-z)\\ y^2 & (z+x-y) & 0\\ z^2 & 0 & (x+y-z)\end{vmatrix}$$ 1

Apply $R_1 \to R_1 - R_2 - R_3$

$$=(x+y+z)^2\begin{vmatrix}2yz & -2z & -2y\\ y^2 & (z+x-y) & 0\\ z^2 & 0 & (x+y-z)\end{vmatrix}$$

	Apply $C_2 \to y\,C_2$ and $C_3 \to z\,C_3$ $=\frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & -2yz & -2yz \\ y^2 & (yz+yx-y^2) & 0 \\ z^2 & 0 & (zx+zy-z^2) \end{vmatrix}$ Apply $C_2 \to C_2+C_1$ and $C_3 \to C_3+C_1$ $=\frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & 0 & 0 \\ y^2 & (yz+yx) & y^2 \\ z^2 & z^2 & (zx+zy) \end{vmatrix}$ expanding along R_1 $=\left(\frac{(x+y+z)^2}{yz}\right)2yz[(yz+yx)(zx+zy)-y^2z^2]$ $=2(x+y+z)^2[xyz^2+x^2yz+xy^2z+y^2z^2-y^2z^2]$ $= 2xyz(x+y+z)^2(x+y+z)$ $= 2xyz(x+y+z)^3$ **OR**	1 1 1
	** A = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ $\lvert A\rvert = 2(-2)-3(2-0)+4(1-0) = -6 \neq 0$ $\therefore A^{-1}$exists Cofactors $A_{11} = -2 \quad A_{12} = -2 \quad A_{13} = 1$ $A_{21} - 2 \quad A_{22} = 4 \quad A_{23} = -2$ $A_{31} = 4 \quad A_{32} = 4 \quad A_{33} = -5$ $Adj\ A = \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}'$ $Adj\ A = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ $A^{-1} = \frac{Adj\ A}{\lvert A\rvert} = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ System of equations can be written as $AX = B$ Where A = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$ Now $AX = B$ $\Rightarrow X = A^{-1}B$ $\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$	1 2 1 $\frac{1}{2}$

	$\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -34-6+28 \\ -34+12+28 \\ 17-6-35 \end{bmatrix}$ $\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$ $\Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow x = 2, \quad y = -1, \quad z = 4$	$1\frac{1}{2}$
34	$x^2 + y^2 = 1$……..(1) $x + y = 1$……..(2) solving (1) and(2) $x^2 + (1-x)^2 = 1$ $x^2 + x^2 - 2x + 1 = 1$ $2x^2 - 2x = 0$ $2x(x-1) = 0$ $x = 0 \; or \; x = 1$  Required area = shaded area ACBDA =area(OACBO)– area(OADBO) $=\int_0^1 (y_{circle} - y_{line})dx$ $\int_0^1 \sqrt{1-x^2}dx - \int_0^1 (1-x)dx$ $=\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$ $\left[\left(0 + \frac{1}{2}.\frac{\pi}{2}\right) - 0\right] - \left[\left(1 - \frac{1}{2}\right)\right]$ $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ square units	1 1 1 $1\frac{1}{2}$ $1\frac{1}{2}$
35	Let r be the radius and $h$be the height of half cylinder Volume $=\frac{1}{2}\pi r^2 h = V$(constant)……..(1)	$\frac{1}{2}(fig)$

	 Total surface area of half cylinder is S=$2\left(\frac{1}{2}\pi r^2\right) + \pi rh + 2rh$……(2) From (1) put the value of h in (2) S=$(\pi r^2) + \pi r\left(\frac{2V}{\pi r^2}\right) + 2r\left(\frac{2V}{\pi r^2}\right)$ S=$(\pi r^2) + \left(\frac{1}{r}\right)\left[\frac{4V}{\pi} + 2V\right]$ $\frac{ds}{dr} = (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right]$ ……..(3) For maxima/minima $\frac{ds}{dr} = 0$ $\Rightarrow (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right] = 0$ $\Rightarrow (2\pi r) = \left(\frac{1}{r^2}\right)\left[\frac{4V + 2V\pi}{\pi}\right]$ $\Rightarrow \pi r^3 = V\left[\frac{2+\pi}{\pi}\right]$ $\Rightarrow$ V=$\frac{\pi^2 r^3}{\pi+2}$……..(4) From (1) and (4) $\Rightarrow \frac{1}{2}\pi r^2 h = \frac{\pi^2 r^3}{\pi + 2}$ $\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi + 2}$ $\Rightarrow height{:}\,diameter = \pi{:}\,\pi + 2$ Differentiating (3) with respect to r $\frac{d^2s}{dr^2} = (2\pi) + \left(\frac{2}{r^3}\right)\left[\frac{4V}{\pi} + 2V\right]$=positive (as all quantities are +ve) so S is minimum when $height{:}\,diameter = \pi{:}\,\pi + 2$ OR	$1\frac{1}{2}$ 1 1 1
	Let 2r be the base and h be the height of triangle ,which is inscribed in a circle of radius R Area of triangle=$\frac{1}{2}(base)(height)$ A $=\frac{1}{2}(2r)(h) = rh$……(1)  Area being positive quantity, A will be maximum or minimum if A^2 is	1 $\frac{1}{2}(fig)$

	maximum or minimum. $Z = A^2 = r^2h^2$……..(2) Now In triangle OLB $BL^2 = OB^2 - OL^2$ In ΔOBD $Z = A^2 = r^2 h^2$ $r^2 = R^2 - (h-R)^2 \Rightarrow r^2 = 2hR - h^2$ $put\ in$ (2) $Z = h^2(2hR - h^2)$ $\Rightarrow Z = (2h^3R - h^4)$ $\Rightarrow \frac{dZ}{dh} = 6h^2R - 4h^3$……(3) For maxima/minima $\frac{dZ}{dh} = 0$ $\Rightarrow 6h^2R - 4h^3 = 0$ $\Rightarrow 6R = 4h(h \neq 0)$ $\Rightarrow h = \frac{3R}{2}$ differentiating (3) w.r.t. h $\Rightarrow \frac{d^2Z}{dh^2} = 12hR - 12h^2$ $\Rightarrow \left.\frac{d^2Z}{dh^2}\right]_{h=\frac{3R}{2}} = 12\left(\frac{3R}{2}\right)R - 12\left(\frac{3R}{2}\right)^2$ $= 18R^2 - 27R^2 = -ve$ so Z=A^2 is maximum when $h = \frac{3R}{2}$ $\Rightarrow A$ is maximum when $h = \frac{3R}{2}$ when $h = \frac{3R}{2}$, $r^2 = 2hR - h^2 = 2R.\frac{3R}{2} - \left(\frac{3R}{2}\right)^2$ $r^2 = \frac{3R^2}{4}$ $r = \frac{\sqrt{3}R}{2}$ $\tan\theta = \frac{h}{r} = \frac{\frac{3R}{2}}{\frac{\sqrt{3}R}{2}} = \sqrt{3}\theta = \frac{\pi}{3}$ Triangle ABC is equilateral triangle	$\frac{1}{2}$ 1 1 1 1
36	Let $P(x,y,z)$ be any point on the plane in which $A(2,1,2)$ and $B(4,-2,1)$lie. $\therefore \overrightarrow{AP}\ and\ \overrightarrow{AB}$lie on required plane. Also required plane is perpendicular to given plane $\vec{r}.(\hat{i} - 2\hat{k}) = 5$ ∴normal to given plane $\overrightarrow{n_1} = (\hat{i} - 2\hat{k})$ lie on required plane. $\Rightarrow \overrightarrow{AP}, \overrightarrow{AB}and\overrightarrow{n_1}$ are coplanar. Where$\overrightarrow{AP} = (x-2)\hat{i} + (y-1)\hat{j} + (z-2)\hat{k}$ $\overrightarrow{AB} = =2\hat{i} - 3\hat{j} - \hat{k}$ ⇒Scaler triple product $[\overrightarrow{AP}\quad \overrightarrow{AB}\quad \overrightarrow{n_1}] = 0$	1 1 1

	$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-2 \\ 2 & -3 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 0$ $\Rightarrow (x-2)(6-0)-(y-1)(-4+1)+(z-2)(0+3)=0$ $\Rightarrow 6x-12+3y-3+3z-6=0$ $\Rightarrow 2x+y+z=7$……(1) Line passing through points $L(3,4,1)$and $M(5,1,6)$ is $\Rightarrow \frac{x-3}{2}=\frac{y-4}{-3}=\frac{z-1}{5}=\lambda$……(2) $\Rightarrow$General point on the line is $Q(2\lambda+3,-3\lambda+4,5\lambda+1)$ As line (2) crosses plane (1) so point Q should satisfy equation(1) $\therefore 2(2\lambda+3)+(-3\lambda+4)+(5\lambda+1)=7$ $4\lambda+6-3\lambda+4+5\lambda+1=7$ $6\lambda=-4$ $\lambda=-\frac{2}{3}$ $Q(-\frac{4}{3}+3,2+4,-\frac{10}{3}+1)=Q\left(\frac{5}{3},6,-\frac{7}{3}\right)$	1 1 1

CLASS XII

MATHEMATICS -041
PRACTICE PAPER-I (2019-20)

Time: 3 Hrs. Maximum Marks: 80
निर्धारित समय : 3 घंटे अधिकतम अंक : 80

General Instructions:

(i) All the questions are compulsory.
(ii) The question paper consists of 36 questions divided into 4 sections A, B,C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprisesof 6 questions of 2 marks each. Section C comprises of 6 questions of 4marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been providedin three questions of 1 mark each, two questions of 2 marks each, twoquestions of 4 marks each, and two questions of 6 marks each. You haveto attempt only one of the alternatives in all such questions.
(v)Use of calculators is not permitted

सामान्य निर्देश:

1.सभी प्रश्न अनिवार्य हैं

2. प्रश्न पत्र में 36 प्रश्न हैं जो चार खंडों में विभक्त है ।

3. खंड अ में 20 प्रश्न है प्रत्येक प्रश्न 1 अंक का है ।खंड ब में6 प्रश्न हैं प्रत्येक प्रश्न 2 अंक का है ।खंड समें 6 प्रश्न है प्रत्येक प्रश्न 4 अंक का है ।खंड द में 4 प्रश्न है प्रत्येक प्रश्न 6 अंक का है ।

4. प्रश्न पत्र में समग्र पर कोई विकल्प नहीं है तथापि 1 अंक के 3 प्रश्न में , 2 अंक के 2 प्रश्नों में , 4 अंक के 2 प्रश्नों में और 6 अंक के 2 प्रश्नों में विकल्पदिया गया है । ऐसे सभी प्रश्नों में एक ही विकल्प हल करना है ।

5.केलकुलेटर के प्रयोग की अनुमति नहीं है ।

SECTION A/ खण्ड अ

Question Number 1-10 Are Of Multiple Choice Type Questions.Select the Correct Option

(1) The matrix A = $\begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a

(A) scalar matrix (B) diagonal matrix (C) unit matrix (D) square matrix

आव्यूहA = $\begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$है

(A) आदिश आव्यूह (B) विकर्ण आव्यूह (C) तत्समक आव्यूह (D)वर्ग आव्यूह

(2) If A and B are square matrices of the same order then (A+B)(A-B) is equal to
यदि A और B सामान कोटि के दो आव्यूह हैं तो (A+B)(A-B) बराबर हैं

(A) $A^2 - B^2$ (B) $A^2 - BA - AB - B^2$
(C) $A^2 - B^2 + BA - AB$ (D) $A^2 - BA + B^2 + AB$

(3) The value of λ for which two vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are perpendicular is
λ का वह मान जिसके लिए सदिश $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ और सदिश $3\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ लम्बवत हैं

(A) 2 (B) 4 (C) 6 (D)8

(4) You are given that A and B are two events such that P(B)=3/5 P(A/B)=1/2 AND P(AUB)=4/5 then P(A) equals to:

आपको ऐसी दो घटनाएंA तथा Bदी हुई हैकि P(B)=3/5 ,P(A/B)=1/2 और P(AUB)=4/5 तो P(A) बराबर है।

(A) 3/10 (B)1/5 (C) 1/2 (D) 3/5

(5) Reflection of the point (α,β,γ) in the xy plane is

xy सम्तल में बिंदू (α,β,γ) का परावर्तन है

(A)(α,β,0) (B)(0,0,γ) (C) $|(-\alpha, -\beta, \gamma)|$ (D) (α,β,-γ)

(6)If cos $(\sin^{-1}\frac{2}{5}\cos^{-1}x)$ =0, then x equals to

यदि cos $(\sin^{-1}\frac{2}{5}\cos^{-1}x)$=0, तो x बराबर है

(A) $\frac{1}{5}$ (B)$\frac{2}{5}$ (C) 0 (D) 1

(7) In a box containing 100 bulbs , 10 are defective .The probability that out of a sample of 5 bulbs , none is defective, is

एक बक्से में 100 बल्ब हैं जिसमें से 10 खराब हैं 5 बल्बों के नमूने में से कोई बल्ब खराब ना होने की प्रायिकता होगी ।

(A) $(9/10)^5$ (B)$\frac{9}{10}$ (C) 1/10 (D) None of these

(8)$\int \frac{dx}{\sin^2 x \cos^2 x}$equals to

$\int \frac{dx}{\sin^2 x \cos^2 x}$बराबर है

(A) $\tan x + \cot x + c$ (B) $(\tan x + \cot x)^2 + c$ (C) $\tan x - \cot x + c$ (D) $(\tan x - \cot x)^2 + c$

(9) The coordinates of the foot of perpendicular drawn from the point (2,5,7) on the x axis are given by

बिंदु (2,5,7) से x अक्ष पर डाले गए लंब पाद के निर्देशक है

(A) (2,0,0) (B) (0,5,0) (C) (0,0,7) (D) (0,5,7)

(10) Distance in (units) between two planes 2x+3y+4z=4 and 4x+6y+8z=12 is:

(A)2 unit (B) 4 units (C)8units (D) $\frac{2}{\sqrt{29}}$ units

दो सम्तलो 2x+3y+4z=4 तथा 4x+6y+8z=12 के बीच की दूरी है:

(A)2 इकाई (B) 4 इकाई (C) इकाई (D)$\frac{2}{\sqrt{29}}$ इकाई

(Q 11-Q15) Fill in the blanks

(रिक्त स्थान भरिये)

(11) Let the relation R be defined in N by aRb if 2a+3b=30. Then R=----------

मान लीजिए कि N में एक संबंध R, aRb यदि 2a+3b=30 द्वारा परिभाषित है तो R =----------

(12)If f(x)=$\begin{cases} ax+1 & if x \geq 1 \\ x+2 & if x < 1 \end{cases}$ is continuous then 'a' should be equal to ----------

यदिf(x)=$\begin{cases} ax+1 & if x \geq 1 \\ x+2 & if x < 1 \end{cases}$संतत है तो'a'---------- के बराबर मान होना चाहिए ।

(13)The equation of normal to the curve y=tanx at (0,0) is----------

वक्र y=tanx के (0,0) पर अभिलम्ब का समीकरन ----- है

OR

The value of 'a' for which function f(x)=sinx-ax+b increases on R are_____

A के वे मान के लिये फलन f(x)=sinx-ax+b, R में वर्धमान है ----------हैं

(14)If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \end{bmatrix}$, then value of y is ----------

यदि $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ हो तो y का मान ----------है

(15)Projection vectors of $\vec{a}$ on$\vec{b}$ is ----------

सदिश $\vec{a}$का $\vec{b}$ पर प्रक्षेप----------है

OR

$Direction\ cosines\ of\ the$ vector $(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$ are_____

सदिश$(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$ की दिक्कोज्याए---------- हैं

(Q16-Q20) Answer the following questions

(Q16-Q20) निम्न प्रश्नो के उत्तर लिखिये

(16) If $\Delta = \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix}$, then show that Δ is equals to zero

यदि $\Delta = \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix}$, तो दिखाइए कि Δ=0 है,

(17) Evaluate

$$\int \log x dx$$

$\int \log x dx$ ज्ञात कीजिए।

(18)

Evaluate $\int_{-2}^{2}(x^3+1)dx$

$\int_{-2}^{2}(x^3+1)dx$ ज्ञात कीजिए।

OR

Evaluate

$\int \frac{x+\sin x}{1+\cos x}$dx

$\int \frac{x+\sin}{1+\text{co}}$ ज्ञात कीजिए।

(19) Evaluate

$$\int e^x(\cos x - \sin x)dx$$

$\int e^x(\cos x - \sin x)dx$ ज्ञात कीजिए।

(20)Find the general solution of differential equation

$\frac{dy}{dx}+\frac{y}{x}=$

अवकल समीकरण

$\frac{dy}{dx}+\frac{y}{x}=1$का व्यापक हल ज्ञात कीजिये ।

SECTION B

Question 21-26 each question carries 2 marks

प्रश्न संख्या 21-26 प्रत्येक प्रश्न दो अंक का है|

(21) Find the unit vector in the direction of the sum of vectors

$\vec{a}=2\hat{\imath} -\hat{\jmath} +2\hat{k}$ and $\vec{b}= -\hat{\imath} +\hat{\jmath} +3\hat{k}$

सदिशों $\vec{a}=2\hat{\imath} -\hat{\jmath} +2\hat{k}$तथा $\vec{b}= -\hat{\imath} +\hat{\jmath} +3\hat{k}$ के योग के अनुदिश मात्रक सदिश ज्ञात कीजिये ।

OR

Find the vector joining the points P(2,3,0) and Q(-1,-2,-4) directed from P to Q

बिन्दु P(2,3,0) तथा Q(-1,-2,-4) को मिलाने वाले सदिश ज्ञात कीजिये जो P से Q की तरफ अनुदिशित हैं ।

(22) show by examples that the relation R in R, defined by R=$\{(a,b): a \leq b^3\}$ is neither reflexive nor transitive.

उदाहरणों द्वारा दर्शाइए कि R में R=$\{(a,b): a \leq b^3\}$द्वारा परिभाषित संबंध R न तो सवतुल्य है न ही संक्रामक है।

OR

Evaluate $cos\left[\sin^{-1}\frac{1}{4}+\sec^{-1}\frac{4}{3}\right]$

$cos\left[\sin^{-1}\frac{1}{4}+\sec^{-1}\frac{4}{3}\right]$ज्ञात कीजिए।

(23) If y=$e^{a\cos^{-1}x}$,show that (1-x^2)$\frac{d^2y}{dx^2}$ -x$\frac{dy}{dx}$-a^2y=0

यदि y=$e^{a\cos^{-1}x}$ है , तो दर्शाइए कि (1-x^2)$\frac{d^2y}{dx^2}$ -x$\frac{dy}{dx}$-a^2y=0

(24) A balloon which always remains spherical has a variable diameter $\frac{3}{2}$(2x+1). Then, find the rate of change of its volume with respect to x.

एक गुब्बारा , जो सदैव गोलाकार रहता है, का परिवर्तनशील व्यास $\frac{3}{2}$(2x+1) है | x के सापेक्ष आयतन के परिवर्तन कि दर ज्ञात कीजिये |

(25)Find the angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$

रेखाओं $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ तथा $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ के बीच का कोण ज्ञात कीजिये |

(26) A die is thrown 6 times .If getting an odd number is a success. Then what is the probability of 5 successes?

एक पासे को 6 बार उछाला जाता है | यदि' पासे पर विषम संख्या प्राप्त होना' एक सफलता है तो 5 सफलताओं कि प्रायिकता ज्ञात कीजिये |

SECTION-C

(27) Prove that the relation R in the set A= {1, 2, 3, 4, 5, 6, 7} given by R= {(a,b): |a-b| is even} is an equivalence relation.

सिद्ध कीजिये कि समुच्चय A= {1,2,3,4,5,6,7} में R= {(a,b): |a-b| सम है} द्वारा प्रदत संबंध R एक तुल्यता संबंध है |

(28) Solve the differential equation :

$\frac{dy}{dx}=\frac{x+y}{x-y}$

OR

Solve the differential equation:

(1+x^2)dy+2xy dx+ cotx dx

अवकल समीकरण $\frac{dy}{dx}=\frac{x+y}{x-y}$ को हल कीजिये |

अथवा

अवकल समीकरण(1+x^2)dy+2xy dx+ cot x dx हल कीजिये|

(29) If x$\sqrt{1+y}$ +y$\sqrt{1+x}$ =0 and x≠y, prove that $\frac{dy}{dx}=\frac{-1}{(x+1)^2}$

यदि x$\sqrt{1+y}$ +y$\sqrt{1+x}$ =0 और x≠y है, तो सिद्ध कीजिये कि $\frac{dy}{dx}=\frac{-1}{(x+1)^2}$

OR

If $\cos x)^y=(\sin y)^x$, find $\frac{dy}{dx}$.

यदि $\cos x)^y=(\sin y)^x$ है, तो $\frac{dy}{dx}$ज्ञात कीजिये |

(30) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and hence evaluate

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$$

सिद्ध कीजिये कि $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

अतः

$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$ का मूल्यांकन कीजिये |

(31) Two biased dice are thrown together. For the first die P (6)=1/2 , the other scores being equally likely while for the second die, P (1) =2/5 and the other scores are equally likely.
Find the probability distribution of 'the number of ones seen'.

दो अभिनत पासे एक साथ फेंके जाते है| पहले पासे के लिए P (6)=1/2 अन्य स्कोर सम संभाव्य है ; जबकि दुसरे पासे के लिए P (1) =2/5 तथा अन्य स्कोर सम संभाव्य है "1 के प्रकट होने कि संख्या" का प्रायिकता बंटन ज्ञात कीजिये |

(32) In a mid-day meal programme, an NGO wants to provide vitamin rich diet to the students of an MCD school. The dietician of the NGO wishes to mix two types of food in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs Rs.50/- per kg to purchase Food 1 and Rs.70/- per kg to purchase Food 2. Formulate the problem as LPP and solve it graphically for the minimum cost of such a mixture.

एक मिड-डे मील कार्यक्रम में, एक एनजीओ एक एमसीडी स्कूल के छात्रों को विटामिन से भरपूर आहार देना चाहता है। एनजीओ के आहार विशेषज्ञ दो प्रकार के भोजन को इस तरह मिलाना चाहते हैं कि मिश्रण की विटामिन सामग्री में विटामिन ए की कम से कम 8 इकाइयाँ और विटामिन सी की 10 इकाइयाँ हों। भोजन 1 में 2 यूनिट प्रति किग्रा विटामिन ए और 1 इकाई शामिल हैं। विटामिन सी प्रति किग्रा। खाद्य 2 में 1 यूनिट प्रति किग्रा विटामिन ए और 2 यूनिट प्रति किग्रा विटामिन सी होता है। भोजन की खरीद के लिए रु50 / - प्रति किग्रा है। खाद्य 2 की खरीद के लिए 1 और रु 70 प्रति किग्रा। एलपीपी के रूप में समस्या को हल करें और ऐसे मिश्रण की न्यूनतम लागत के लिए इसे ग्राफिक रूप से हल करें।

SECTION-D

(33) Using integration, find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

समाकलन के प्रयोग से, दीर्घवृत्त $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ का क्षेत्रफल ज्ञात कीजिये|

OR/ अथवा

Evaluate integral of $\int_1^3 (x^2+x+e^x)dx$ as the limit of a sum.

योगफल कि सीमा विधि द्वारा $\int_1^3 (x^2+x+e^x)dx$ का मान ज्ञात कीजिये|

(34) Show that for the matrix A=$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, $A^3-6A^2+5A+11I=0$

Hence find A^{-1}.

आव्यूह A=$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ के लिए दर्शाइए कि $A^3 - 6A^2 + 5A + 11I = 0$ है| इसकी सहायता से A^{-1} ज्ञात कीजिये|

(35)

Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these lines.

OR

Find the coordinates of the foot of the perpendicular Q drawn from P(3, 2, 1) to the plane $2x - y + z + 1 = 0$.

Also, find the distance PQ and the image of the point P treating this plane as a mirror.

बिंदु (2,3,2) से होकर जाने वाली सदिश समीकरण ज्ञात कीजिये जो $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ रेखा के समान्तर है| इन दोनों रेखाओ के बीच कि दूरी भी ज्ञात कीजिये|

अथवा

बिंदु P (2,3,2) से समतल 2x-y+z+1=0 पर खींचे गए लम्ब के पाद Q के निर्देशांक ज्ञात कीजिये | लम्बवत दूरी PQ तथा समतल को दर्पण लेते हुए इस बिंदु P का प्रतिभिम्ब भी ज्ञात कीजिये|

(36) An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

त्रिज्या a वाले वृत्त के अन्दर समद्विबाहु त्रिभुज बना है जिसका शीर्ष कोण 2θ दर्शाइए कि त्रिभुज का क्षेत्रफल अधिकतम होगा जब $\theta = \frac{\pi}{6}$ होगा|

SOLUTION/ ANSWER KEY OF PRACTICE PAPER -I

CLASS XII MATHEMATICS

2019-20

Q NO	VALUE POINTS
1	(D) square matrix
2	(C) $A^2 - B^2 + BA - AB$
3	(D)8
4	(C) 1/2
5	(D) (α,β,-γ)
6	(B) $\frac{2}{5}$
7	(B) 9/10
8	(C)$\tan x - \cot x + c$
9	(A) (2,0,0)
10	(D)$\frac{2}{\sqrt{29}}$units
11	R= {(3,8),(6,6).(9,4),(12,2)}
12	a=2
13	X+y=0 OR $(-\infty, -1)$
14	y=2
15	$\left(\frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$ OR 2/3,2/3,-1/3
16	-Interchanging rows and column we get $\Delta = \begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ Taking (-1) common from R_1, R_2, R_3 we get $\Delta = (-1)^3 \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix} = -\Delta$ therefore $2\Delta = 0 \therefore \Delta = 0$
17	$x\log x - x + c$
18	4 OR

	$x\tan\frac{x}{2}+c$
19	$e^x \cos x + c$
20	$yx=\frac{x^2}{2}+c$
21	Let $\vec{c}$ denote the sum of $\vec{a}$&$\vec{b}$ we have $\vec{c}=(2\hat{\imath}-\hat{\jmath}+2\hat{k})+(-\hat{\imath}+\hat{\jmath}+3\hat{k})=\hat{\imath}+5\hat{k}$ now $\lvert\hat{\vec{c}}\rvert=\sqrt{1^2+2^2}=\sqrt{26}$ Required unit vector is $\hat{c}=\frac{\vec{c}}{\lvert\vec{c}\rvert}=\frac{1}{\sqrt{26}}(\hat{\imath}+5\hat{k})=\frac{1}{\sqrt{26}}\hat{\imath}+\frac{5}{26}\hat{k}$ OR P(2,3,0) and Q(-1,-2,-4) $\overrightarrow{PQ}=(-1,-2)\hat{\imath}+(-2,-3)\hat{\jmath}+(-4,-0)\hat{k}$ $=-3\hat{\imath}-5\hat{\jmath}-4\hat{k}$ ∴ Vector joining P and Q given by $\overrightarrow{PQ}=-3\hat{\imath}-5\hat{\jmath}-4\hat{k}$
22	a=1/2 not reflexive 1/2≤1/2 so R is not Reflexive A=9, b=4, c=2 , not transitive OR $cos\left[\sin^{-1}\frac{1}{4}+\sec^{-1}\frac{4}{3}\right]=cos\left[\sin^{-1}\frac{1}{4}+\cos^{-1}\frac{4}{3}\right]$ $=\cos(\sin^{-1}\frac{1}{4})\cos(\cos^{-1}\frac{3}{4})-\sin\sin^{-1}\frac{1}{4}\sin\cos^{-1}\frac{3}{4}$ $=\frac{3}{4}\sqrt{1-(\frac{1}{4})^2}-\frac{1}{4}\sqrt{1-(\frac{3}{4})^2}$ $=\frac{3\sqrt{15}-\sqrt{7}}{16}$
23	$y=e^{a\cos^{-1}x}\Rightarrow\frac{dy}{dx}=e^{a\cos^{-1}x}(-a)\frac{-a}{\sqrt{1-x^2}}$ Therefore ,$\sqrt{1-x^2}\frac{dy}{dx}=-ay$------(1) Differentiating again w.r.t x , we get $\sqrt{1-x^2}\frac{d^2y}{dx^2}-\frac{-x}{\sqrt{1-x^2}}\frac{dy}{dx}=-a\frac{dy}{dx}$ $\Rightarrow(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-a\sqrt{1-x^2}\frac{dy}{dx}$ $=-a(-ay)$ hence $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$
24	Given, diameter of the balloon= $\frac{3}{2}(2x+1)$ ∴Radius of the balloon= $\frac{Diameter}{2}$ $=\frac{1}{2}[\frac{3}{2}(2x+1)]=\frac{3}{4}(2x+1)$ For the volume V, the balloon is given by $V=\frac{4}{3}\pi(\text{radius})^3=\frac{4}{3}\pi[\frac{3}{4}(2x+1)]^3=\frac{9\pi}{16}(2x+1)^3$ For the rate of change of volume, differentiate w.r.t x, we get

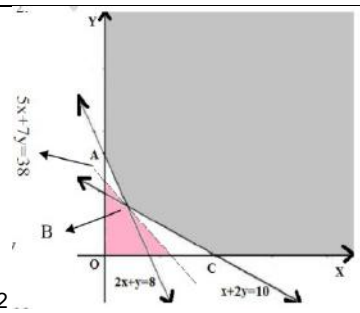
	$\frac{dV}{dx}=\frac{9\pi}{16}\times 3(2x+1)^2\times 2=\frac{27\pi}{8}(2x+1)^2$ Thus, the rate of change of volume is $\frac{27\pi}{8}(2x+1)^2$.
25	Given equations of lines are $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ here direction ratios of two lines are (2,2,1) and (4,1,8) Let θ be the acute angle between the given lines, then $\cos\theta=\frac{\lvert a_1a_2+b_1b_2+c_1c_2\rvert}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$ $\cos\theta=\frac{\lvert 2\times4+2\times1+1\times8\rvert}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$ $=\frac{\lvert 8+2+8\rvert}{\sqrt{4+4+1}\sqrt{16+1+64}}$ $=\frac{18}{\sqrt{9}\sqrt{81}}$ $=\frac{18}{3\times9}=\frac{2}{3}\Rightarrow\theta=cos^{-1}\left(\frac{2}{3}\right)$
26	Given n=6 and p= $\frac{Number\ of\ odd\ number\ in\ one\ die}{Total\ number\ in\ one\ die}=\frac{3}{6}=\frac{1}{2}$ $\therefore q=1-p=1-\frac{1}{2}$ So . P(getting a 5 success in six trials)= P(X=5)=$6c_5p^5q^1=1x(\frac{1}{2})^5(\frac{1}{2})^1=1/64$
27	(i) Reflexive: ∀ a ∈ A, |a-a|= 0 which is even ⇒ (a,a) ∈ R, hence R is reflexive. (ii) Symmetric: Let (a,b) ∈ R ⇒|a-b| is even ⇒ |-(b-a)| is even ⇒|(b-a)| is even So, (b,a) ∈ R Hence, R is symmetric. (iii) Transitive: Let (a,b), (b,c) ∈ R So, |a-b| is even and |b-c| is even ⇒a-b=2λ, b-c=2μ where λ, μ ∈ Z Now, a-c= (a-b)+(b-c)=2(λ+μ) ⇒ |a-c| is even, hence R is transitive. Since R is reflexive, symmetric, transitive Therefore, it is an equivalence relation.

28	$\frac{dy}{dx}=\frac{x+y}{x-y}=\frac{1+\frac{y}{x}}{1-\frac{y}{x}}$ Put y/x = v so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore\ v+x\frac{dv}{dx}=\frac{1+v}{1-v}\ \Rightarrow\ x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v+v^2}{1-v}$ $\Rightarrow \int\frac{1-v}{1+v^2}dv=\int\frac{dx}{x}\ \Rightarrow \int\frac{1}{1+v^2}dv-\frac{1}{2}\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}$ $\Rightarrow\ \tan^{-1}v=\frac{1}{2}\log\lvert 1+v^2\rvert+\log\lvert x\rvert+c$ $\Rightarrow\ \tan^{-1}\left(\frac{y}{x}\right)=\frac{1}{2}\log\left\lvert\frac{x^2+y^2}{x^2}\right\rvert+\log\lvert x\rvert+c$ or $\tan^{-1}\left(\frac{y}{x}\right)=\frac{1}{2}\log\lvert x^2+y^2\rvert+c$ OR $(1+x^2)dy+2xy\,dx=\cot x.dx.$ $\Rightarrow\ \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{\cot x}{1+x^2}$ I.F. = $e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=(1+x^2)$ $\therefore$ Solution is, $y\cdot(1+x^2)=\int\cot x\,dx=\log\lvert\sin x\rvert+c$ or $y=\frac{1}{1+x^2}\cdot\log\lvert\sin x\rvert+\frac{c}{1+x^2}$
29	$\Rightarrow\ \frac{dy}{dx}\cdot\log(\cos x)+y(-\tan x)=\log(\sin y)+x\cdot\cot y\cdot\frac{dy}{dx}$ $\Rightarrow\ \frac{dy}{dx}=\frac{y\cdot\tan x+\log(\sin y)}{\log(\cos x)-x\cot y}$

30	RHS $= \int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt$, where $a+b-x=t$, $dx=-dt$ $= \int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \text{LHS}$ Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ...(i) $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ...(ii) adding (i) and (ii) to get $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\cdot dx = x\Big]_{\pi/6}^{\pi/3} = \pi/6.$ $\Rightarrow I = \frac{\pi}{12}$
31	For the first die : P(6)=1/2 , P(6′)=1/2 i.e., P(6′)=P(1)+P(2)+P(3)+P(4)+P(5)=1/2 ⇒ P(1)=1/10, P(1′)=9/10 [$\therefore P(1)=P(2)=P(3)=P(4)=P(5)$] For the second die: P(1)=2/5 , P(1′)=3/5 Let X: number of ones seen $\therefore X = 0,1,2$ P(X=0)=P(not 1 from 1st die).P(not 1 from 2nd die)= $\frac{9}{10}\times\frac{3}{5}=\frac{27}{50}=0.54$ P(X=1)= P(1 from 1st die) P(not 1 from 2nd die)+ P(not 1 from 1st die) P(1 from 2nd die) $=\frac{1}{10}\times\frac{3}{5}+\frac{9}{10}\times\frac{2}{5}=\frac{21}{50}=0.42$ P(X=2)= P(1 from 1st die)P(1 from 2nd die)=$\frac{1}{10}\times\frac{2}{5}=\frac{2}{50}=0.04$ The table for probability distribution is shown as below:

X	0	1	2
P(X)	0.54	0.42	0.04

32



Let x kg of food 1 be mixed with y kg of food 2

To minimize Z = ₹ (50x + 70y)

Subject to the constraints :

$2x + y \geq 8,\ x + 2y \geq 10, x \geq 0, y \geq 0$

Corner Points	Value of Z (in ₹)
A(0, 8)	560
B(2, 4)	380 ← Min. value
C(10, 0)	500

Since feasible region is unbounded so, 380 may or may not be minimum value of z.

To check, draw $50x + 70y < 380$ i.e., $5x + 7y < 38$.

As in the half plane $5x + 7y < 38$, there is no point common with the feasible region.

Hence minimum value of Z is ₹ 380.

33

$$\text{Area of ellipse} = 4\left(\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx\right)$$

$$= 4\left|\left(\frac{b}{a}\left(\frac{x}{2}\sqrt{(a^2 - x^2)} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)\right)\right|_0^a$$

$$= 4\frac{b}{a}\left(\frac{\pi a^2}{4}\right)$$

$$= \pi ab$$

OR

$a = 1,\ b = 3,\ nh = 2$

$$\int_3^3 (x^2 + x + e^x)\,dx = \lim_{h\to 0} h(f(1) + f(1+h) + \ldots + f(1 + (n-1)h)$$

$$= \lim_{h\to 0} = h(2 + e + (1+h)^2 + (1+h) + e^{1+h} + \ldots + (1 + (n-1)h)^2 + (1 + (n-1)h) + e^{1+(n-1)h})$$

$$= \lim_{h\to 0} = h(2 + e + 2 + 3h + h^2 + e^{1+h} + \ldots + (1)^2 + (n-1)^2 h^2 + 2(n-1)h + 1 + (n-1)h + e^{1+(n-1)h})$$

34

$A^3 - 6A^2 + 5A + 11I = O$, Pre-multiplying by A^{-1}

$$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = O \Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$$

$$\therefore\ A^{-1} = \begin{bmatrix} -3/11 & 4/11 & 5/11 \\ 9/11 & -1/11 & -4/11 \\ 5/11 & -3/11 & -1/11 \end{bmatrix}$$

35

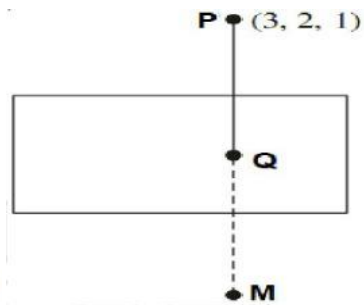
As the d.r.'s of parallel lines are proportional so, the equation of line passing through (2, 3, 2) and parallel to $\vec{r} = (-2\hat{i}+3\hat{j}) + \lambda(2\hat{i}-3\hat{j}+6\hat{k})$ is : $\vec{r} = (2\hat{i}+3\hat{j}+2\hat{k}) + \lambda(2\hat{i}-3\hat{j}+6\hat{k})$.

Now $\vec{a}_1 = -2\hat{i}+3\hat{j}$, $\vec{a}_2 = 2\hat{i}+3\hat{j}+2\hat{k}$, $\vec{b} = 2\hat{i}-3\hat{j}+6\hat{k}$

$$\Rightarrow \vec{a}_2 - \vec{a}_1 = (2\hat{i}+3\hat{j}+2\hat{k}) - (-2\hat{i}+3\hat{j}) = 4\hat{i}+2\hat{k} \text{ and } (\vec{a}_2-\vec{a}_1)\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix} = 6\hat{i}-20\hat{j}-12\hat{k}$$

$$\therefore \text{S.D.} = \frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|}$$

$$\Rightarrow \quad = \frac{|6\hat{i}-20\hat{j}-12\hat{k}|}{|2\hat{i}-3\hat{j}+6\hat{k}|} = \frac{\sqrt{36+400+144}}{\sqrt{4+9+36}} = \frac{\sqrt{580}}{7} \text{ Units.}$$



OR

The d.r.'s of normal to the plane are 2, –1, 1.

Since PQ is perpendicular to the plane so, its equation is

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda.$$

The coordinates of any random point on the line PQ :

$Q(2\lambda+3, -\lambda+2, \lambda+1)$.

$\because$ Q lies on the plane so, $2(2\lambda+3) - (-\lambda+2) + (\lambda+1) + 1 = 0$

$\Rightarrow 6\lambda + 6 = 0 \quad \therefore \lambda = -1$

$\therefore$ Foot of perpendicular : $Q(1,3,0)$.

Distance PQ $\sqrt{(3-1)^2+(2-3)^2+(1-0)^2} = \sqrt{6}$ Units.

Let $M(\alpha, \beta, \gamma)$ be the image of P in the plane.

So, Q will be mid-point of PM.

That is, $Q(1,3,0) = Q\left(\frac{\alpha+3}{2}, \frac{\beta+2}{2}, \frac{\gamma+1}{2}\right)$

On comparing the coordinates, we get: α =-1, β=4, γ=-1.

Therefore, the Image is M (–1, 4, –1).

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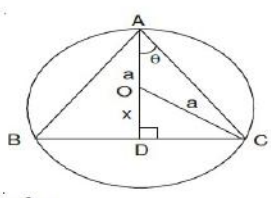


$$CD = \sqrt{a^2 - x^2}$$

$$\text{Area, } A = \frac{1}{2} \times 2\sqrt{a^2-x^2}\,(a+x)$$

$$Z = A^2 = (a-x)(a+x)^3$$

$$\frac{dZ}{dx} = 2(a+x)^2(a-2x)$$

$$\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{2}$$

$$\frac{d^2Z}{dx^2} = -12(a+x)x$$

$$\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{2}} = -9a^2 < 0$$

	$\therefore$ Z is maximum when $x = \frac{a}{2}$ i.e., Area is maximum when $x = \frac{a}{2}$ For maximum area $\tan\theta = \frac{CD}{AD} = \frac{\sqrt{a^2 - \frac{a^2}{4}}}{a + \frac{a}{2}} = \frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{6}$

CLASS XII
MATHEMATICS -041
PRACTICE PAPER II (2019-20)

Time: 3 Hrs. Maximum Marks: 80
निर्धारितसमय : 3 घंटे अधिकतमअंक : 80

General Instructions:
(1) All the questions are compulsory.
(2) The question paper consists of 36 questions divided into 4 sections A, B, C and D.
(3) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(4) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(5) Use of calculators is not permitted
समान्य निर्देश:

1.सभी प्रश्न अनिवार्य हैं

2. प्रश्न पत्र में 36 प्रश्न हैं जो चार खंडों में विभक्त है अ,ब,स,द

3. खंड अ में 20 प्रश्न हैप्रत्येक प्रश्न 1 अंक का है।

खंड बमैं 6 प्रश्न हैं प्रत्येक प्रश्न 2 अंक का है।

खंड स में 6 प्रश्न है प्रत्येक प्रश्न 4 अंक का है।

खंडद मैं 4 प्रश्न है प्रत्येक प्रश्न 6 अंक का है।

4. प्रश्न पत्र में समग्र पर कोई विकल्प नहीं है तथापि 1 अंक के 3 प्रश्न में , 2 अंक के 2 प्रश्नों में , 4 अंक के 2 प्रश्नों में और 6 अंक के 2 प्रश्नों में विकल्प दिया गया है। ऐसे सभी प्रश्नों में एक ही विकल्प हल करना है।

5.केलकुलेटर के प्रयोग की अनुमति नहीं है।

Question Number 1-10 Are Of Multiple Choice Type Questions .
Select the Correct Option

(1) If A = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$ then

(A) Only AB is defined (B) Only BA is defined (C) AB and BA both are defined
(D) AB and BA both are not defined

यदि A =$\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ तथा B= $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$तो ,

(A) केवल AB परिभाषित है (B) केवल BA परिभाषित है (C)AB तथा BA दोनों परिभाषित हैं
(D) AB तथा BA दोनों परिभाषित नहीं हैं |

(2) If A =$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$(C) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (D $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

यदि A =$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, तो A^2 बराबर है

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (D $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(3) The value of λ for which the vectors $\vec{a} = 2\hat{\imath}+\lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath}+3\hat{k}$ are orthogonal is
(A) 0 (B) 0 (C) 3/2 (D)-5/2

यदि सदिश $\vec{a} = 2\hat{\imath}+\lambda\hat{\jmath} + \hat{k}$ और सदिश $\vec{b} = \hat{\imath} + 2\hat{\jmath}+3\hat{k}$लाम्बिक (orthogonal) हों तो λ का मान है

(A) 0 (B) 0 (C) 3/2 (D)-5/2

(4) Let A and B be two events such that P(A)=0.6, P(B)=0.2 and P(A/B)=0.5 then P(A'/B') equals

(A) 1/10 (B)3/10 (C) 3/8 (D) 6/7

मान लीजिए कि A तथा B दो घटनाएंऐसी हैंकि P(A)=0.6,P(B)=0.2 तथा P(A/B)=0.5 P(A'/B') बराबर होगा

(A)1/10 (B)3/10 (C) 3/8 (D) 6/7

(5) Distance of the point (α,β,γ) from y axis is

(A)β (B)$|\beta|$ (C) $|\beta + |\gamma||$ (D) $\sqrt{\alpha^2 + \gamma^2}$

बिंदु (α,β,γ) की y अक्ष से दूरी है

(A) β (B)$|\beta|$ (C) $|\beta + |\gamma||$ (D) $\sqrt{\alpha^2 + \gamma^2}$

(6) If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then $\cot^{-1} x + \cot^{-1} y$ *equals*

(A) $\frac{\pi}{5}$ (B) $\frac{2\pi}{5}$ (C) $\frac{3\pi}{5}$ (D)π

यदि $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, तो $\cot^{-1} x + \cot^{-1} y$ बराबर हैं

$(A)\frac{\pi}{5}$ (B) $\frac{2\pi}{5}$ (C) $\frac{3\pi}{5}$ (D)π

(7) A bag contains 5 red and 3 blue balls three balls drawn at random without replacement the probability of getting exactly one ball is

(A) 45 /196 (B)135/392 (C)5/56 (D) 15/29

एक थैले में 5 लालतथा 3 नीली गेंद है । यदि 3 गेंदबिना प्रतिस्थापन के निकाली जाती है।एक लाल रंग की गेंद निकालने कीप्रायिकता--

(A) 45 /196 (B)135/392 (C) 15/56 (D) 15/29

(8) $\int e^x(\cos x\text{-}\sin x)$ dx equals to

(A)$e^x \cos x + c$ (B)$e^x \sin x + c$

(C)$-e^x \cos x + c$ (D) $- e^x \sin x + c$

$\int e^x(\cos x\text{-}\sin x)$ dx का मान है

(A)$e^x \cos x + c$ (B)$e^x \sin x + c$

(C)$-e^x \cos x + c$ (D) $- e^x \sin x + c$

(9) Reflection of the point (α,β,γ) in the xy plane is

(A)(α,β,0) (B)(0,0,γ) (C) $|(-\alpha, -\beta, \gamma)|$ (D) (α,β,-γ)

xy सम्तल में बिंदू (α,β,γ) का परावर्तन है

(A)(α,β,0) (B)(0,0,γ) (C) $|(-\alpha, -\beta, \gamma)|$ (D) (α,β,-γ)

(10)Planes 2x-y+4z=5 and 5x-2.5y+10z =6 are:

(A)Perpendicular to each other (B) Parallel

(C)Intersect at y axis (D) passes through point (0.0.5/4)

समतल 2x-y+4z=5 तथा 5x-2.5y+10z =6 है :

(A) परस्पर लम्ब (B) समांतर

(C) y अक्ष पर प्रतिवेदन करते हैं (D)बिंदु (0.0.5/4)से गुजरते हैं

(Q 11-Q15) Fill in the blanks

(Q 11-Q15) रिक्त स्थान भरिये

(11)The domain of the function f:R→ R defined by f(x)=$\sqrt{x^2+3x+2}$ is ----------

f(x)=$\sqrt{x^2+3x+2}$द्वारा परिभाषित फलनf:R→ Rका प्रांत------- है

(12)The function f(x)=$\begin{cases} \frac{\sin x}{x}+\cos x, & If\, x \neq 0 \\ k, & \text{If } x=0 \end{cases}$ is continuous at x=0, then the value of k is --------

यदिफलनf(x)=$\begin{cases} \frac{\sin x}{x}+\cos x, & \text{यदि } x \neq 0 \\ k, & \text{यदि } x=0 \end{cases}$ बिंदु x=0 पर संतत है

तो k का मान है----------

(13)If f(x) =, $\frac{1}{4x^2+2x+1}$then its maximum value is----------

यदिf(x) =, $\frac{1}{4x^2+2x+1}$, तो इसका उच्तम मान – – – – – – – – – – –है

OR

The equation of normal to the curve y=sinx at (0,0) is_____

वक्र y=sinxके (0,0) पर अभिलम्ब का समीकरन _____ है

(14)If A is a matrix of order 3x3, then $|3A| = ---------$

यदि A एक 3X3 कोटि का आव्यूह है तो $|3A|$= ----------

(15)Cartesian equation of the plane $\vec{r}.(\hat{\imath}+\hat{\jmath}-\hat{k})$=2 is----------

सम्तल $\vec{r}.(\hat{\imath}+\hat{\jmath}-\hat{k})$=2 का कार्तीये समीकरन ---------- है.

OR

The value of the expression $|\vec{a}\times\vec{b}|^2(\vec{a}.\vec{b})^2$is_____

व्यंजक$|\vec{\boldsymbol{a}}\times\vec{\boldsymbol{b}}|^2(\vec{\boldsymbol{a}}.\vec{\boldsymbol{b}})^2$ मान है ----------

(Q16-Q20) Answer the following questions

(Q16-Q20) निम्न प्रश्नो के उत्तर लिखिये

(16) If cos2θ=0 then find the value of $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$

यदिcos2θ=0 तब$\begin{vmatrix} 0 & cos\theta & sin\theta \\ cos\theta & sin\theta & 0 \\ sin\theta & 0 & cos\theta \end{vmatrix}^2$ का मान ज्ञात कीजिए

(17)Evaluate $\int \frac{1}{(2x+xlogx)}$dx

$\int \frac{1}{(2x+xlogx)}$ dx का मान ज्ञात कीजिए।

(18)Find $\int \frac{3+3\,\mathrm{co}}{x+\mathrm{si}}$dx

$\int \frac{3+3\,\mathrm{co}}{x+\sin x}$ ज्ञात कीजिए।

OR

Evaluate$\int \frac{(1+\cos x)}{x+\sin}$dx

$\int \frac{(1+\mathrm{co}\quad)}{x+\sin x}$ ज्ञात कीजिए।

(19) Evaluate

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$\int \frac{dx}{\sin^2 x \cos^2 x}$ ज्ञात कीजिए।

(20)Find the general solution of differential equation

x$\frac{dy}{dx}$+2y=x^2

अवकल समीकरण

x$\frac{dy}{dx}$+2y=x^2 का व्यापक हल ज्ञात कीजिये ।

SECTION B

Question 21-26 each question carries 2 marks

प्रश्न संख्या 21-26 प्रत्येक प्रश्न दो अंक का है

(21) If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ then find the value of $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a}$
यदि $\vec{a}+\vec{b}+\vec{c}=0$ तथा $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ तो $\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a}$ का मान ज्ञात कीजिये ।

OR

For the given vectors, $\vec{a}=3\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $\vec{b}=-2\hat{\imath}+\hat{\jmath}-\hat{k}$ find the unit vector in the direction of vector $\vec{a}+\vec{b}$.
सदिशों$\vec{a}=3\hat{\imath}-\hat{\jmath}+2\hat{k}$तथा$\vec{b}=-2\hat{\imath}+\hat{\jmath}-\hat{k}$, $(\vec{a}+\vec{b})$के अनुदिश मात्रक सदिश ज्ञात कीजिये ।

(22) Write $\tan^{-1}(\frac{\cos x}{1-\sin x})$, $\frac{-\pi}{2}<x<\frac{3\pi}{2}$ in the simplest form
$\tan^{-1}(\frac{\cos x}{1-\sin x})$, $\frac{-\pi}{2}<x<\frac{3\pi}{2}$को सलत्तम रूप में लिखिए।

OR

Find gof and fog ,if f:R→R and g:R→R are given by f(x)=cosx and g(x)=$3x^2$ then show that gof≠fog

यदि f:R→R तथा g:R→R फलन क्रमश: f(x)=cosxतथा g(x)=$3x^2$द्वारा परिभाषित है तो gof और fog ज्ञात किजिये । सिद्ध कीजिए कि gof≠fog

(23) If y=A sin x.+ B cos x , then prove that $\frac{d2y}{dx2}+y=0$.

यदि y=A sin x.+ B cos x तो सिद्ध कीजिये कि $\frac{d2y}{dx2}+y=0$.

.(24) A particle moves along the curve $6y=x^3+2$. Find the points on the curve at which the y ordinate is changing 2 times as fast as the x coordinate.

एक कण वक्र $6y=x^3+2$ के अनुगत गति कर रहा हैं | वक्र पर उन बिन्दुओं को ज्ञात कीजिये जबकि x निर्देशांक कि तुलना में y निर्देशांक दुगुनी तीव्रता से बदल रहा है |

(25) Find the angle between the lines

$\vec{r}=(2\hat{\imath}-5\hat{\jmath}+\hat{k})+\lambda(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$ and $\vec{r}=(7\hat{\imath}-6\hat{k})+\mu(\hat{\imath}+2\hat{\jmath}+2\hat{k})$

रेखाओं $\vec{r}=(2\hat{\imath}-5\hat{\jmath}+\hat{k})+\lambda(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$ तथा $\vec{r}=(7\hat{\imath}-6\hat{k})+\mu(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ के बीच का कोण ज्ञात कीजिये |

(26) Four cards are drawn successively without replacement from a well shuffled deck of 52 playing cards. What is the probability that 'only 2 cards are spades'?

52 पत्तों कि एक गड्डी में से यद्रिचाच्या बिना प्रतिस्तापित किये गए 4 पत्ते निकाले गए | दोनों पत्तों के कुदाल कार्ड होने की प्रायिकता ज्ञात कीजिये |

SECTION- C

(27) Show that the function f in A= R-$\left\{\frac{2}{3}\right\}$defined as f(x) =$\frac{4x+3}{6x-4}$ is one-one and onto.

Hence, find f^{-1}.

दर्शाइए कि A= R-$\left\{\frac{2}{3}\right\}$ में , f(x)=$\frac{4x+3}{6x-4}$ द्वारा परिभाषित फलन एकैकी और आचादक है | अतः f^{-1} ज्ञात कीजिये |

(28)If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)$=$\cot^{-1} a$ find $\frac{d^2y}{dx^2}$

यदि$\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)$=$\cot^{-1} a$ तो $\frac{d^2y}{dx^2}$ज्ञातकीजिये|

OR

If $\sin y$=x $\sin(a+y)$, prove that $\frac{dy}{dx}$=$\frac{sin^2(a+y)}{\sin a}$

यदि $\sin y$=x$\sin(a+y)$ है, तो सिद्ध कीजिये कि $\frac{dy}{dx}$=$\frac{sin^2(a+y)}{\sin a}$

(29) Solve the differential equation:-

$x\frac{dy}{dx}$= $y - xtan\left(\frac{y}{x}\right)$

अवकल समीकरण

$x\frac{dy}{dx}$ = $y - xtan\left(\frac{y}{x}\right)$को हल कीजिये :

(30) Find: $\int \frac{3x+5}{x^2+3x-1}dx$

ज्ञात कीजिये $\int \frac{3x+5}{x^2+3x-18}dx$

(31) Four cards are drawn one by one with replacement from a well shuffled deck of playing cards. Find the probability that at least three cards are of diamonds.

अच्छी प्रकार से फैंटी गयी ताश की गड्डी में से एक के बाद एक चार पत्ते प्रतिस्थापना सहित निकले गए | प्रायिकता ज्ञात कीजिये कि कम से कम तीन पत्ते ईंट के आये |

OR

The probability of two students A and B coming to school on time are $\frac{2}{7}$and $\frac{4}{7}$ respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent. Find the probability of only one of them coming to school on time.

दो विद्यार्थियों A और B के समय पर आने कि प्रयिक्तायें क्रम्शः $\frac{2}{7}$ और $\frac{4}{7}$ हैं | मानिए कि, $'A$ समय पर आताहै $'$ और $'B$समय पर आताहै $'$ स्वतन्त्र घटनाएं हैं , तो प्रायिकता ज्ञात कीजिये कि उनमे से एक ही विद्यालय में समय पर आता है |

(32) A manufacturer produces nuts and bolts. It takes one hour of work on machine A and three hours on machine B to produce a package of nuts. It takes three hours on machine A and one hour on machine B to produce package of bolts. He earns a profit of Rs. 35 per package of nuts and Rs. 14 per package of bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates each machine for almost 12 hour a day? Convert it into an LPP and solve graphically.

एक निर्माणकर्ता नट और बोल्ट का निर्माण करता है| एक पाकेट नटों के निर्माण में मशीन A पर एक घंटा और मशीन B पर तीन घंटे काम करना पड़ता है |जबकि एक पैकेट बोल्ट के निर्माण में तीन घंटे मशीन A पर और एक घंटा मशीन B पर काम करना पड़ता है | वह नटों से Rs. 35 प्रति पैकेट और बोल्टों पर Rs. 14 प्रति पैकेट लाभ कमाता है | यदि प्रतिदिन मचिनों का अधिकतम उपयोग 12 घंटे किया जाए तोह प्रत्येक (नट और

बोल्ट) के कितने पैकेट उत्पादित किये जाएँ ताकि अधिकतम लाभ कमाया जा सके |LPP में बदलकर ग्राफ द्वारा हल कीजिये |

SECTION-D

(33) Using elementary row transformation find the inverse of the matrix

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

प्रारंभिक पंक्ति रूपान्तरणों द्वारा आव्यूह

$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ का व्युत्क्रम ज्ञात कीजिये |

OR/ अथवा

Using matrices solve the following system of linear equations:-

2x+3y+10z=4

4x-6y+5z=1

6x+9y-20z=2

आव्यूहों का प्रयोग कर निम्नलिखित रैखिक समीकरण निकाय को हल कीजिये :

2x+3y+10z=4

4x-6y+5z=1

6x+9y-20z=2

(34) Using the method of integration find the area of the region bounded by the lines

$3x-2y+1=0$, $2x+3y-21=0$ and $x-5y+9=0$.

समाकलन विधि का उपयोग करते हुए ऐसे क्षेत्र का क्षेत्रफल ज्ञात कीजिये जो कि रेखाओं

$3x-2y+1=0$, $2x+3y-21=0$ तथा $x-5y+9=0$ से घिरा हुआ है |

(35)Show that the height of the cylinder, which is open at the top, having a given surface

area and greatest volume is equal to the radius of its base.

दिखाइए की अधिकतम आयतन के और दिए गए पृष्ठीय क्षेत्रफल के बेलन (जिसका उपरी भाग खुला हो) कि उचाई बेलन के आधार की त्रिज्या के बराबर होगी |

OR / अथवा

The sum of the perimeters of circle and a square is K, where K is some constant. Prove that the sum of their areas is least when the side of the square is twice the radius of the circle.

एक वृत्त और एक वर्ग के परिमापों का योगफल K है, जहाँ K एक अचर है |सिद्ध कीजिये कि उनके क्षेत्रफलों का योगफल निम्रतम है , जब वर्ग कि भुजा वृत्त कि त्रिज्या कि दुगुनी है |

(36) Find the coordinates of the foot Q of the perpendicular drawn from the point P (1,3,4) to the plane $2x-y+z+3=0$. Find the distance PQ and the image of P treating the plane as a mirror.

बिंदु P (1,3,4) से समतल $2x-y+z+3=0$ पर खींचे गए लम्ब के पाद Q के निर्देशांक ज्ञात कीजिये | लम्बवत दूरी PQ तथा समतल को दर्पण लेते हुए इस बिंदु P का प्रतिभिम्ब भी ज्ञात कीजिये|

SOLUTION/ ANSWER KEY OF PRACTICE PAPER-2

CLASS XII MATHEMATICS

2019-20

Q NO	VALUE POINTS
1	(C) AB and BA both are defined
2	(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3	(D)-5/2
4	(A)1/10
5	(D) $\sqrt{\alpha^2+\gamma^2}$
6	(A) $\frac{\pi}{5}$
7	(C) 15/56
8	(A) $e^x \cos x + c$
9	(D) (α,β,-γ)
10	(B)Parallel
11	Domain = $(-\infty, 1] \cup [2, \infty)$
12	2
13	Maximum value is $\frac{4}{3}$ ORx+y=0
14	$27\|A\|$
15	x+y-z=2 OR $\|\vec{a}\|^2\|\vec{b}\|^2$
16	$\frac{-3}{\sqrt{1-x^2}}$
17	$\log\|2+\log x\|$+c
18	$3\log\|(x+\sin x)\|$+c OR $\log\|x+\sin x\|$+c
19	$\tan x + \cot x + c$
20	y=$\frac{x^4+c}{4x^2}$
21	$(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$ $\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=0$ $\Rightarrow \|\vec{a}\|^2+\|\vec{b}\|^2+\|\vec{c}\|^2+2(\vec{a}\vec{b}+\vec{b}\vec{c}+\vec{c}\vec{a})=0$

	$\Rightarrow \overrightarrow{(a\vec{b}}+\vec{b}\vec{c}+\vec{c}\vec{a})=\frac{-83}{2}$ OR Given vectors are $\vec{a}$=3$\hat{\imath}$ -$\hat{\jmath}$ +2$\hat{k}$ and $\vec{b}$= -2$\hat{\imath}$ +$\hat{\jmath}$ -$\hat{k}$, therefore , $\vec{a}+\vec{b}$.=(3-2)$\hat{\imath}$+(-1+1)$\hat{\jmath}$+(2-1)$\hat{k}$ $\Rightarrow \vec{a}+\vec{b}$=.=1. $i+\ 0.\hat{\jmath}$+1.$\hat{k}$=$\hat{\imath}$+$\hat{k}$ Hence unit vector in the direction of ($\vec{a}+\vec{b}$) is , $\frac{(\vec{a}+\vec{b})}{\lvert(\vec{a}+\vec{b})\rvert}$ =$\frac{\hat{\imath}+\hat{k}}{\sqrt{2}}$ =$\frac{1}{\sqrt{2}}\hat{\imath}+\frac{1}{\sqrt{2}}\hat{k}$
22	$\tan^{-1}(\frac{\cos x}{1-\sin x})$ =$\tan^{-1}\left[\frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{cos^2\frac{x}{2}+sin^2\frac{x}{2}-2si\ \frac{x}{2}cos\frac{x}{2}}\right]$ =$\tan^{-1}(\frac{cos\frac{x}{2}+sin\frac{x}{2}}{cos\frac{x}{2}-sin\frac{x}{2}})$ =$\tan^{-1}[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}]$ =$\tan^{-1}[\tan(\frac{\pi}{4}+\frac{x}{2})]$ =$\frac{\pi}{4}+\frac{x}{2}$ OR f(x)=cosx , g(x)=3x^2 f(x)=cosx f(x)=cosx⇒f(g(x))=cosg(x) ⇒fog(x)=cos 3x^2 g(x)=3x^2⇒g(f(x))=3f(x)^2 gof(x)=3cos^{2}x Hence gof≠fog
23	$\frac{dy}{dx}$=Acos x+ B(-sinx)=Acos x _Bsin x $\frac{d2y}{dx2}$= A(-sin x) –B cos x=-A sin x –B cos x
24	According to to question, we have to find out the point on the curve at which the y coordinate is changing 2 times as fast as the x - coordinate.ie ; $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$=2=>$\frac{dy}{dx}$=2 , now equation of the curve 6y = x^3 + 2 =>6$\frac{dy}{dx}$=3x^2=>2$\frac{dy}{dx}$=x^2

	now, put x = 4 in $6y = x^3 + 2 \Rightarrow 2x2=x^2$ $\Rightarrow x=\pm 2$ When x=2 $6y = 2^3 + 2 = 10$ $\Rightarrow y = 10/6$ $\Rightarrow y=5/3$ hence, a point on the curve is (2,5/3) When x=-2 $6y =(- 2)^3 + 2 = -8+2=-6$ $\Rightarrow y = -1$ $\Rightarrow$points on the curve will be (-2,-1), (2,5/3)
25	Given lines are $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\vec{r}= (7\hat{i}-6\hat{k}) + \mu(\hat{i} + 2\hat{j} +2\hat{k})$ on compairing with $\vec{r}=\vec{a}_1+ \lambda\vec{b}_1$ and $\vec{a}_2 +\mu\vec{b}_2$ we get $\vec{b}_1=3\hat{i}+2\hat{j}+6\hat{k}$ and $\vec{b}_2=\hat{i} + 2\hat{j} +2\hat{k}$ angle between the lines is given by $\cos\theta = \left\lvert\frac{\vec{b1}\vec{b2}}{\lvert\vec{b1}\rvert\lvert\vec{b2}\rvert}\right\rvert = \left\lvert\frac{(3\hat{i}+2\hat{j}+6\hat{k})(\hat{i} + 2\hat{j} +2\hat{k}}{\sqrt{49}\sqrt{9}}\right\rvert = \frac{19}{\sqrt{49}\sqrt{9}} = \frac{19}{7x3} = \cos^{-1}\frac{19}{21}$
26	*No. of spade cards in a pack of 52 playing cards =13* *Let E: getting a spade* *$\therefore$ P (E) =$\frac{13}{52}$, P (E) =$\frac{39}{52}$* *Therefore, P (only 2 cards are spades) = $^4C_2\frac{13}{52}.\frac{13}{52}.\frac{39}{52}.\frac{39}{52} = \frac{54}{256}$*

27	For x_1, x_2 ϵA, let $f(x_1)=f(x_2)$ $\frac{4x_1+3}{6x_1-4}=\frac{4x_2+3}{6x_2-4}$ $(4x_1+3)(6x_2-4)=(6x_1-4)(4x_2+3)$ $\Rightarrow x_1=x_2$ Hence, f is one-one. For any y ϵA s.t $y=\frac{4x+3}{6x-4}$ $\exists$ x such that 6xy-4y=4x+3 $\Rightarrow x=\frac{4y+3}{6y-4} \in A$ Also, $f(x) = f\left(\frac{4y+3}{6y-4}\right) = \frac{4\left(\frac{4y+3}{6y-4}\right)+3}{6\left(\frac{4y+3}{6y-4}\right)-4} = y$ $\Rightarrow$f(x) is onto. Since, f(x) is one-one and onto, therefore f^{-1} exists and $f^{-1}(y) = \frac{4y+3}{6y-4}$.
28	Here $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\cot^{-1} a$ $\Rightarrow\frac{x^2-y^2}{x^2+y^2} = \cos(\cot^{-1} a)$ Applying componendo and dividendo, we get :$\frac{x^2-y^2+x^2+y^2}{x^2-y^2+x^2-y^2}=\frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}$ $\Rightarrow\frac{2x^2}{-2y^2}=\frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}$ $\Rightarrow \frac{d}{dx}\left(\frac{x^2}{y^2}\right)=\frac{d}{dx}\left(-\frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}\right)$ $\Rightarrow \frac{y2x\ 2x-x2x2y\frac{dy}{dx}}{(y^2)^2}=0$

	$\Rightarrow$y-x$\frac{dy}{dx} = 0 \therefore \frac{dy}{dx}=\frac{y}{x}$---------(i) Now $\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{y}{x}\right)\Rightarrow\frac{d^2y}{dx^2}=\frac{x\frac{dy}{dx}-yX1}{x2}$ $\Rightarrow \frac{d^2y}{dx^2}=\frac{x\frac{y}{x}-y}{x2} \quad =\frac{0}{x2}$ by (i) hence $\frac{d^2y}{dx^2}$=0 OR sin y = x sin(a+y) $\Rightarrow$ x = $\frac{\sin y}{\sin(a+y)}$ Differentiating w.r.t y, we get, $\frac{dx}{dy}=\frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{sin^2(a+y)}$ $\frac{dx}{dy}=\frac{\sin(a+y-y)}{sin^2(a+y)}=\frac{\sin a}{sin^2(a+y)}$ $\therefore \frac{dy}{dx} = \frac{sin^2(a+y}{\sin a}$
29	The given differential equation can be written as $\frac{dy}{dx}=\frac{y}{x}-tan\left(\frac{y}{x}\right)$ Put $\frac{y}{x} = v$ and$\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get $v + x\frac{dv}{dx} = v - tanv \Rightarrow x\frac{dv}{dx} = -tanv$ $\Rightarrow \cot v\, dv = \frac{-1}{x}dx, integrating\ both\ sides\ we\ get$ $\log\lvert sinv\rvert = -\log\lvert x\rvert + logc$

	$\Rightarrow \log\lvert sinv\rvert = log\left\lvert\frac{c}{x}\right\rvert$ Solution of differential equation is: $sin\left(\frac{y}{x}\right)=\frac{c}{x}$ or $xsin\left(\frac{y}{x}\right) = c$
30	$= \int \frac{3x+5}{x^2+3x-18}dx = \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{x^2+3x-18}dx$ $$= \frac{3}{2}\int \frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}dx$$ $$= \frac{3}{2}\log\lvert x^2+3x-18\rvert + \frac{1}{18}log\left\lvert\frac{x-3}{x+6}\right\rvert + c$$
31	P (at least 3 are diamonds) P(3)+ P(4)= ${}^4C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right) + {}^4C_4\left(\frac{1}{4}\right)^4$ $= \left(\frac{1}{4}\right)^4(12+1) = \frac{13}{256}$ OR P (only one on time) =P (A) P ($\bar{B}$)+P ($\bar{A}$) P (B) $$= \frac{2}{7}\times\frac{3}{7} + \frac{5}{7}\times\frac{4}{7}$$ $$= \frac{26}{49}$$

32

Let no. of packages of nuts be x units and no. of packages of bolts be y units.

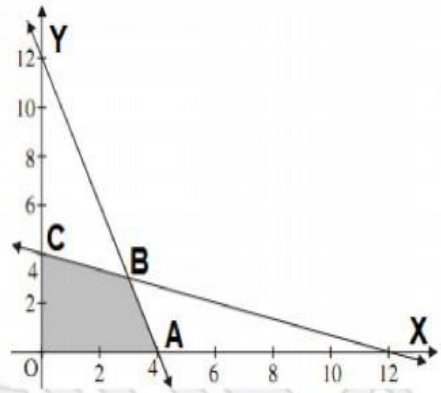
To maximize : $Z = ₹(35x + 14y)$

Subject to constraints :

$x \geq 0,\ y \geq 0,$

$x + 3y \leq 12,$

$3x + y \leq 12$



Corner Points	**Value of Z (in ₹)**
A(4, 0)	140
B(3, 3)	147 ← Maximum
C(0, 4)	56

Hence, maximum profit of ₹147 is obtained when no. of packages of nuts = x = 3 units and, no. of packages of bolts = y = 3 units are produced.

33

Let $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Then A = IA

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - R_2)$$

where $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$|A| = 1200 \neq 0$

$\Rightarrow A^{-1}$ exists.

$X = A^{-1}B$

$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 9 & -12 & 9 \end{bmatrix}$$

OR

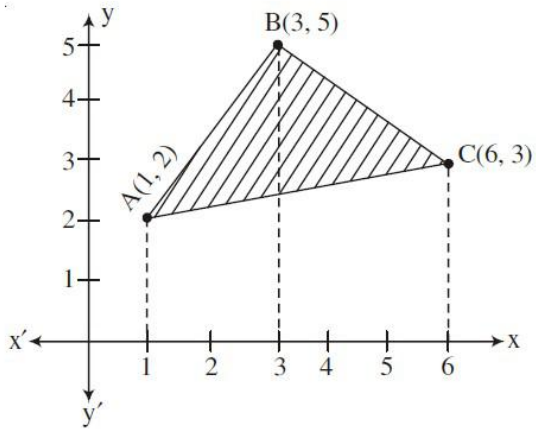
The given system of equations is

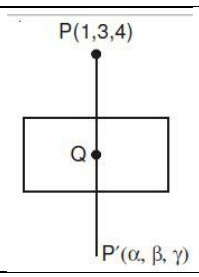
AX = B,

$$X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$$

34	 Required Area $= \int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{21-2x}{3}dx - \int_1^6 \frac{x+9}{5}dx$ $= \left(\frac{3x^2}{4}+\frac{x}{2}\right)\Big\|_1^3 + \left(7x-\frac{x^2}{3}\right)\Big\|_3^6 - \left(\frac{x^2}{10}+\frac{9x}{5}\right)\Big\|_1^6$ $= 7+12-\frac{25}{2}$ $= \frac{13}{2}$
35	Let Given surface area of open cylinder be S. Then $S = 2\pi rh + \pi r^2$ $\Rightarrow h = \frac{S-\pi r^2}{2\pi r}$ Volume $V = \pi r^2 h$ $V = \pi r^2\left[\frac{S-\pi r^2}{2\pi r}\right] = \frac{1}{2}[Sr-\pi r^3]$ $\frac{dV}{dr} = \frac{1}{2}[S-3\pi r^2]$ $\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2$ or $2\pi rh + \pi r^2 = 3\pi r^2$

	$\Rightarrow\ 2\pi rh = 2\pi r^2 \qquad \Rightarrow h = r$ $\dfrac{d^2V}{dr^2} = -6\pi r < 0$ $\therefore$ For volume to be maximum, height = radius Let x be the radius of circle and y be the side of square $2\pi x + 4y = k$ $A = \pi x^2 + y^2$ $A = \pi x^2 + \left(\dfrac{k-2\pi x}{4}\right)^2 = \dfrac{16\pi x^2 + k^2 + 4\pi^2 x^2 - 4\pi kx}{16}$ $\dfrac{dA}{dx} = \dfrac{1}{16}(32\pi x + 8\pi^2 x - 4\pi k)$ $\dfrac{dA}{dx} = 0 \Rightarrow 32\pi x + 8\pi^2 x - 4\pi k = 0$ $\Rightarrow\ x = \dfrac{k}{8+2\pi}$ $\left.\dfrac{d^2A}{dx^2}\right]_{x=\frac{k}{8+2\pi}} = \dfrac{1}{16}[32\pi + 8\pi^2] > 0 \Rightarrow$ Sum of areas is minimum OR $2\pi\left(\dfrac{k}{8+2\pi}\right) + 4y = k \Rightarrow y = \dfrac{k}{4+\pi} \Rightarrow y = 2x$
36	 Equation of line PQ is $\dfrac{x-1}{2} = \dfrac{y-3}{-1} = \dfrac{z-4}{1} = \lambda$ The coordinates of Q are $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

	$\because$ Q lies on plane $2x - y + z + 3 = 0$ $\therefore\ 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$ $\Rightarrow 6\lambda + 6 = 0$ i.e., $\lambda = -1$ The coordinates of Q are $(-1, 4, 3)$ Let $P'(\alpha, \beta, \gamma)$ be the image of P. then $\frac{\alpha+1}{2} = -1,\ \frac{\beta+3}{2} = 4,\ \frac{\gamma+4}{2} = 3$ $\Rightarrow \alpha = -3,\ \beta = 5,\ \gamma = 2$ $\therefore$ the image P' is $(-3, 5, 2)$